

# **EFFECT OF MEAN FREE PATH ON NONLINEAR LOSSES OF TRAPPED VORTICES DRIVEN BY A RF FIELD\***



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# ABSTRACT

We report extensive numerical simulations on nonlinear dynamics of a trapped elastic vortex under rf field, and its dependence on electron mean free path l<sub>i</sub>. Our calculations of the field-dependent residual surface resistance R<sub>i</sub>(H) take into account the vortex line tension, the linear Bardeen-Stephen viscous drag and random distributions of pinning centers. We showed that R<sub>i</sub>(H) decreases significantly at small fields as the material gets dirtier while showing field independent behavior at higher fields for clean and dirty limit. At low frequencies  $R_i(H)$  increases smoothly with the field amplitude at small H and levels off at higher fields. The mean free path dependency of viscosity and pinning strength can result in a nonmonotonic mean free path dependence of  $R_i(H)$ , which decreases with  $l_i$  at higher fields and weak pinning strength.

# **GENERATION OF TRAPPED VORTICES** [1]

• Vortex state is favorable at  $B > B_{c1}(T)$  but because  $B_{c1}(T_c) \rightarrow 0$ , vortices can be generated in the cavity during the cool-down through T<sub>c</sub>

### **NUMERICAL RESULTS**

Taking  $\lambda_0/\xi_0=1$ ,  $\lambda=40$  nm,  $\rho_n=10^{-9} \Omega m$ ,  $U_n=1.4$  meV/nm [2,3] for a clean Nb at f=1

- Most of the vortices exit as T it reduced below T<sub>c</sub> but some vortices get pinned by materials defects.
- Trapped vortices contribute to the residual surface resistance
- How does the vortex residual resistance depend on B, mean free path, frequency (f), pinning distribution and their strength?

H(t)





R<sub>i</sub> (H) at different mean free paths

at f ~ 1 GHz,  $\kappa_0=2$ 



y'(l,t) = z'(l,t) = 0

Dynamic equation for the local velocity normal to a flexible vortex line:

 $M\frac{\partial^2 \mathbf{R}}{\partial t^2} + \eta \frac{\partial \mathbf{R}}{\partial t} = \epsilon \frac{\partial^2 \mathbf{R}}{\partial X^2} - \nabla U(X, \mathbf{R}) - \hat{y}Fe^{-X/\lambda}\sin\omega t$ 

M-vortex mass, η- viscosity, ε- line tension, R=[Y(X,t),Z(X,t)] is the ac displacement of the vortex with the  $\varepsilon$ , F- amplitude of the rf driving force

• The core pinning of the vortices is modeled by:

 $f_0 = \frac{H_{c10}\rho_{n0}}{H_{c20}\lambda_0^2\mu_0},$ 

$$U(X, \mathbf{R}) = -\sum_{n=1}^{N} \frac{U_n}{1 + [(X - X_n)^2 + |\mathbf{R} - \mathbf{R}_n|^2]/\xi^2}$$

• U<sub>n</sub> are determined by the gain in the condensation energy in the vortex core at the pin

Dynamic equations for the dimensionless vortex displacement  $y = Y(X,t)/\lambda_0$ ,  $z = Z(X,t)/\lambda_0$ 

$$\gamma \dot{y} = y'' - \sum_{n=1}^{N} A_n(x, \mathbf{r})(y - y_n) + \beta \sin(2\pi t) e^{-x/\Gamma},$$

$$N$$



### **SUMMARY**

Field dependent R<sub>i</sub>(H) strongly depends on the mean free path, pinning density, strength, and their statistical distributions at small fields.

 $\gamma \dot{z} = z'' - \sum A_n(x, \mathbf{r})(z - z_n),$ 

- $\lambda_0$  London penetration depth in clean limit time is in units of rf period *l* is in units of  $\lambda_0$
- $N_v$  number of vortices
- $\rho_0$  normal state resistivity in clean limit
- $H_{c10}$ ,  $H_{c20}$  Lower and upper critical fields in clean limit

**Dimensionless surface resistance:** 

 $\zeta_{n0} = 2\kappa_0^2 U_n / \epsilon_0,$ 

 $g_0 = \ln \frac{\lambda_0}{\xi_0} + \frac{1}{2}, \qquad g = \ln \frac{\lambda_0 \Gamma^2}{\xi_0} + \frac{1}{2}.$ 

 $\Gamma = (1 + \xi_0 / l_i)^{1/2}, \qquad \beta = \frac{g_0 \Gamma H}{g H_{c10}},$ 

 $= \frac{g_0 \Gamma^5 \zeta_{n0}}{g \left[1 + \Gamma^2 \kappa_0^2 (x - x_n)^2 + \Gamma^2 \kappa_0^2 |\mathbf{r} - \mathbf{r}_n|^2\right]^2},$ 

 $\gamma = \frac{g_0 \Gamma^4 l_i f}{g \xi_0 f_0},$ 

$$r_{i}(\beta) = \frac{\gamma_{0}}{g_{0}\Gamma N_{v}\beta^{2}} \sum_{k=1}^{N_{v}} \int_{0}^{1} dt \int_{0}^{l} \beta_{t} e^{-x} \dot{y}_{k}(x,t) dx$$



 $B_0$ -Field due to trapped flux

- Low-field global surface resistance averaged over statistical realizations of random pinning potential increases nearly linearly with H and levels off at higher fields.
- $R_i(H)$  at strong RF field becomes a nonmonotonic function of H which decreases with H at higher frequencies if we consider the Larkin-Ovchinnikov decrease of  $\eta(v)$ [1,2]. This can contribute to the negative Q(H) slope in SRF cavities.
- Overheating can mask the descending field dependence of  $R_i(H)$  as frequency increases.
- More details can be found in our previous work in [1] and [2].

[1] W.P.M.R. Pathirana and A. Gurevich, Nonlinear dynamics and dissipation of a curvilinear vortex driven by a strong time-dependent Meissner current. Phys. Rev. B, 101, 064504 (2020). [2] W.P.M.R. Pathirana and A. Gurevich, Effect of random pinning on nonlinear dynamics and dissipation of a vortex driven by a strong microwave current. Phys. Rev. B, 103, 184518 (2021). [3] Checchin, M, Martinello, M, Grassellino, A, Romanenko, A, Zasadzinski J F. Electron mean free path dependence of the vortex surface impedance, Supercond. Sci. Technol. 30, 034003 (2017).

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