BASIC PRINCIPLES OF RF SUPERCONDUCTIVITY

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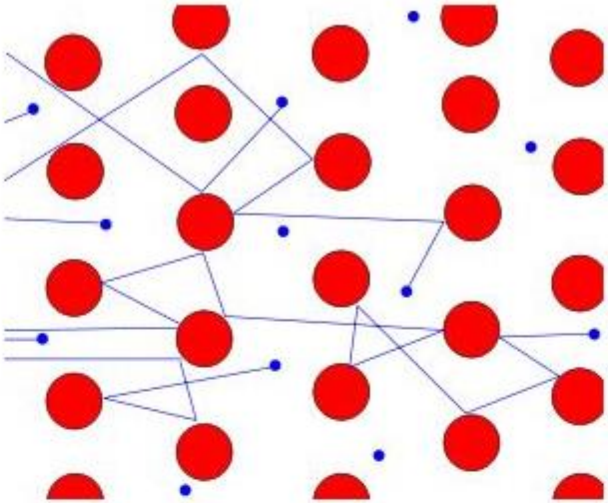
Outline

- Electrodynamics of normal conductors
- Superconductivity
  - Type-I and type-II superconductors
  - Intro to BCS and GL theories
- Surface resistance of superconductors
- Field dependence of surface resistance
- Intro to performance limitations

DC electrical conduction: resistance



Drude Model electrons (shown here in blue) constantly bounce between heavier, stationary crystal ions (shown in red).

Average momentum of an electron in an electric field within the time between collision, τ

$$\langle p \rangle = -eE\tau$$

$\tau = l/v_F \approx 10^{-14}$ s is the electrons' scattering time

$$\mathbf{j} = -en\mathbf{v} = \frac{e^2 \tau n}{m}\mathbf{E} = \sigma \mathbf{E}$$

Ohm's law, <u>local</u> relation between J and E

Electrodynamics of normal conductors

$E = E_0 e^{i\omega t}$ For accelerator applications, the rate of oscillation of the e.m. field is in the **radio-frequency (RF)** range (3 kHz – 300 GHz)

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \qquad \nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \cdot B = 0 \qquad \nabla \times H = J + \frac{\partial D}{\partial t}$$

Maxwell's equations

+

$$D = \varepsilon_0 \varepsilon \; E$$

$$B = \mu_0 \mu \; H$$

$$J = f\,(E)$$

(linear and isotropic) material's equations

- From Drude's model:

$$\frac{\partial J}{\partial t} + \frac{J}{\tau} = \frac{n e^2}{m} E \qquad J = \frac{\sigma}{(1 + i\omega\tau)} E = \sigma E$$

$\omega\tau \ll 1$ at RF frequencies

Skin depth

- For a good conductor at RF frequencies, $\omega\varepsilon << \sigma \rightarrow \partial D/\partial t \sim 0$

$$\nabla \times \nabla \times H = \nabla(\nabla \cdot H) - \nabla^2 H = \sigma \nabla \times E = -i\mu_0 \mu \sigma \omega H$$

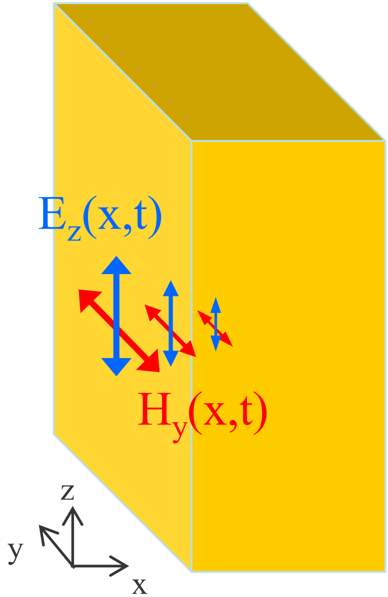
$\Rightarrow$ $\nabla^2 H = i\sigma\mu_0\mu\omega H$ similar equations for *E* and *J*

- Solution (semi-infinite slab):

$$H_y = H_0 e^{-x/\delta} e^{-ix/\delta}$$

$$E_z = -\frac{(1+i)}{\sigma\delta} H_y$$

$$\delta = \sqrt{\frac{2}{\mu_0 \mu \sigma \omega}}$$



Surface Impedance

- The surface impedance is defined as:

$$Z = \frac{|E_{\|}|}{\int_0^{\infty} J(x)dx} = \frac{E_{\|}}{H_{\|}} = R_s + iX_s$$

surface reactance

surface resistance

- For the semi-infinite plane conductor:

$$Z_n = \frac{|E_z(0)|}{H_y(0)} = \frac{1+i}{\sigma\delta}$$

$$R_s = X_s = \frac{1}{\sigma\delta} = \sqrt{\frac{\mu_0 \mu \omega}{2\sigma}}$$

- The impedance of vacuum is: $Z_0 = \left(\frac{\mu_0}{\varepsilon_0}\right)^{1/2} \simeq 377\Omega$

Example

Surface resistance of Cu at 300 K, 1.5 GHz:

$\sigma(300\text{ K}) = 5.8\times10^{7}\ 1/\Omega\text{m}$
$\mu_0 = 1.26\times10^{-6}\ \text{Vs/Am}$
$\mu = 1$

⇨ $\delta = 1.7\ \mu\text{m},\ R_s = 10\ \text{m}\Omega$

What happens at low temperature?

- $\sigma(T)$ increases, δ decreases ➡ The skin depth (the distance over which fields vary) can become less than the mean free path of the electrons (the distance they travel before being scattered) ➡ $J(x) \neq \sigma E(x)$

- Introduce a new relationship where *J* is related to *E* over a volume of the size of the mean free path (*l*)

$$\vec{J}(\vec{r},t) = \frac{3\sigma}{4\pi l}\int_V d\vec{r}'\,\frac{\vec{R}\left[\vec{R}\cdot\vec{E}(\vec{r}',t-\vec{R}/v_F)\right]}{R^4}e^{-R/l} \quad \text{with} \quad \vec{R}=\vec{r}'-\vec{r}$$

Effective conductivity $\sigma_{eff} \approx \frac{\delta}{l}\sigma = \frac{\delta}{l}\frac{ne^2 l}{m v_F}$, with $\frac{l}{v_F} = \tau$

Contrary to the DC case higher purity (longer *l*) does not increase the conductivity → anomalous skin effect

Anomalous skin effect

$$Z_n = \frac{4}{9}\left(\frac{\mu_0^2}{2\pi}\sqrt{3}\right)^{1/3}\left(\frac{l}{\sigma}\right)^{1/3}\omega^{2/3}(1+\sqrt{3}i) \qquad l >> \delta$$

- $l/\sigma = mv_F/e^2n = \hbar(3\pi^2 n)^{1/3}/e^2 n$ is a constant for each material = $(1.27\times10^4\ \Omega)/n^{2/3}$
- Independent of temperature and of purity of the material

G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. A 195 (1948) 336.

Example

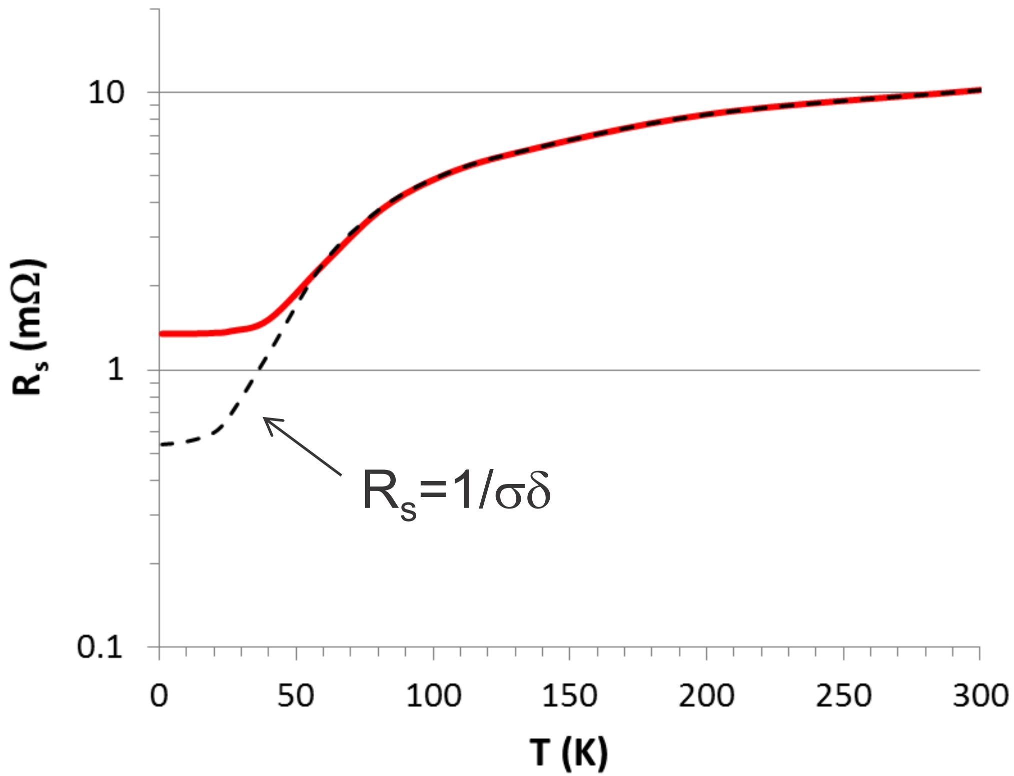
Surface resistance of Cu at 1.5 GHz as a function of temperature

$\rho l = 6.6\times10^{-16}\ \Omega m^2$

$\rho(273\ K) = 1.55\times10^{-8}\ \Omega m$

$RRR = \rho(300K)/\rho(4\ K) = 300$

$R_s(4\ K) \cong 1.3\ m\Omega$



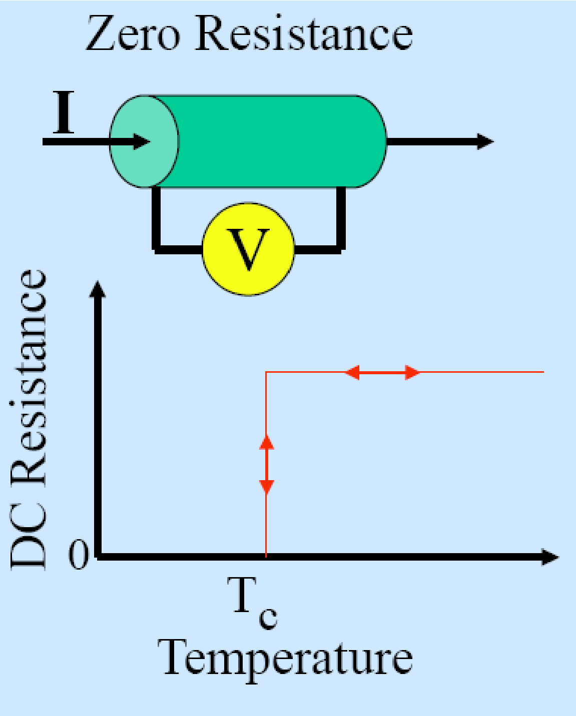
…in spite of the resistivity decreasing by a factor 300 from 300 K to 4 K, R_s only decreases by a factor of ~8!

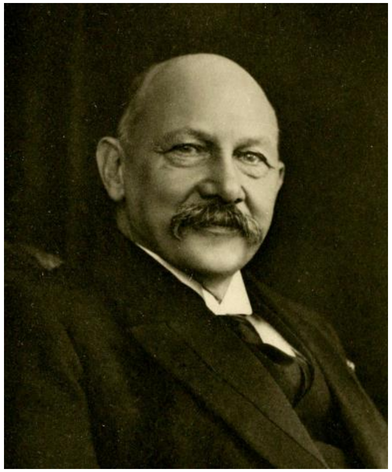
Superconductivity

The 3 Hallmarks of Superconductivity

- Zero resistance
- Complete diamagnetism
- Flux quantization

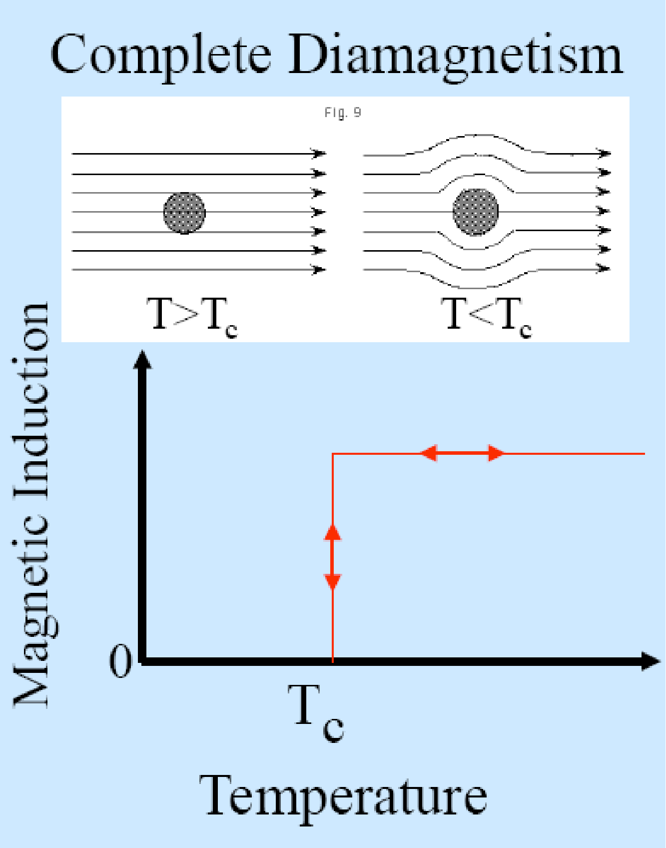
Zero Resistance





Kammerlingh-Onnes, 1911

Complete Diamagnetism



Meissner

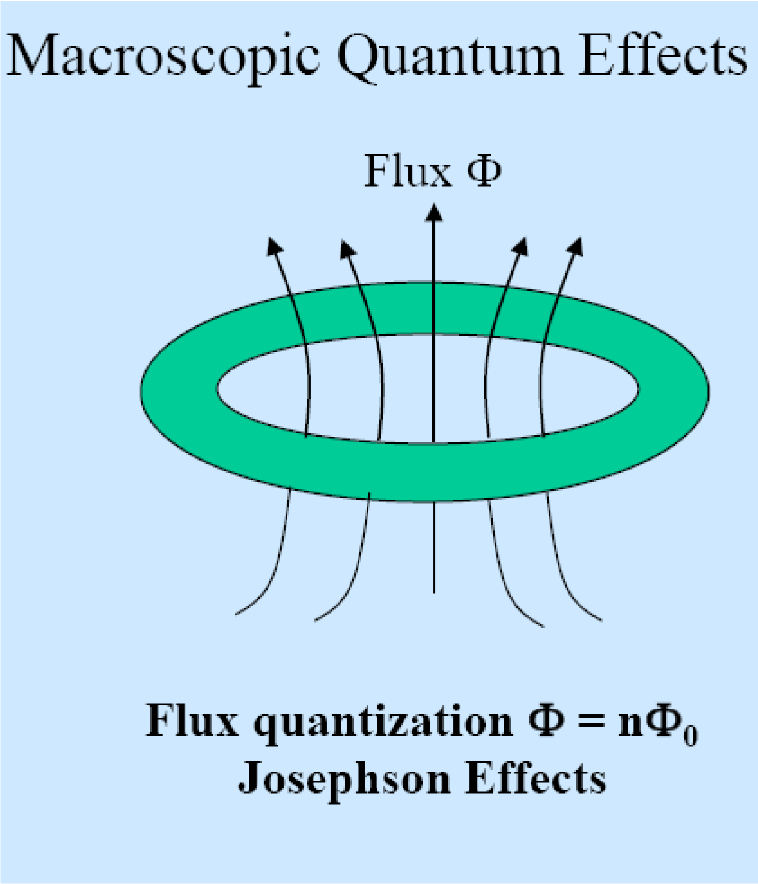
and

Ochsenfeld,

1933

"Meissner effect"

Flux Quantization



Deaver

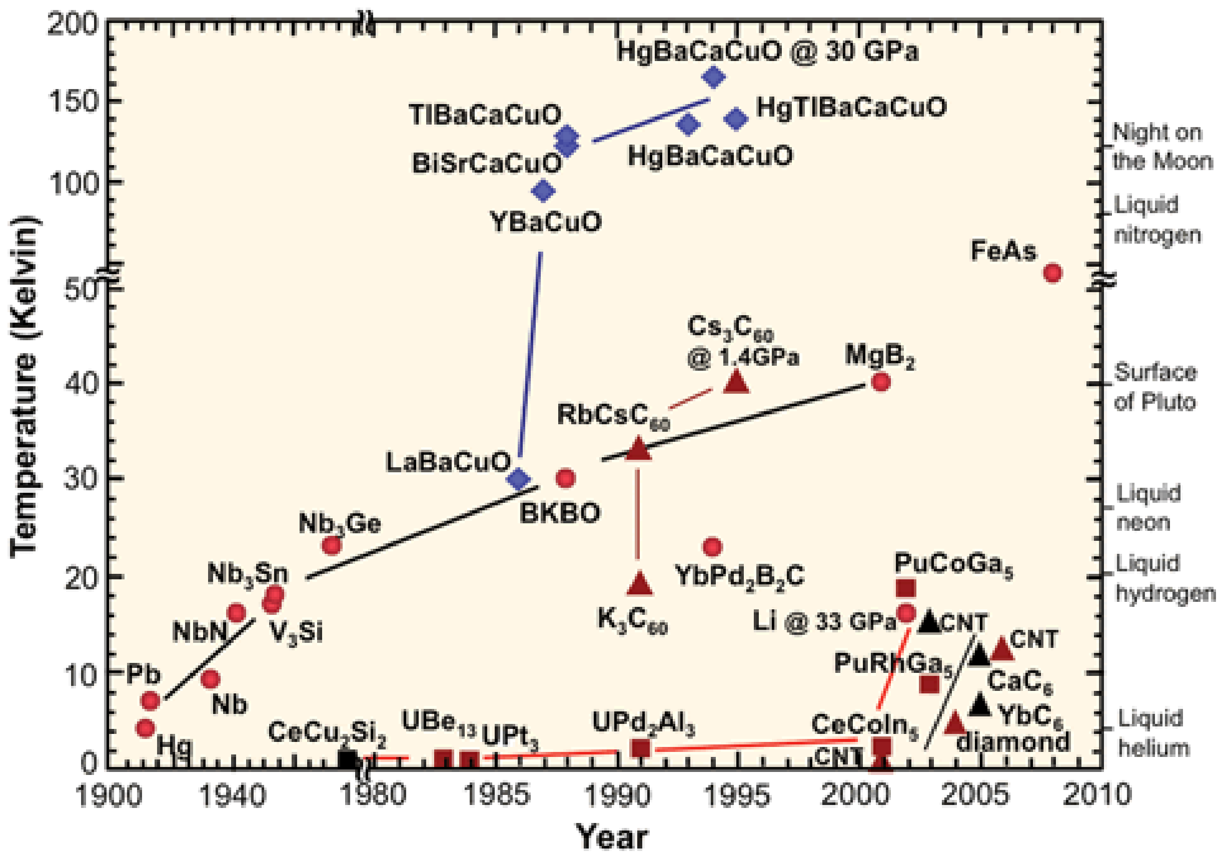
and

Fairbank,

1961

Critical Temperature

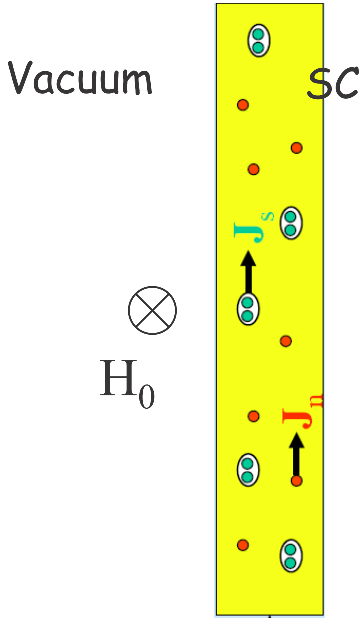
- "Isotope effect" (1950): $T_c \propto 1/\sqrt{M}$, M=isotope mass



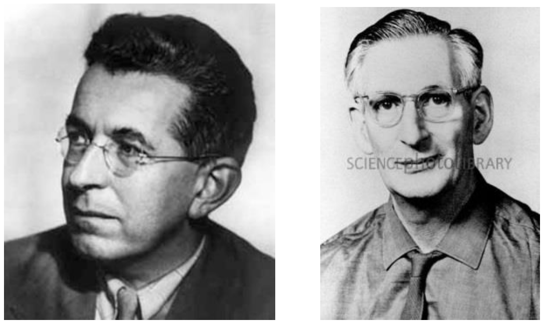
Two-fluid model

- Gorter and Casimir (1934) two-fluid model: charge carriers are divided in two subsystems, superconducting carriers of density n_s and normal electrons of density n_n.
- The normal current J_n and the supercurrent J_s are assumed to flow in parallel. J_s flows with no resistance.

$$J = J_n + J_s$$



London equations (I)



F. and H. London, 1935

- Superelectrons accelerate steadily in the presence of a constant electric field

$$m\frac{d\overrightarrow{v_s}}{dt} = e\vec{E} \quad \Rightarrow \quad \frac{d\overrightarrow{J_s}}{dt} = \frac{n_s e^2}{m}\vec{E}$$

$$\uparrow$$

$$J_s = n_s e v_s$$

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

London penetration depth

$$\frac{d\vec{J}_s}{dt} = \frac{1}{\mu_0 \lambda_L^2}\vec{E}$$

- **E**=0: J_s goes on forever
- **E** is required to maintain an AC current

London equations (II)

$$\vec{\nabla} \times \dot{\vec{J_s}} = \frac{1}{\mu_0 \lambda_L^2} \vec{\nabla} \times \vec{E} \quad \Rightarrow \quad \vec{\nabla} \times \dot{\vec{J_s}} = -\frac{1}{\mu_0 \lambda_L^2} \dot{\vec{B}}$$

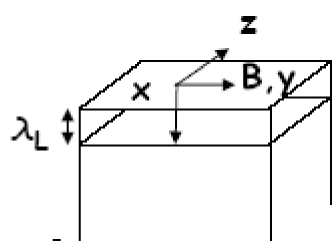
$$\nabla \times E = -\dot{B}$$

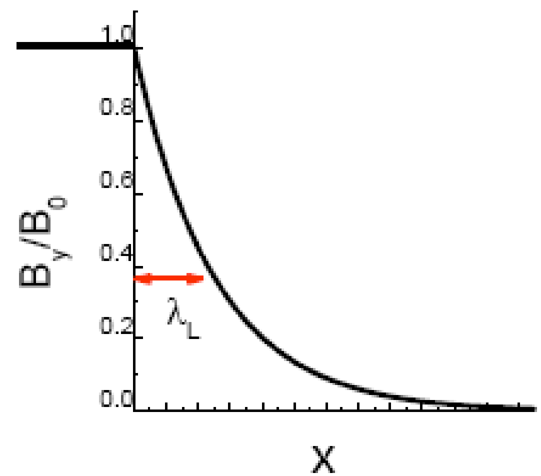
$$\vec{\nabla} \times \vec{J}_s = -\frac{1}{\mu_0 \lambda_L^2} \vec{B}$$

- **B** is the source of J_s
- Spontaneus flux exclusion

$$\nabla \times B = \mu_0 J_s$$

$$\nabla^2 B = \frac{B}{\lambda_L^2}$$





Coherence length

$$\vec{J}_s = -\frac{1}{\lambda_L^2}\vec{A}.$$

Local condition between current and field. Valid if $\xi_0 << \lambda_L$ or $l << \lambda_L$

Nonlocal generalization proposed by Pippard in 1953:

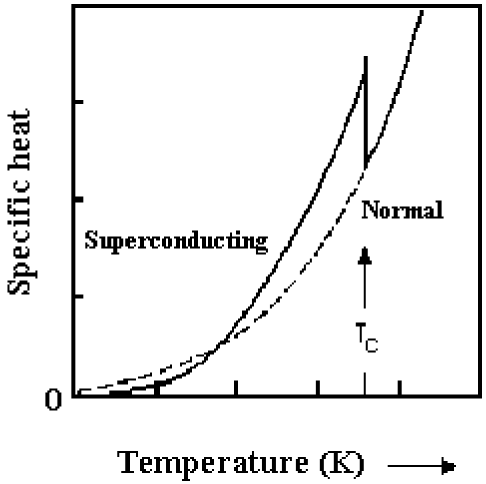
$$\vec{J}_s(\vec{r}) = -\frac{3}{4\pi\xi_0\lambda_L^2}\int_V \frac{\vec{R}\vec{R}\cdot\vec{A}(\vec{r}')e^{-R/\xi}}{R^4}d\vec{r}' \qquad \vec{R} = \vec{r} - \vec{r'}$$

ξ: "**coherence length**", characteristic dimension of the superelectrons wave-function

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{l}$$

$$\xi_0 \propto \frac{\hbar v_F}{kT_c}$$ for a pure material

The energy gap



Measurements of the electronic specific heat (1954):

- Jump at T_c without any latent heat
- Exponential decrease well below T_c

$$C_{es} \propto e^{-bT/T_c} \qquad b{\sim}1.5$$

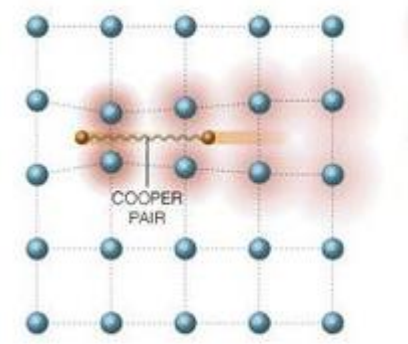
Results of measurements of electromagnetic absorption (1956) also consistent with the existence of an **energy gap Δ**, of order kT_c, between the ground state and the excited state of a superconductor

The BCS theory

Bardeen, Cooper and Schrieffer

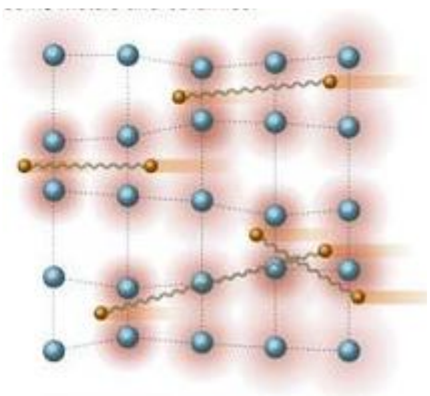
- In 1958 Bardeen, Cooper and Schrieffer published a theory of superconductivity in which
  - There exists an attractive interaction between electrons, forming "Cooper pairs"
  - This interaction occurs through the exchange of a lattice phonon
  - As a results of this interaction, there exists a bound state with energy lower than $2E_F$

Cooper pairs



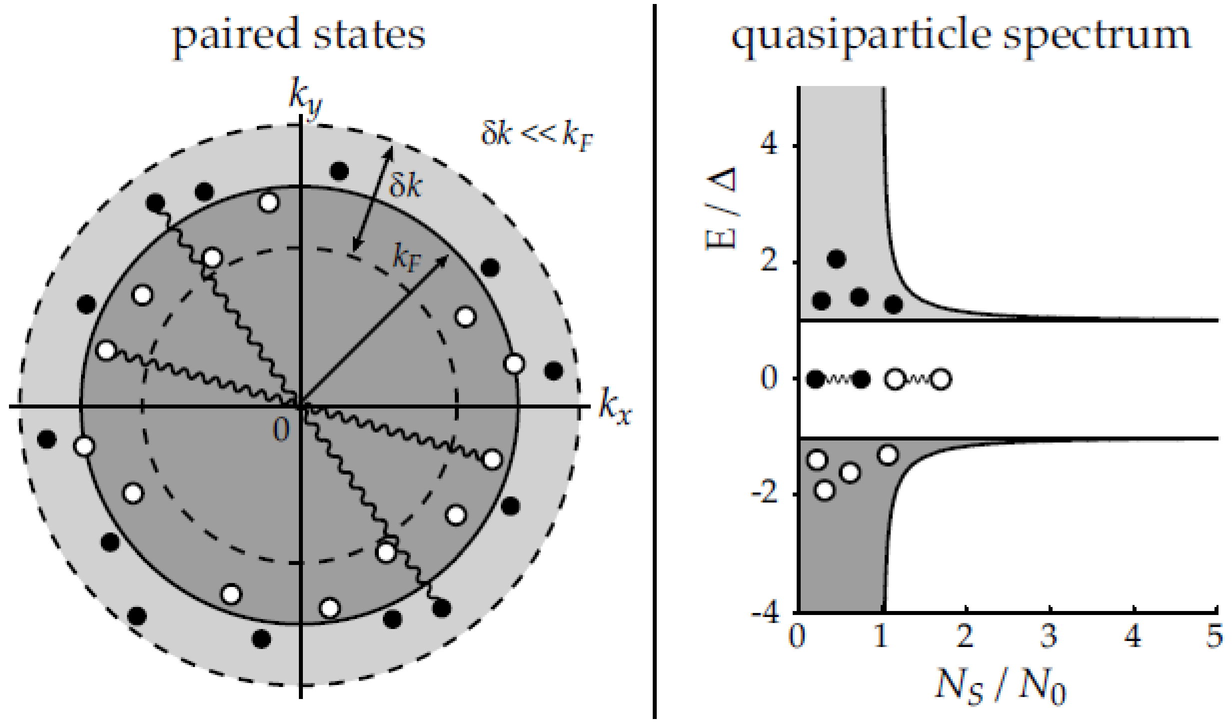
- Positively charged wake due to moving electron attracting nearby atoms
- This wake can attract another nearby electron

  → a Cooper pair is formed



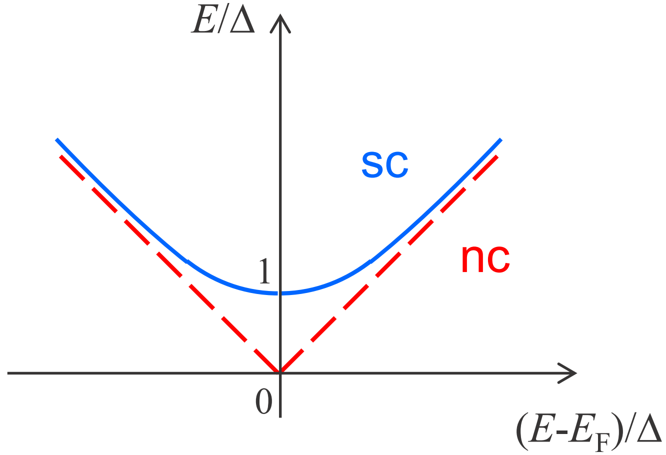
- Cooper pairs are formed by electrons with opposite momentum and spin
- Cooper pairs belong all to the same quantum state and have the same energy
- When carrying a current, each Cooper pair acquires a momentum which is the same for all pairs
- The **total** momentum of the pair remains constant. It can be changed only if the pair is broken, but this requires a minimum energy 2Δ

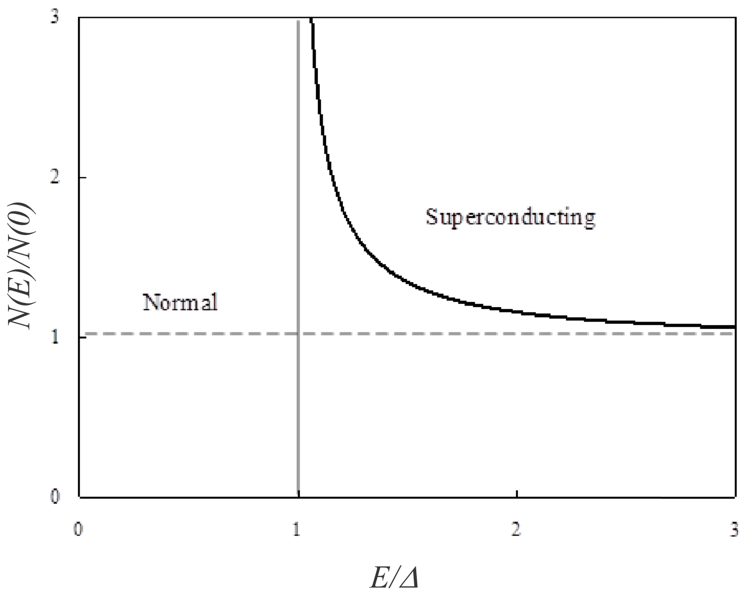
Paired states and quasiparticles



Quasi-particles excitations

- The unpaired electrons behave almost like free electrons and are called "quasi-particles"





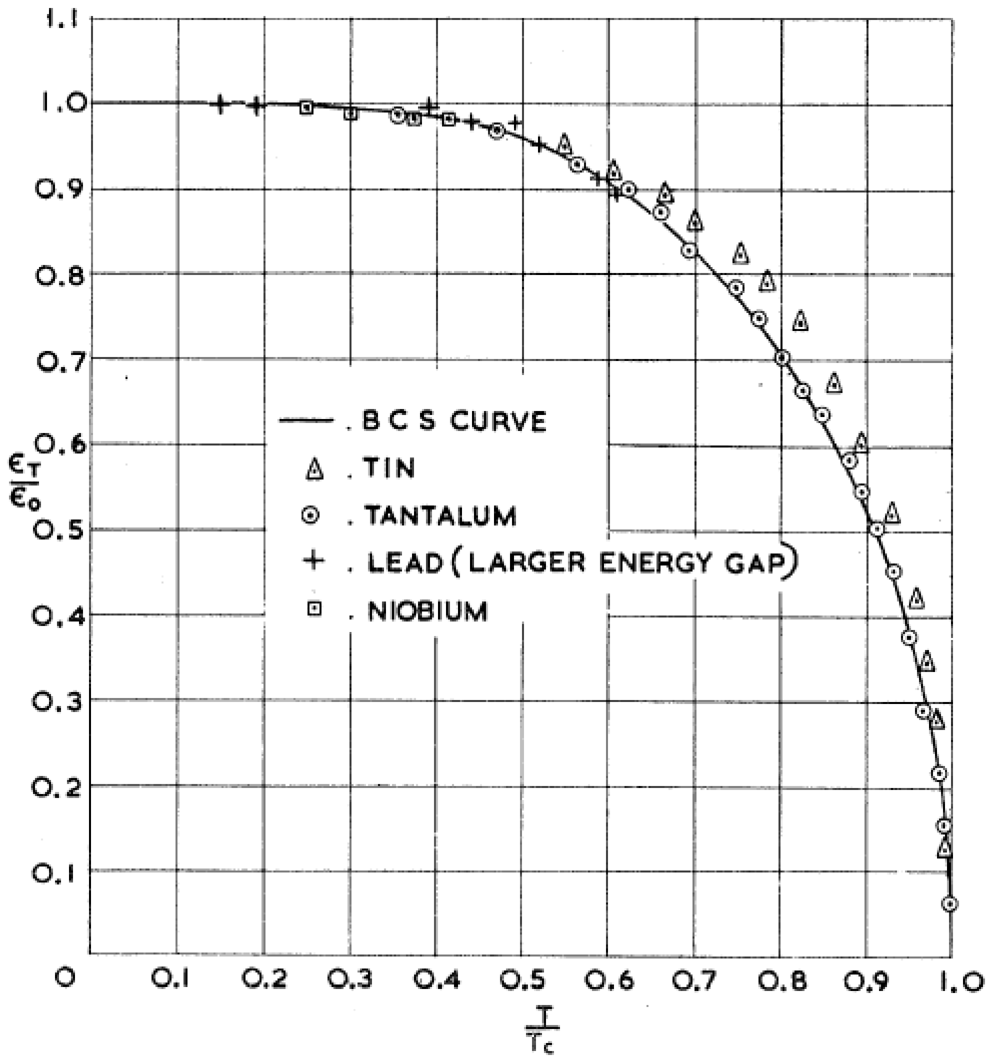
Energy spectrum

$$E^2 = \Delta^2 + (E-E_F)^2$$

Density of States

$$N(E)/N(0) = \begin{cases} E/(E^2-\Delta^2)^{1/2} & E > \Delta \\ 0 & E < \Delta \end{cases}$$

Energy gap



P. Townsend and J. Sutton, Phys. Rev. 128 (1962) 591.

$$\Delta(T) = \Delta(0)\sqrt{\cos\left(\frac{\pi t^2}{2}\right)}$$

$$t = T/T_c$$

$$\Delta(0)/kT_c = 1.764$$

$\Delta(0)$ = 1.55 meV for Nb

Characteristic Lengths

- **Coherence length** $\xi_0 \equiv \frac{\hbar v_F}{\pi\Delta(0)}$: interaction distance between electrons forming a Cooper pair **ξ_0 = 39 nm for Nb**

- **Penetration depth**, $\lambda(T)$: decay length of magnetic field in the superconductor **$\lambda(0)$ = 36 nm for Nb**

$$\lambda(T) = \frac{\lambda_L(0)}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}}$$

Effect of impurities on ξ and λ

- Adding impurities to a superconductor reduces the normal electrons mean free path, so that the electrodynamic response changes from "**clean**" (*l* >> ξ) to the "**dirty**" limit (*l* << ξ).
- Changes in the characteristic lengths of the SC can be approximated as:

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{l}$$

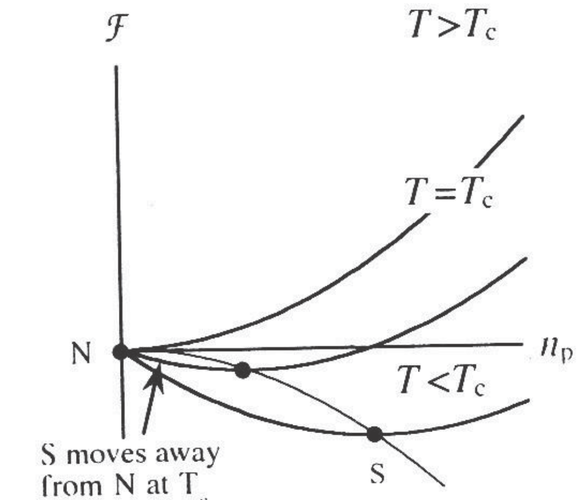
$$\lambda(l,T) = \lambda_L(T)\sqrt{1 + \frac{\xi_0}{l}}$$

Ginzburg-Landau theory

V. Ginzburg

L. Landau

- In 1950 Ginzburg and Landau proposed a theory of SC alternative to the London theory:
  - **<u>Near T_c</u>**, the difference in the Helmholtz free energy density between SC and NC state can be written as a power series of a complex order parameter, $\psi(\vec{r}) = |\psi(\vec{r})| e^{i\phi(\vec{r})}$



$$f_S = f_N + \alpha|\psi|^2 + \beta|\psi|^4$$

$$n_p = |\psi|^2 = n_s/2$$

Ginzburg-Landau equations

$$f_s = f_n + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\nabla - e^*\mathbf{A}\right)\psi\right|^2 + \frac{\mu_0 H^2}{2}$$

$m^* = 2m_e$
$e^* = 2e$

Minimization of f_s with respect to changes in order parameter and magnetic fields results in two equations:

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m^*}\left(-i\hbar\nabla - e^*\mathbf{A}\right)^2\psi = 0$$

$$\mathbf{J} = \frac{e^*}{m^*}\psi^*\left(-i\hbar\nabla - e^*\mathbf{A}\right)\psi$$

with proper boundary conditions. For example $(-i\hbar\nabla - e^*\mathbf{A})\psi|_n = 0$

Characteristic lengths in GL theory

- GL penetration depth: characteristic length for variation of the magnetic field

$$\lambda_{GL} = \sqrt{\frac{m^*}{\mu_0 |\psi|^2 e^{*2}}}$$

- GL coherence length: characteristic length for variation of the order parameter

$$\xi_{GL}(T) = \frac{\hbar}{\sqrt{2m^* |\alpha(T)|}} \propto \frac{1}{\sqrt{1-t}}$$

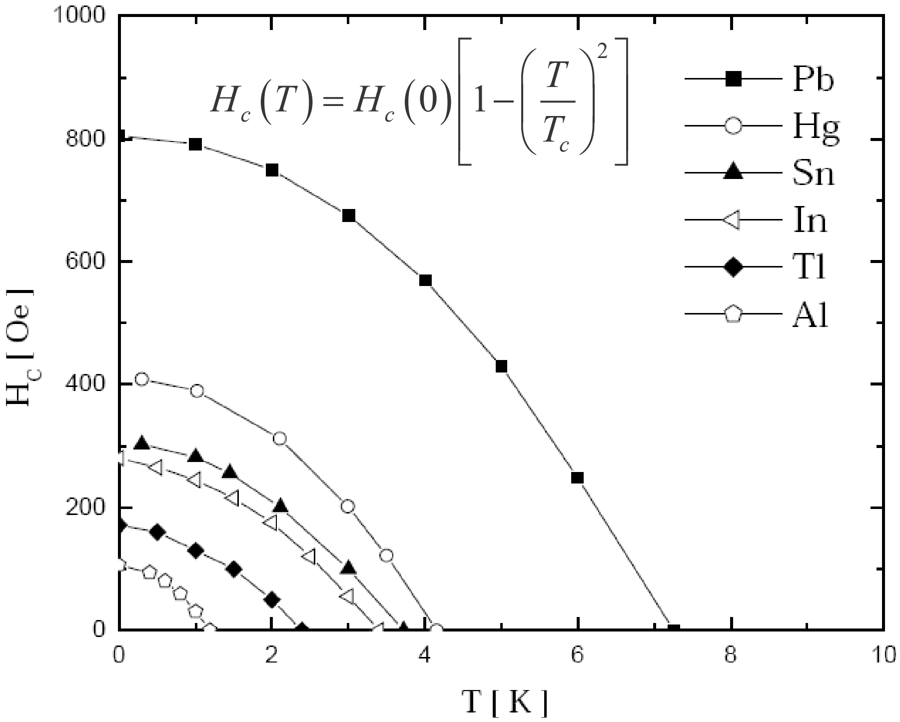
ξ_{GL} is related to the BCS coherence length (ξ_0):

$$\xi_{GL}(T) \propto \frac{\xi_0}{\sqrt{1-t}}$$ Clean limit

$$\xi_{GL}(T) \propto \sqrt{\frac{\xi_0 l}{1-t}}$$ Dirty limit

Thermodynamic critical field

Superconductivity is lost when a magnetic field applied to a SC increases above a critical value.



Gibbs free energy density in a SC with applied magnetic field H_a:

$$g_s(T,H) = g_s(T,0) + \frac{1}{2}\mu_0 H_a^2$$

at $H_a = H_c$, $g_s = g_n$

$$H_c = \sqrt{\frac{2}{\mu_0}[g_n(T,0) - g_s(T,0)]}$$

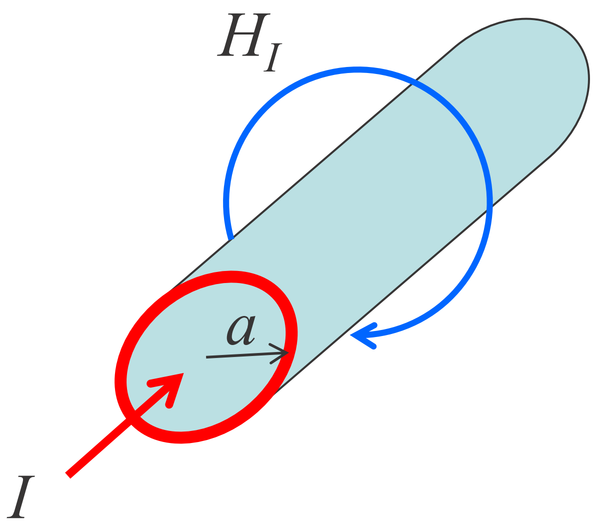
$$H_c(0) = \sqrt{\frac{0.472\gamma}{\mu_0}}T_c$$

from BCS theory

γ is the Sommerfeld gamma = $2/3\pi^2 N_F k_B^2$

Critical current

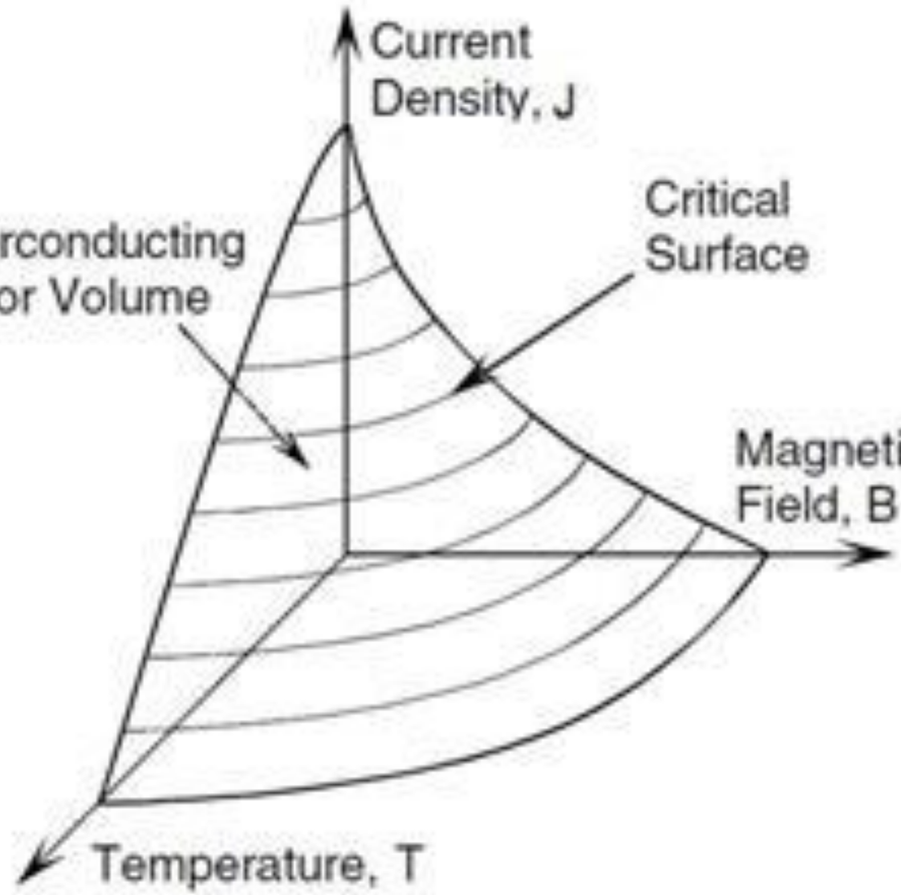
Superconductivity is lost when a current flowing in a SC increases above a critical value.



$$I_c = 2\pi a H_c$$

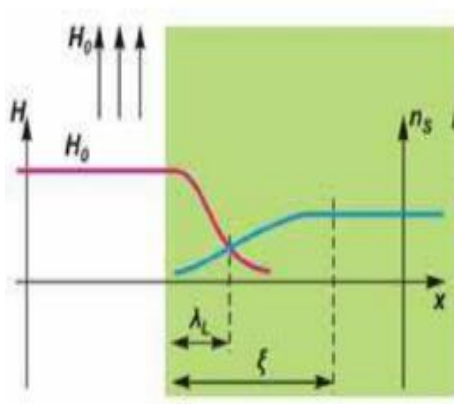
$$J_c = \frac{H_c}{\lambda}$$

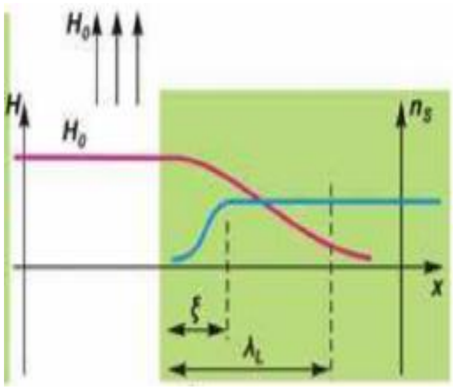
Phase diagram of SC



The NS boundary energy

Ginzburg-Landau parameter: $\kappa_{GL} = \lambda/\xi_{GL}$



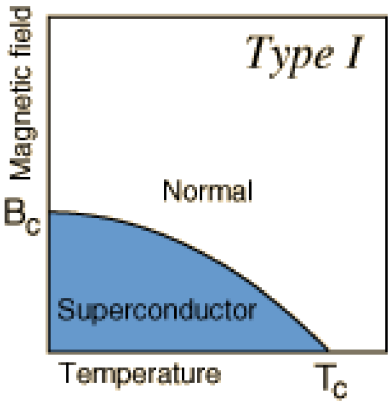


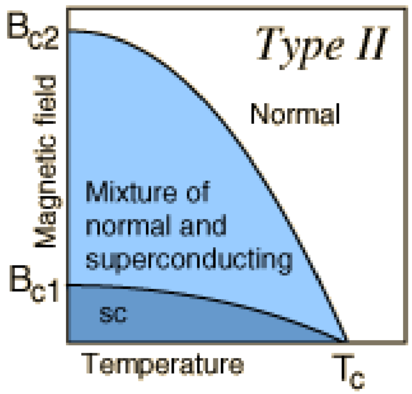
The change in free energy density δf due to the presence of a NS boundary was calculated using GL theory. Qualitatively:

$$\delta f = \frac{\mu_0}{2}(H_0^2\lambda - H_c^2\xi)$$

If $\kappa_{GL} > \frac{1}{\sqrt{2}}, \delta f < 0$ ⇨ it is energetically favorable to create NS boundaries within the SC

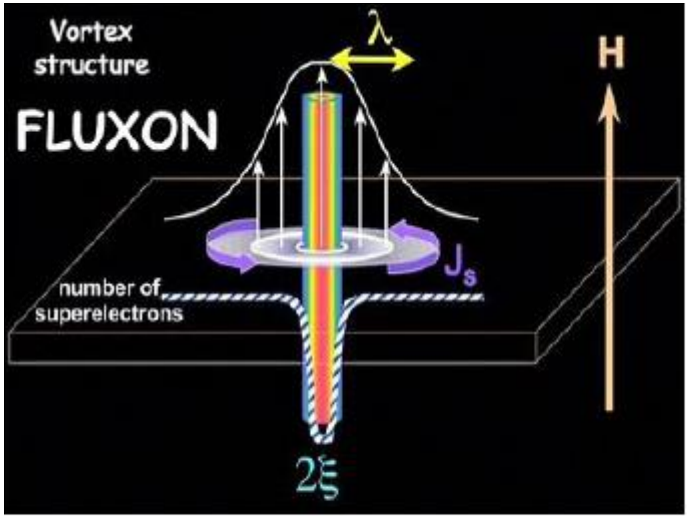
Type-I and Type-II SC

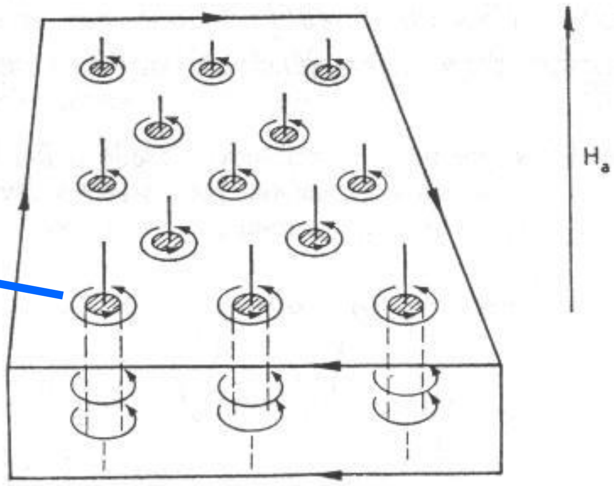




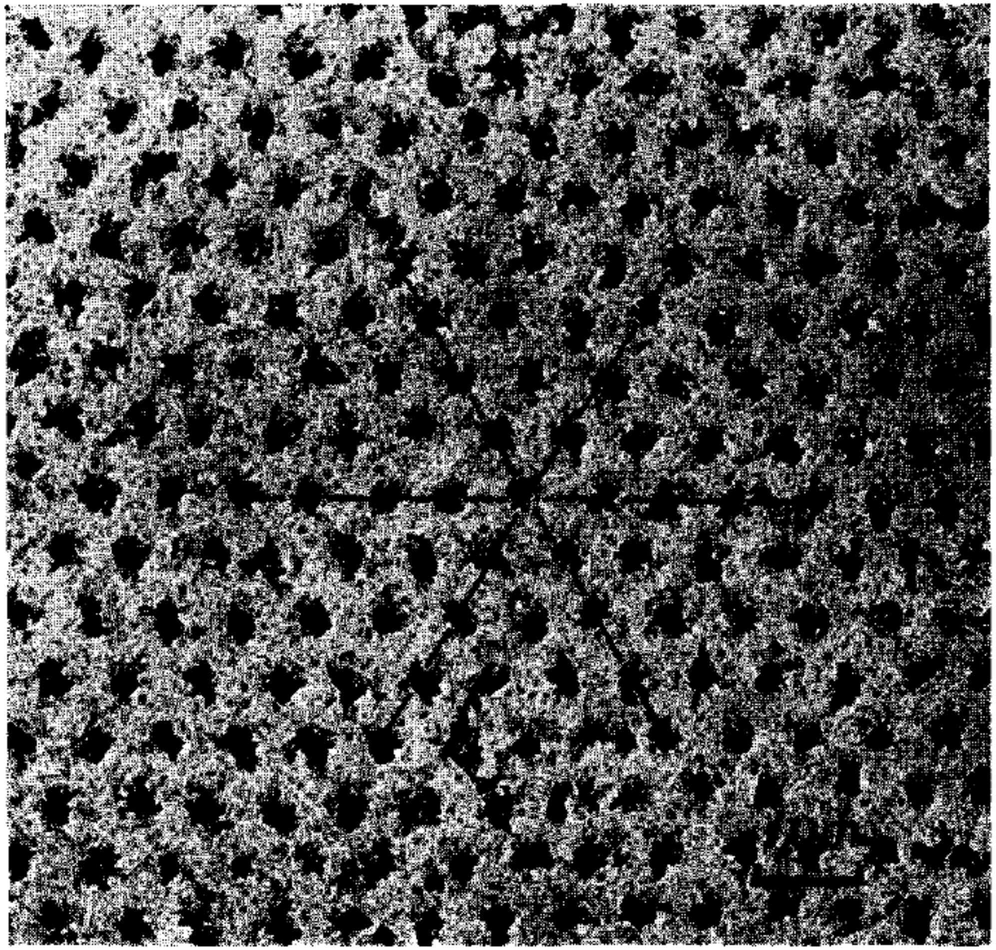
Abrikosov found solutions $\psi(x, y)$ with periodic zeros = lattice of vortices with **quantized magnetic flux**

$$\Phi_0 = \frac{h}{2e} = 2.07 \times 10^{-15}\,\mathrm{Wb}$$





Flux-line lattice



H. Träuble and U. Essmann, *J. Appl. Phys*. **39**, 4052 (1968)

Triangular flux-line lattice penetrating the top surface of a SC lead-indium sample

The points of exit of the flux lines are decorated by small ferromagnetic particles

Critical fields

$$H_c = \frac{\phi_0}{2\pi\sqrt{2}\lambda\xi}$$ Thermodynamic critical field

$$H_{c2} = \sqrt{2}\kappa H_c = \frac{\phi_0}{2\pi\xi^2}$$ Upper critical field

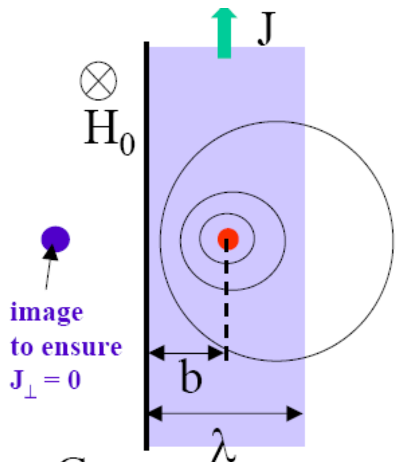
$$H_{c1} \approx \frac{\phi_0}{4\pi\lambda^2}\ln(\kappa + \alpha)$$ Lower critical field

$$\alpha = \frac{1}{2} + \frac{1 + ln2}{2\kappa - \sqrt{2} + 2} = \begin{cases} 1.35, \kappa = 0.71 \\ 0.5, \kappa \gg 1 \end{cases}$$

For Nb, κ ~ 0.85, $B_{c1}(0)$ ~ 180 mT, $B_c(0)$~195 mT, $B_{c2}(0)$ ~ 400 mT

Surface barrier

- Condition for entry of the first vortex, parallel to a planar surface (Bean and Livingston, 1964).

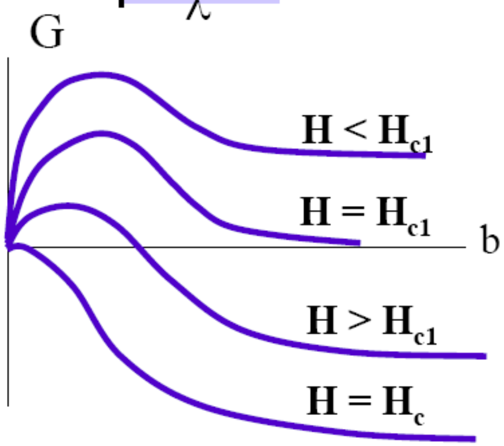


Two forces acting on the vortex at the surface:

- Meissner currents push the vortex in the bulk
- Attraction of the vortex to its antivortex image pushes the vortex outside

Thermodynamic potential G(b) as a function of the position b:

$$G(b) = \phi_0 [\underbrace{H_0 e^{-b/\lambda}}_{\text{Meissner}} - \underbrace{H_v(2b)}_{\text{Image}} + H_{c1} - H_0]$$

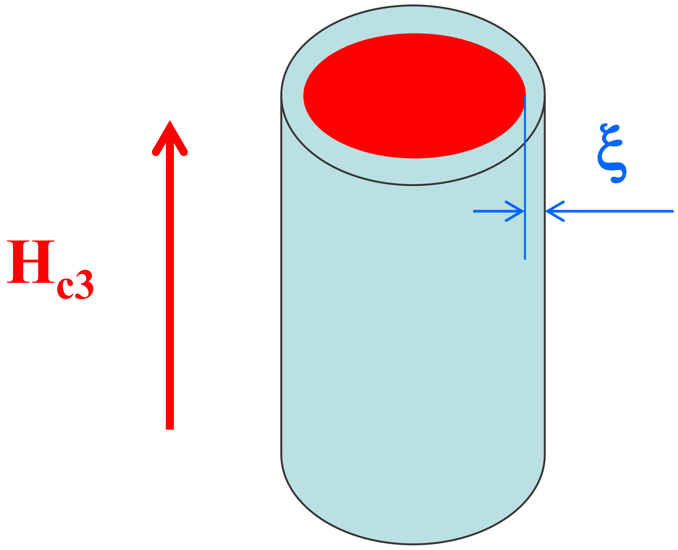


Penetration occurs at

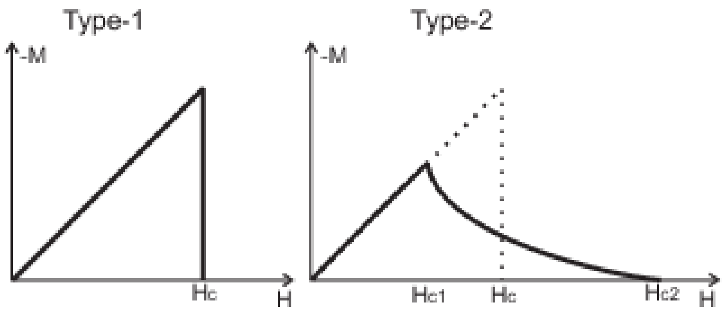
$$B_p \sim B_c > B_{c1}$$

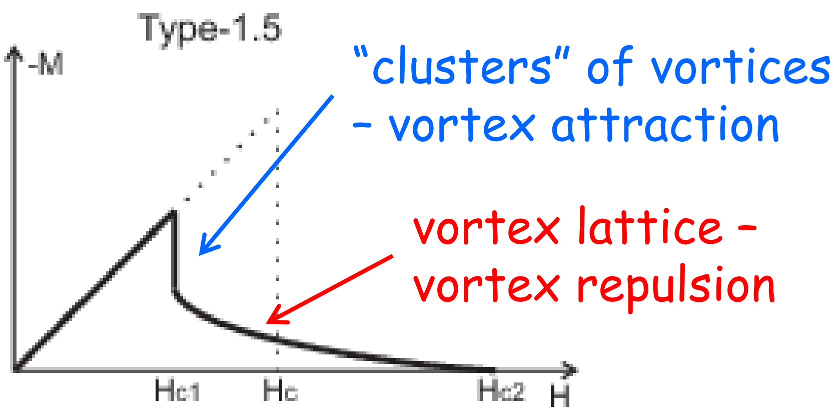
Surface critical field

- Saint-James and de Gennes obtained, using the GL theory, that in a magnetic field parallel to the surface, SC will nucleate in a surface layer of thickness ~ ξ at a field $\boldsymbol{H_{c3} = 1.695 H_{c2}}$, higher than that at which nucleation occurs in the volume of the material



Type 1.5 Superconductors

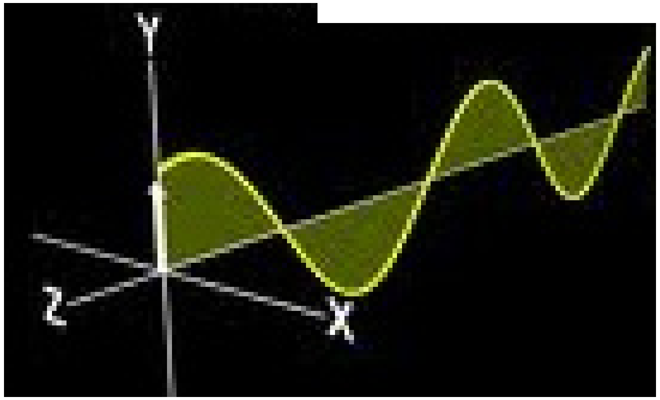




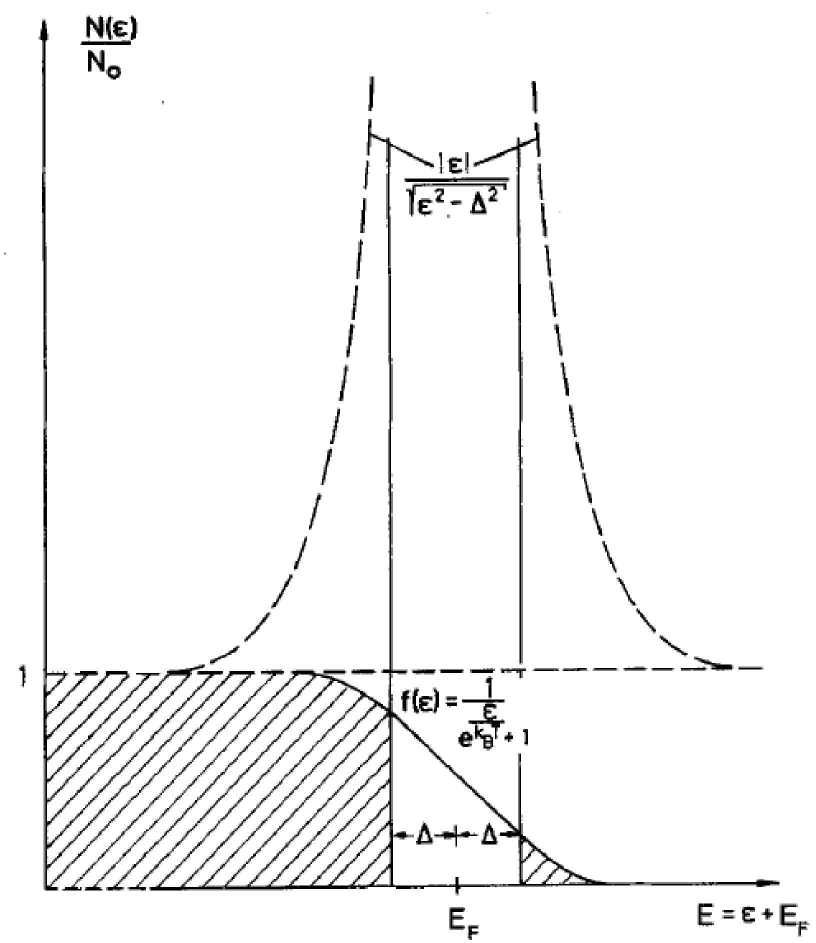
- Multi-component SC with $\xi_1 < \lambda < \xi_2$
- Theoretically, vortices can have long-range attractive, short-range repulsive interaction in such material
- "Semi-Meissner state": vortex clusters coexisting with Meissner domains at intermediate fields

Similar phenomenon was observed in low-κ SC (Nb, TaN, PbIn) with the origin of vortex attraction being related to non-local effects (*Type-IIa* or *Type-II/1*)

Enter RF Superconductivity



Surface resistance of SC



In RF fields, the time-dependent magnetic field in the penetration depth will generate an electric field:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

At $T > 0$ K, there is small fraction of unpaired electrons

$$n_n(T) \propto e^{-\Delta/k_B T}$$

$R_s > 0$

Surface impedance of superconductors

1st London Equation

$$\frac{\partial}{\partial t}\vec{J}_s = \frac{\vec{E}}{\mu_0\lambda_L^2} \quad \Rightarrow \quad J_s = -i\frac{1}{\omega\mu_0\lambda_L^2}E, \qquad \frac{1}{\omega\mu_0\lambda_L^2} = \sigma_2$$

Two-fluid model

$$J = J_n + J_s = \underbrace{(\sigma_1 - i\sigma_2)}_{\sigma}E$$

$$\sigma_1 = \frac{n_n e^2 \tau}{m}, \quad \sigma_2 = \frac{n_s e^2}{m\omega}$$

- Electrodynamics of sc is the same as nc, only that we have to change $\sigma \rightarrow \sigma_1 - i\,\sigma_2$
- Penetration depth: $\delta = \sqrt{\frac{2}{\mu_0\sigma\omega}} = \frac{1}{\sqrt{\mu_0\omega\sigma_2}}\sqrt{\frac{2i}{1-i\sigma_1/\sigma_2}} \cong (1+i)\lambda_L\left(1+i\frac{\sigma_1}{2\sigma_2}\right)$

$\sigma_1 << \sigma_2$ for sc at $T << T_c$

$$H_y = H_0 \exp\left(-\frac{(1+i)}{\delta}x\right) \quad \Rightarrow \quad H_y = H_0 e^{-\frac{x}{\lambda_L}} e^{-i\frac{x}{\lambda_L}\frac{\sigma_1}{2\sigma_2}}$$

For Nb, λ_L = 36 nm, compared to δ = 1.7 μm for Cu at 1.5 GHz

Surface impedance of superconductors

$$Z_s = \sqrt{\frac{i\omega\mu_0}{\sigma}} = \sqrt{\frac{\omega\mu_0}{2\sigma_1}}\left(\varphi_- + i\varphi_+\right)$$

$$\varphi_\pm^2 = \frac{y}{1+y^2}\left(\sqrt{1+y^2} \pm 1\right) \qquad y = \frac{\sigma_1}{\sigma_2}$$

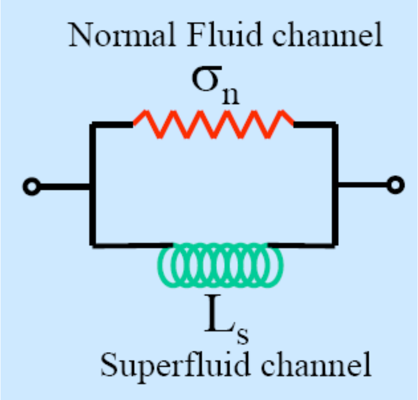
For a sc $\sigma_1 << \sigma_2 \rightarrow y << 1$ ➡ $\varphi_- = \sqrt{\frac{y^3}{2}}, \quad \varphi_+ = \sqrt{2y}$

$$Z_s = R_s + iX_s$$

$$X_s = \omega\mu_0\lambda_L$$

L_s: kinetic inductance ($L_s = \mu_0\lambda_L$)

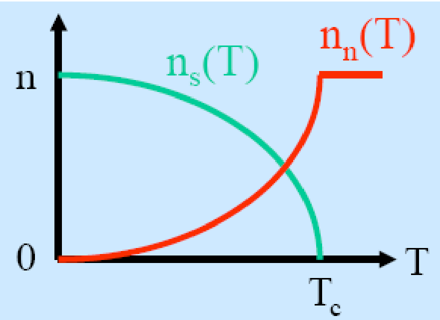
$$R_s = \frac{1}{2}\mu_0^2\omega^2\sigma_1\lambda_L^3$$



Surface resistance of superconductor

$$R_s = \frac{1}{2}\mu_0^2 \omega^2 \sigma_1 \lambda_L^3$$

- $R_s \propto \omega^2 \rightarrow$ use low-frequency cavities to reduce power dissipation
- Temperature dependence:



$n_s(T) \propto 1-(T/T_c)^4$ near T_c

$\sigma_1(T) \propto n_n(T) \propto e^{-\Delta/k_B T}$ at $T \ll T_c$

$$R_s \propto \omega^2 \lambda_L^3 l \exp\left(-\Delta/k_B T\right) \qquad T < T_c/2$$

Material purity dependence of R_s

- The dependence of the penetration depth on l is approximated as

$$\lambda(l) \approx \lambda_L \sqrt{1 + \frac{\xi_0}{l}}$$

- $\sigma_1 \propto l$

$$\Rightarrow \quad R_s \propto \left(1 + \frac{\xi_0}{l}\right)^{3/2} l \quad \Rightarrow$$

$R_s \propto l$ if $l >> \xi_0$ ("clean" limit)

$R_s \propto l^{-1/2}$ if $l << \xi_0$ ("dirty" limit)

R_s has a minimum for $l = \xi_0/2$

BCS surface resistance (1)

- Mattis and Bardeen (1958) calculated the perturbed state function using time-dependent perturbation theory.
- Considered only the linear response to weak fields (only terms linear in **A**) so that the perturbation term is:

$$H_1 = \frac{e}{2m}\sum_i \left(\vec{A}\cdot\hat{p}_i + \hat{p}_i\cdot\vec{A}\right)$$

BCS surface resistance (2)

- The following non-local equation between the total current density **J** and the vector potential **A** produced by the perturbation H_1 was derived

$$\vec{J}(\vec{r}) = \frac{3}{4\pi^2 v_0 \hbar \lambda_{L0}^2} \int_V \frac{\vec{R}\vec{R}\cdot\vec{A}(\vec{r}')I(\omega,R,T)e^{-R/l}}{R^4} d\vec{r}' \qquad \vec{R} = \vec{r} - \vec{r}'$$

$I(\omega,R,T)$ decays over a characteristic length scale $R \sim \min\{\xi_0, l\}$

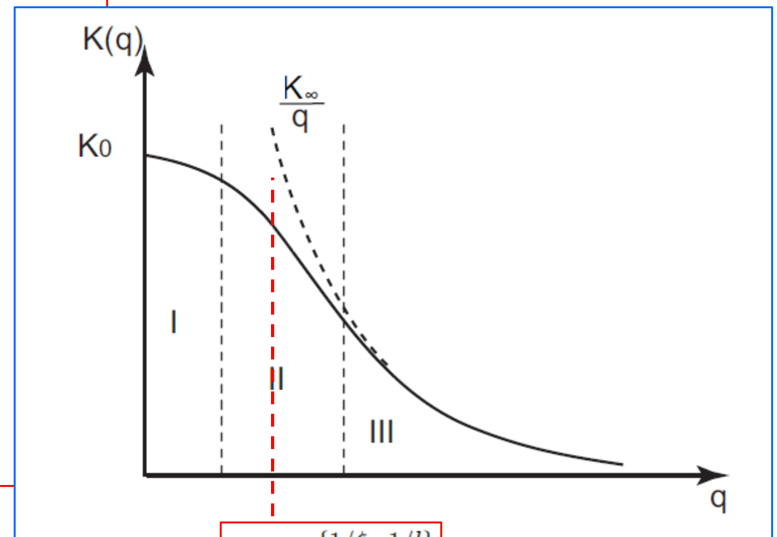
Factor which accounts for impurity scattering

BCS surface resistance (3)

Fourier domain: $J(q,\omega) = -K(q,\omega)A(q,\omega)$

$$\mathrm{Re}\{K(q)\} = \frac{3}{\hbar v_0 \lambda_{L0}^2 q} \times$$
$$\Big\{ \int_{\max\{\Delta-\hbar\omega,-\Delta\}}^{\Delta} [1-2f(E+\hbar\omega)]\{\frac{E^2+\Delta^2+\hbar\omega E}{\sqrt{\Delta^2-E^2}\sqrt{(E+\hbar\omega)^2-\Delta^2}} R(a_2, a_1+b) + S(a_2, a_1+b)\} dE$$
$$+ \frac{1}{2}\int_{\Delta-\hbar\omega}^{-\Delta} [1-2f(E+\hbar\omega)]\{[g(E)+1]S(a^-,b) - [g(E)-1]S(a^+,b)\} dE$$
$$- \int_{\Delta}^{\infty} [1-f(E)-f(E+\hbar\omega)][g(E)-1]S(a^+,b) dE$$
$$+ \int_{\Delta}^{\infty} [f(E)-f(E+\hbar\omega)][g(E)+1]S(a^-,b) dE \Big\}$$

$$\mathrm{Im}\{K(q)\} = \frac{3}{\hbar v_0 \lambda_{L0}^2 q} \times$$
$$\Big\{ -\frac{1}{2}\int_{\Delta-\hbar\omega}^{-\Delta} [1-2f(E+\hbar\omega)]\{[g(E)+1]R(a^-,b) + [g(E)-1]R(a^+,b)\} dE$$
$$+ \int_{\Delta}^{\infty} [f(E)-f(E+\hbar\omega)]\{[g(E)+1]R(a^-,b) + [g(E)-1]R(a^+,b)\} dE \Big\}$$



$q \sim \max\{1/\xi_0, 1/l\}$

$b = 1/ql$, $a^+ = a_1 + a_2$, $a^- = a_2 - a_1$, $a_1 = \Delta_1/(\hbar v_0 q)$, and $a_2 = \Delta_2/(\hbar v_0 q)$.

$$R(a,b) = -\frac{b}{2} + \frac{ab}{4}\ln[\frac{b^2+(1+a)^2}{b^2+(1-a)^2}] + \frac{1}{4}(1+b^2-a^2)\{\arctan[\frac{2b}{b^2+a^2-1}] + n_x\pi\}$$

$$S(a,b) = \frac{a}{2} - \frac{ab}{2}\{\arctan[\frac{2b}{b^2+a^2-1}] + n_x\pi\} + \frac{1}{8}(1+b^2-a^2)\ln[\frac{b^2+(1+a)^2}{b^2+(1-a)^2}]$$

$n_x = 0$ for $b^2+a^2-1 \geq 0$, $n_x = 1$ for $b^2+a^2-1 < 0$.

BCS surface resistance (4)

- The surface impedance can be derived in term of the Kernel $K(q)$:

$$Z_s = \frac{j\mu_0\omega\pi}{\int_0^\infty \ln(1+\frac{K(q)}{q^2})dq}$$

for diffuse scattering of electrons at the metal surface

$$Z_s = \frac{j\mu_0\omega}{\pi}\int_{-\infty}^{\infty}\frac{dq}{q^2+K(q)}$$

for specular scattering

- There are numerical codes (Halbritter (1970)) to calculate Z_s as a function of ω, T and material parameters (ξ_0, λ_L, T_c, Δ, l)
- For example, check http://www.lepp.cornell.edu/~liepe/webpage/researchsrimp.html

BCS surface resistance (5)

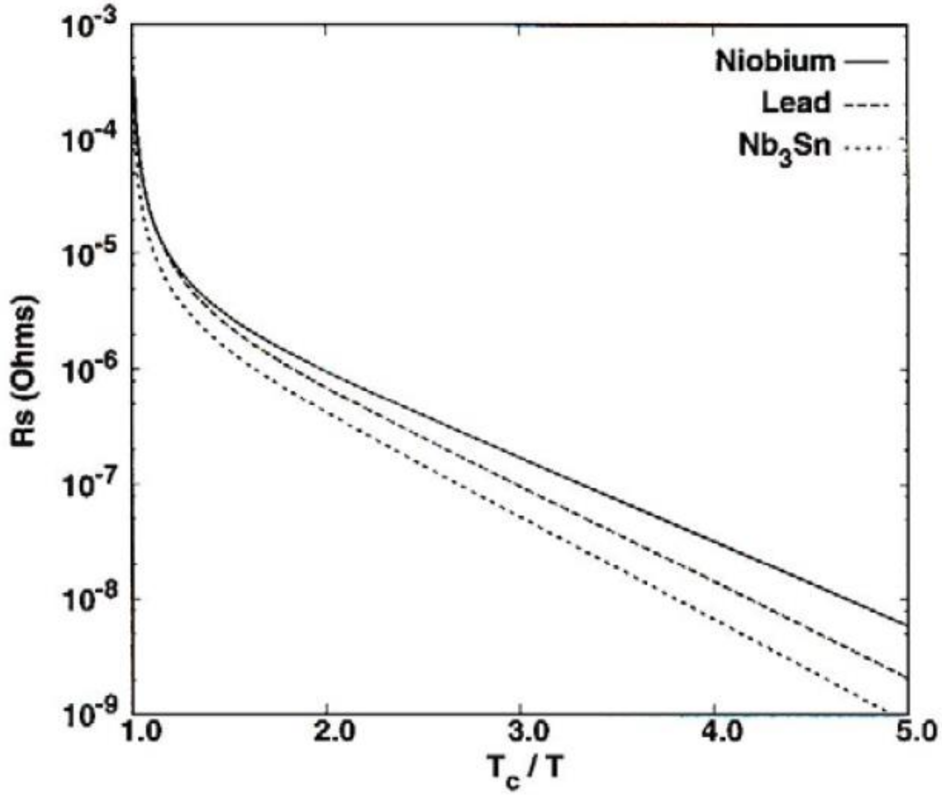
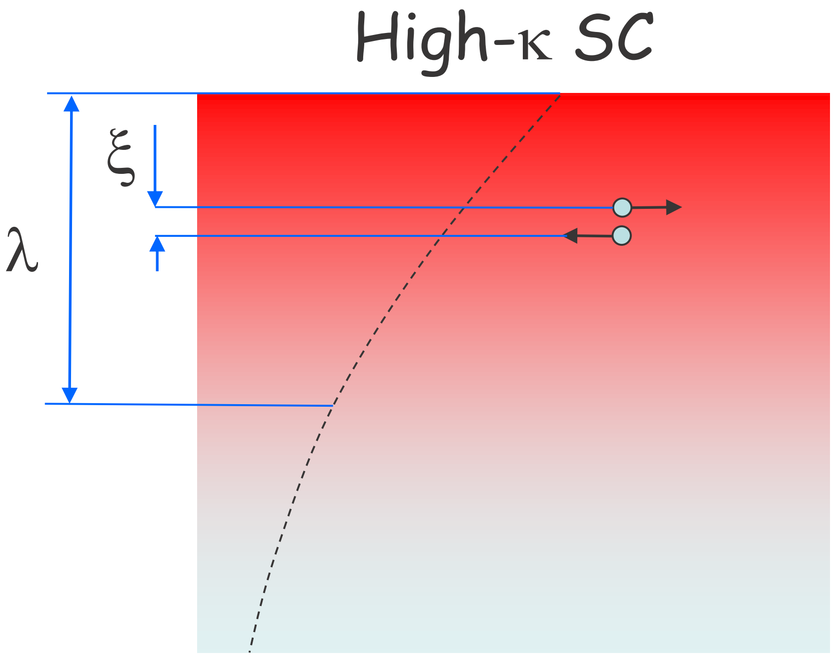
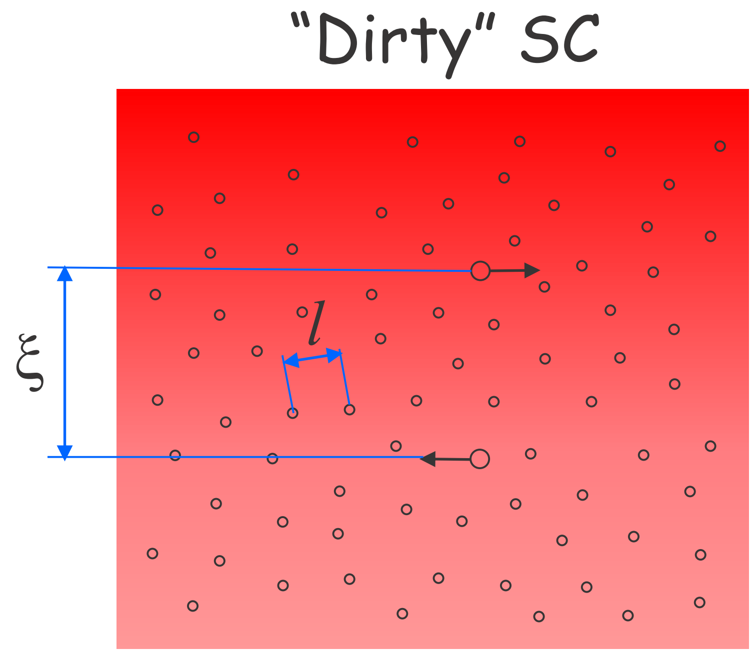


Figure 4.5: Theoretical surface resistance at 1.5 GHz of lead, niobium and Nb_3Sn as calculated from program [94]. The values given in Table 4.1 were used for the material parameters.

Applicability of "local" limit





- A good approximation of R_{BCS} in the dirty limit for $T < T_c/2$ and $\omega < \Delta/\hbar$ is:

$$R_{BCS} \cong \frac{\mu_0^2 \omega^2 \lambda^3 \sigma_n \Delta}{k_B T} \ln\left[\frac{C_1 k_B T}{\hbar\omega}\right] \exp\left[-\frac{\Delta}{k_B T}\right]$$

$C_1 \sim 9/4$

Nb vs Cu

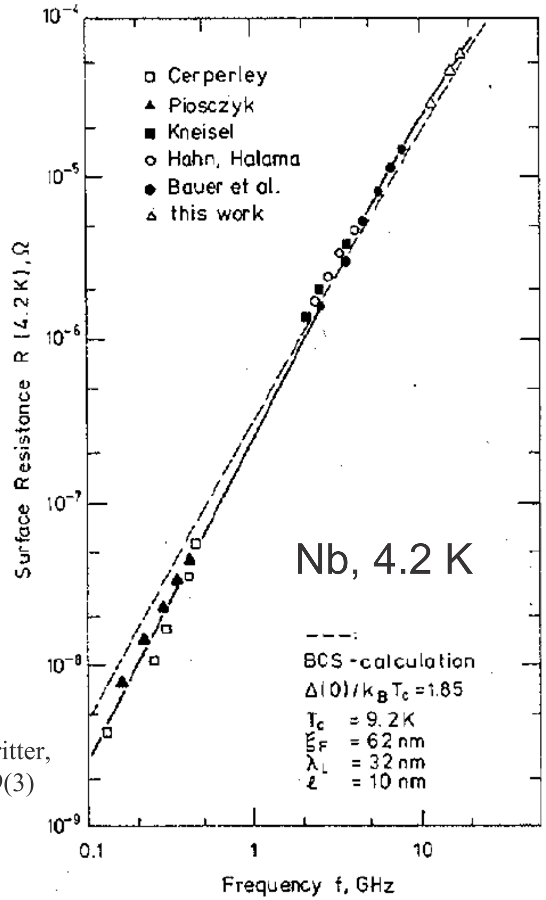
Let's run some numbers: Nb at 2.0 K, 1.5 GHz $\rightarrow \lambda$ = 36 nm, σ_n = 3.3×10⁸ 1/Ωm, $\Delta/k_B T_c$ = 1.85, T_c = 9.25 K

$R_{BCS} \cong 20\ \mathrm{n\Omega}$ $\qquad X_s \cong 0.47\ \mathrm{m\Omega}$

$$\begin{array}{l} \text{Nb} \rightarrow \\ \text{Cu} \rightarrow \end{array} \frac{R_{BCS}(2\,\mathrm{K}, 1.5\,\mathrm{GHz})}{R_s(300\,\mathrm{K}, 1.5\,\mathrm{GHz})} \cong 2\times 10^{-6}$$

Experimental results

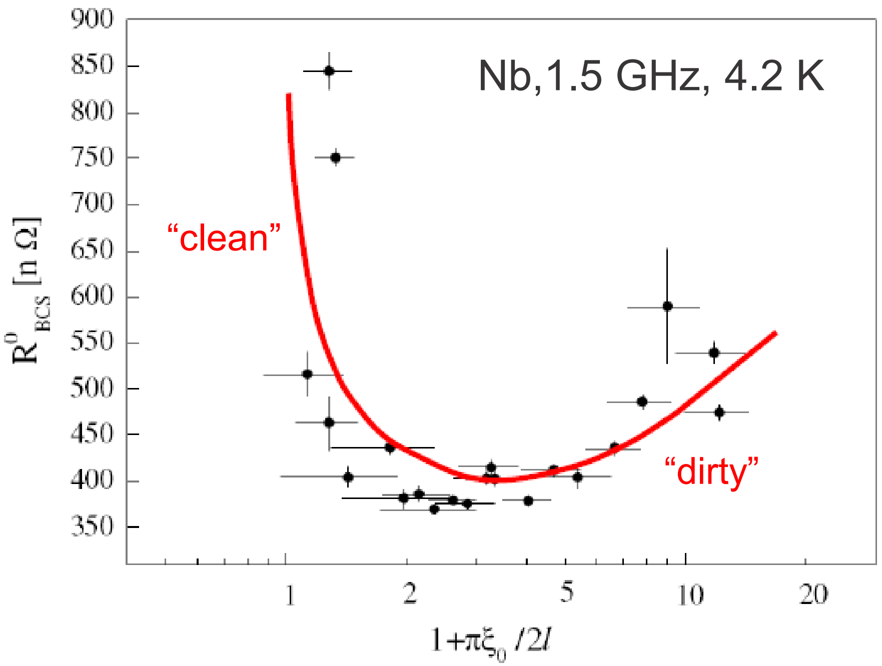
Frequency dependence



A. Phillip and J. Halbritter, IEEE Trans. Magn. **19**(3) (1983) 999.

- Small deviations from BCS theory can be explained by strong coupling effects, anisotropic energy gap in the presence of impurity scattering or by inhomogeneities

Dependence on material purity

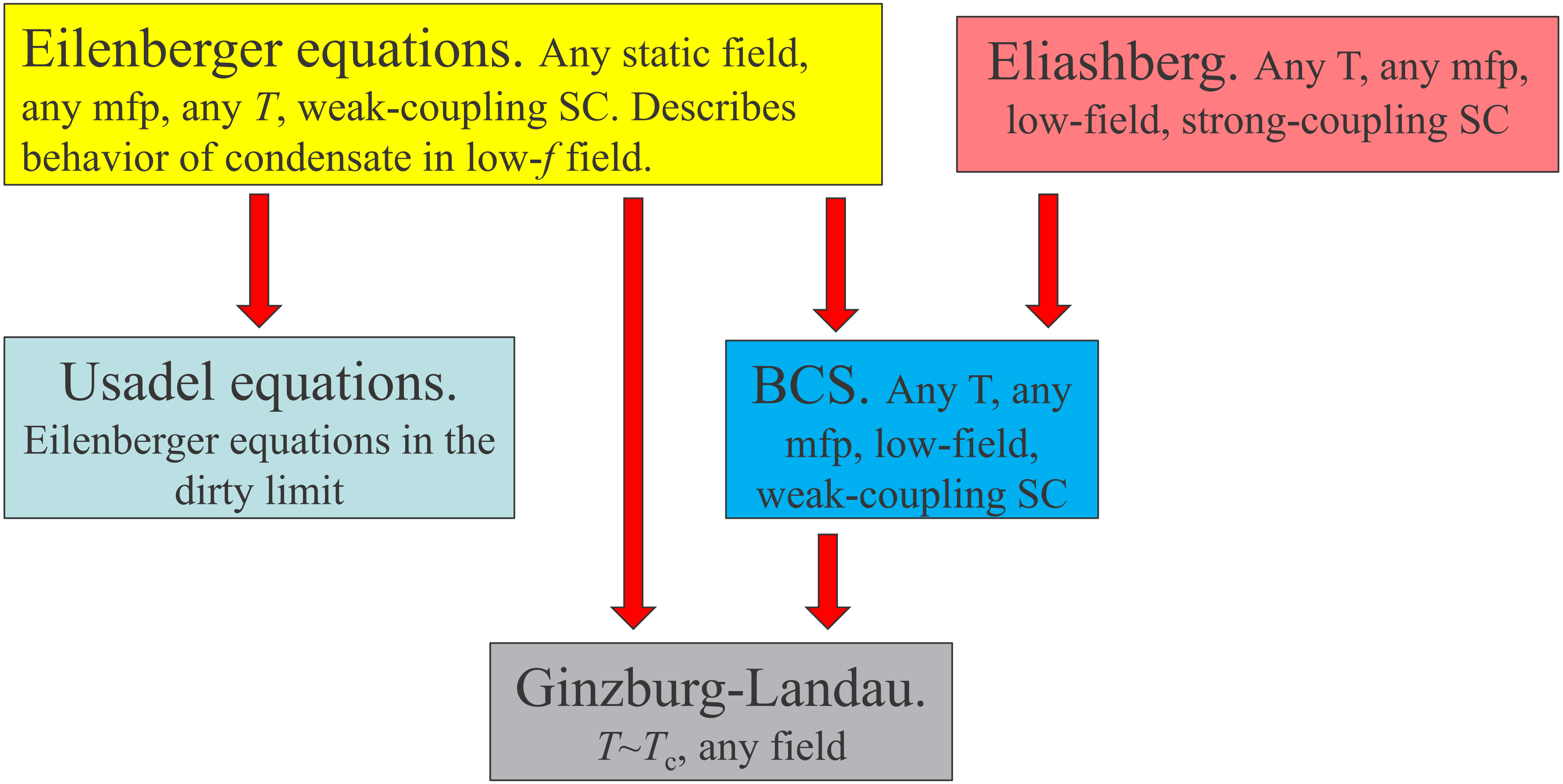


C. Benvenuti et al., Physica C **316** (1999) 153.

- Nb films sputtered on Cu
- By changing the sputtering species, the mean free path was varied

R_{BCS} can be optimized by tuning the density of impurities at the cavity surface.

Theories landscape

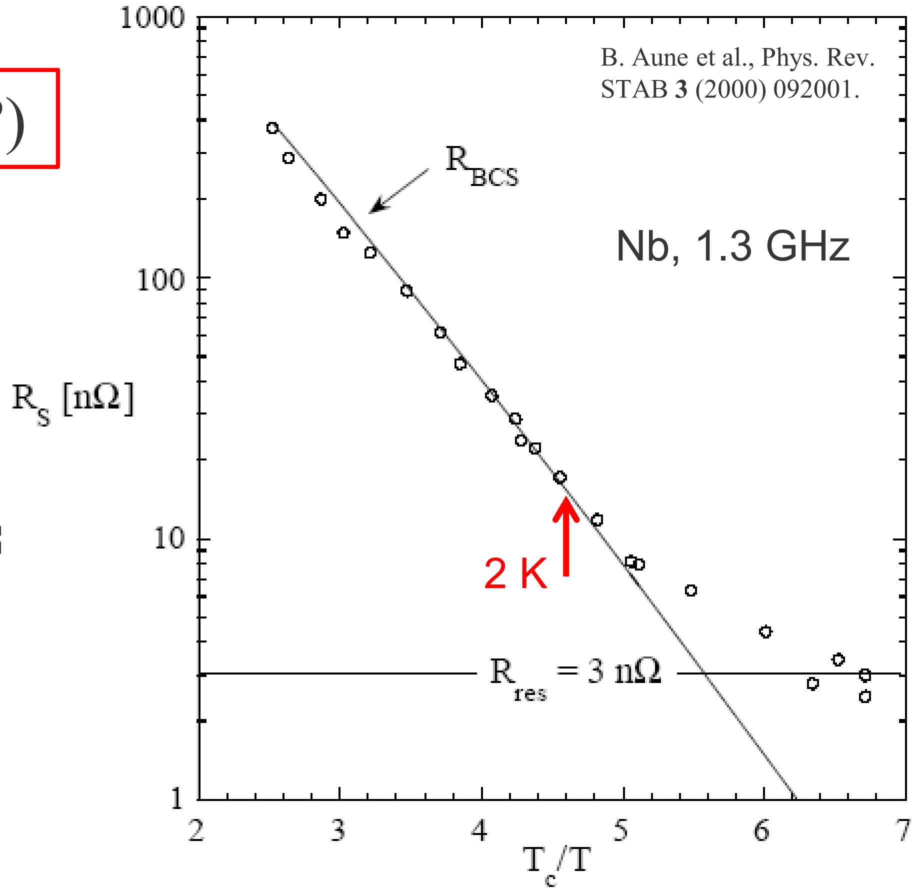


Residual resistance

$$R_s = R_{BCS}(\omega, T, \Delta, T_c, \lambda_L, \xi_0, l) + R_{res}(?)$$

Possible contributions to R_{res}:

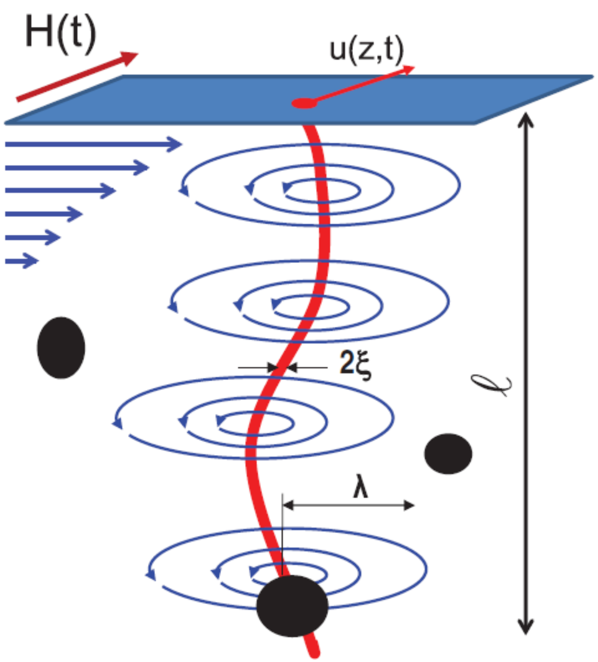
- Trapped magnetic field
- Normal conducting precipitates
- Grain boundaries
- Interface losses
- Subgap states



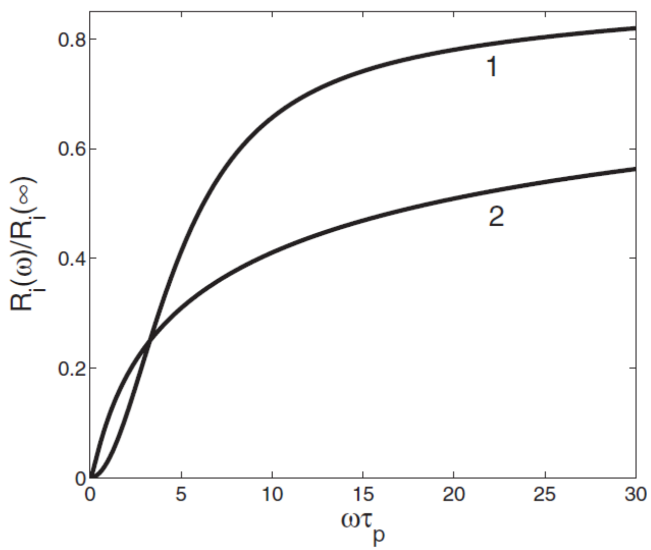
For Nb, R_{res} (~1-10 nΩ) dominates R_s at low frequency (f < ~750 MHz) and low temperature (T < ~2.1 K)

Trapped magnetic flux

- When a cavity is cooled down in an ambient DC magnetic field not all flux is expulsed – Incomplete Meissner effect
- Trapped magnetic field can also result from thermoelectric currents



- Dissipation due to oscillating vortex segments, driven by the RF field



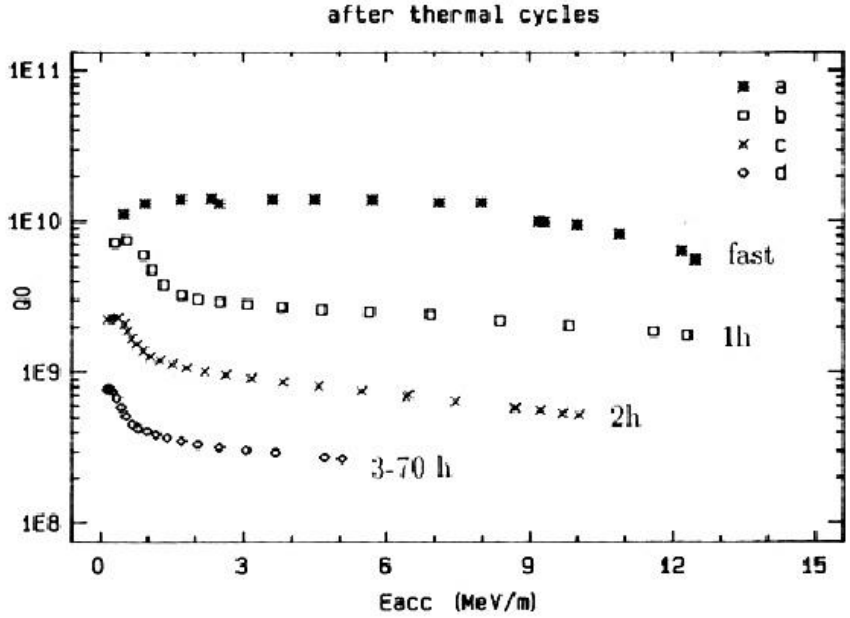
R_{fl} ~ 0.3 – 1 nΩ/mG at ~1 GHz

A. Gurevich and G. Ciovati, Phys. Rev. B **87**, 054502 (2013)

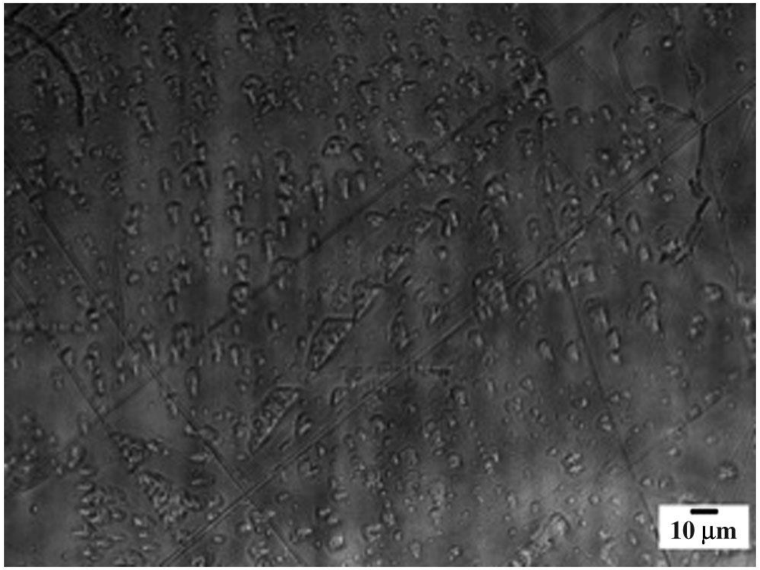
A. Gurevich and G. Ciovati, Phys. Rev. B **77**, 104501 (2008)

Normal conducting precipitates

- Islands of NbH precipitates at the surface
  - Bulk hydrogen conc. > 10 wt.ppm
  - Cooling rate < ~1 K/min between 90 – 150 K



B. Bonin and R. W. Roth, *Proc. 5th SRF Workshop*, Hamburg, Germany, 199, p. 210.

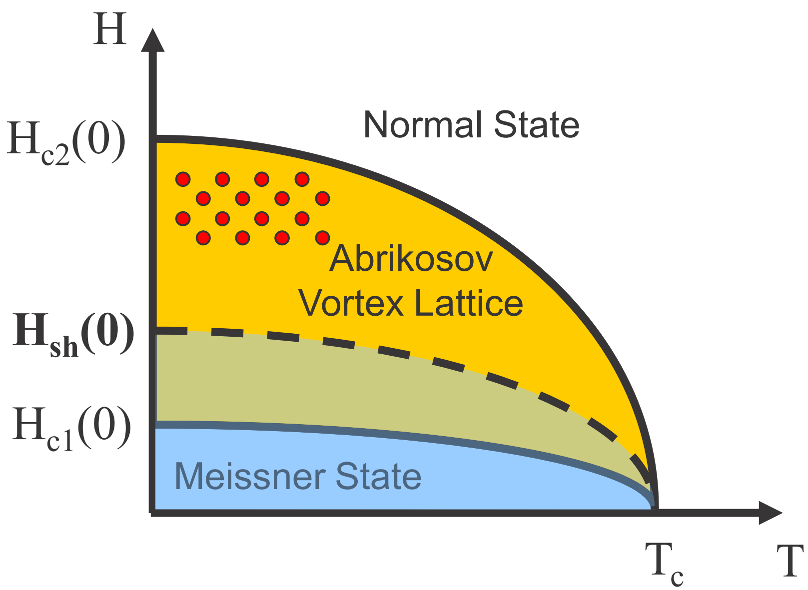


F. Barkov, A. Romanenko, and A. Grassellino, Phys. Rev. ST Accel. Beams **15**, 122001 (2012)

SRF cavities: a unique application in which low R_s at highest RF field is desired

RF critical field: superheating field

Type-II SC



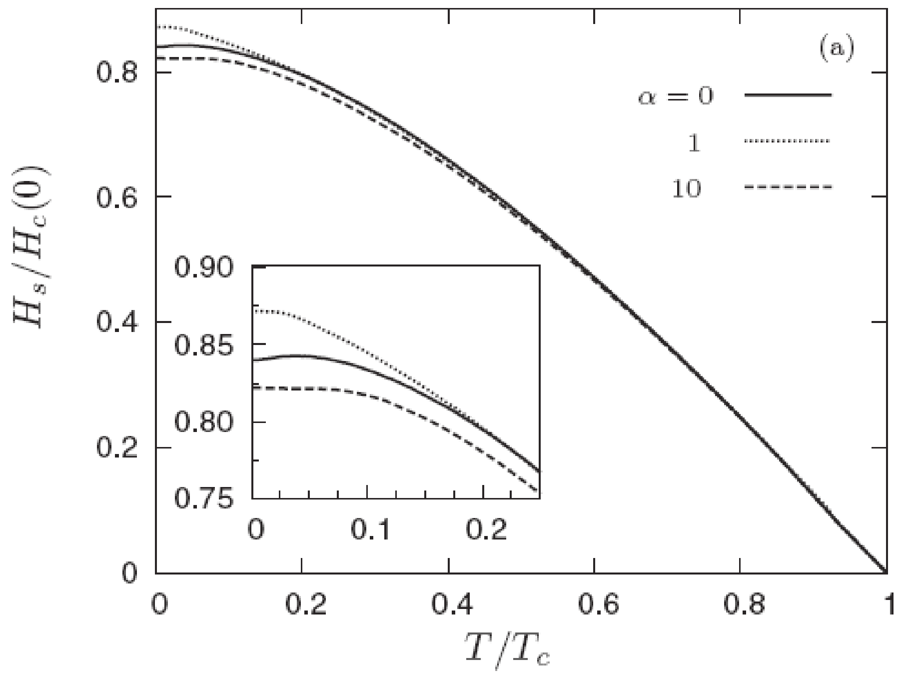
- Penetration and oscillation of vortices under the RF field gives rise to strong dissipation and the surface resistance of the order of R_s in the normal state
- the Meissner state can remain metastable at higher fields, $H > H_{c1}$ up to the superheating field H_{sh} at which the Bean-Livingston surface barrier for penetration of vortices disappears and the Meissner state becomes unstable

H_{sh} is the maximum magnetic field at which a type-II superconductor can remain in a true non-dissipative state not altered by dissipative motion of vortices.

At $H = H_{sh}$ the screening surface current reaches the depairing value $J_d = n_s e\Delta/p_F$

Superheating field: theory

- Calculation of $H_{sh}(T, l)$ for $\kappa \gg 1$ from Eilenberger equations ($0 < T < T_c$):



F. Pei-Jen Lin and A. Gurevich, Phys. Rev. B **85**, 054513 (2012)

$\alpha = \pi\xi_0/l$ Impurity scattering parameter

$$H_{sh} \approx 0.845 H_c$$

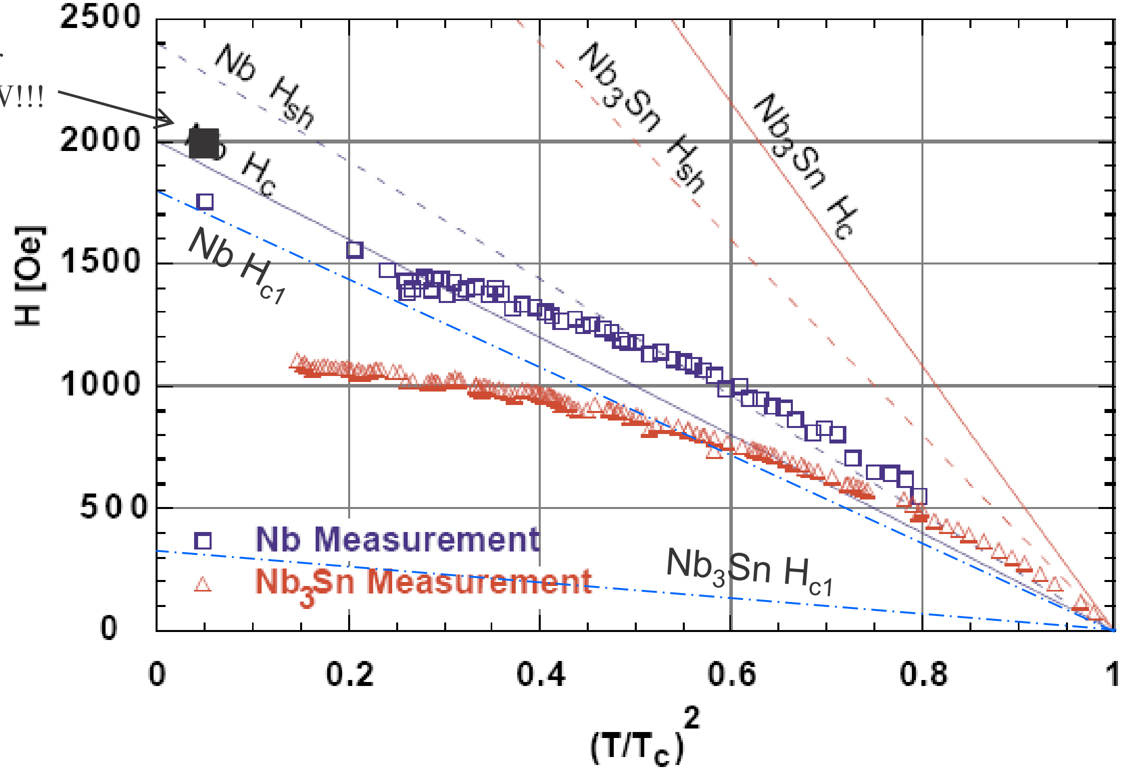
$$H_{sh}(T) \cong H_{sh}(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

- Weak dependence of H_{sh} on non-magnetic impurities

Superheating field: experimental results

- Use high-power (~1 MW) and short (~100 μs) RF pulses to achieve the metastable state before other loss mechanisms kick-in

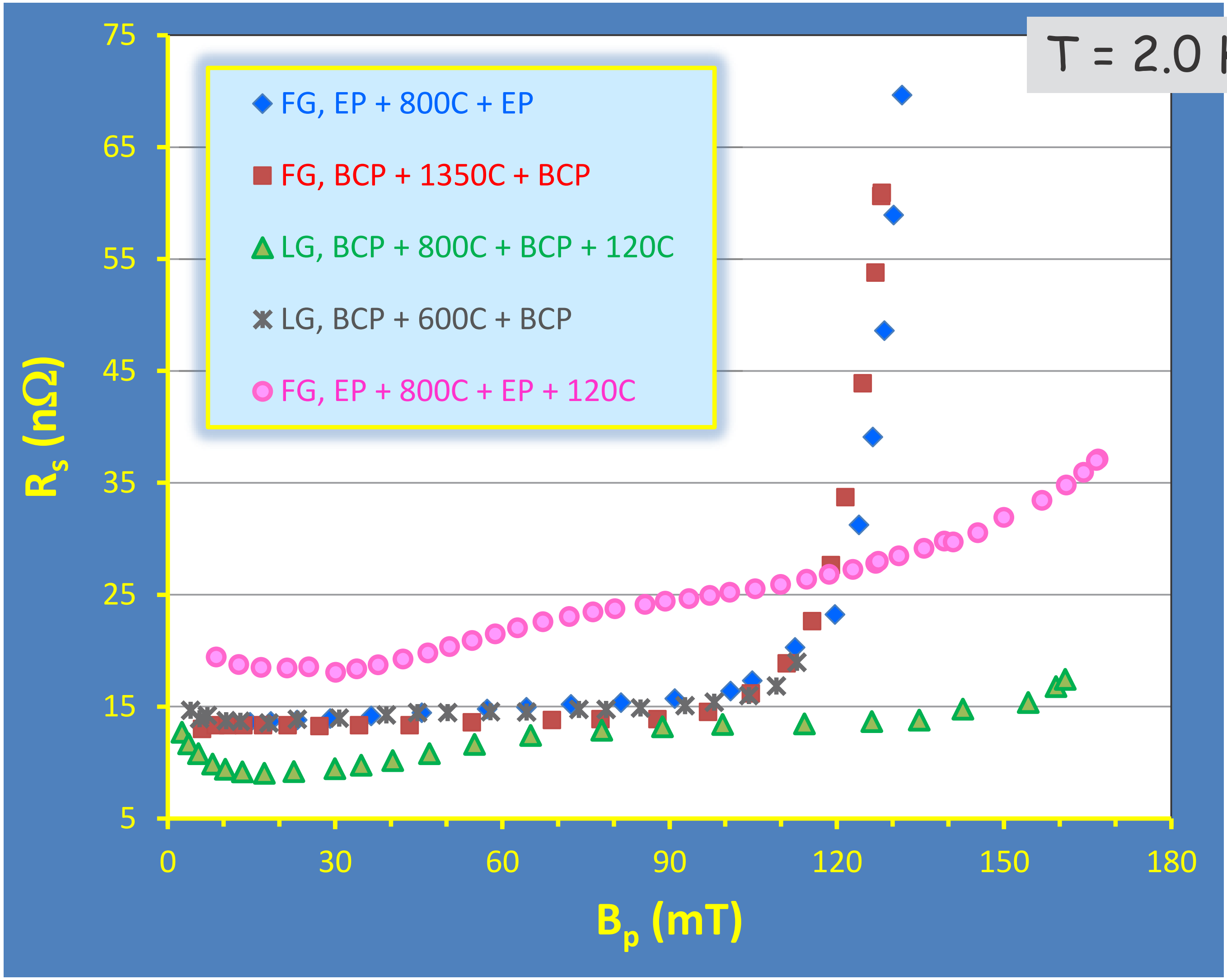
Highest H_p ever measured in CW!!!



T. Hays and H. Padamsee, Proc. 1997 SRF Workshop, Abano Terme, Italy, p. 789 (1997) .

- RF magnetic fields higher than H_{c1} have been measured in both Nb and Nb_3Sn cavities. However max H_{RF} in Nb_3Sn is << predicted H_{sh} at low T

Field dependence of R_s: Experimental results



T = 2.0 K, f = 1.3 GHz

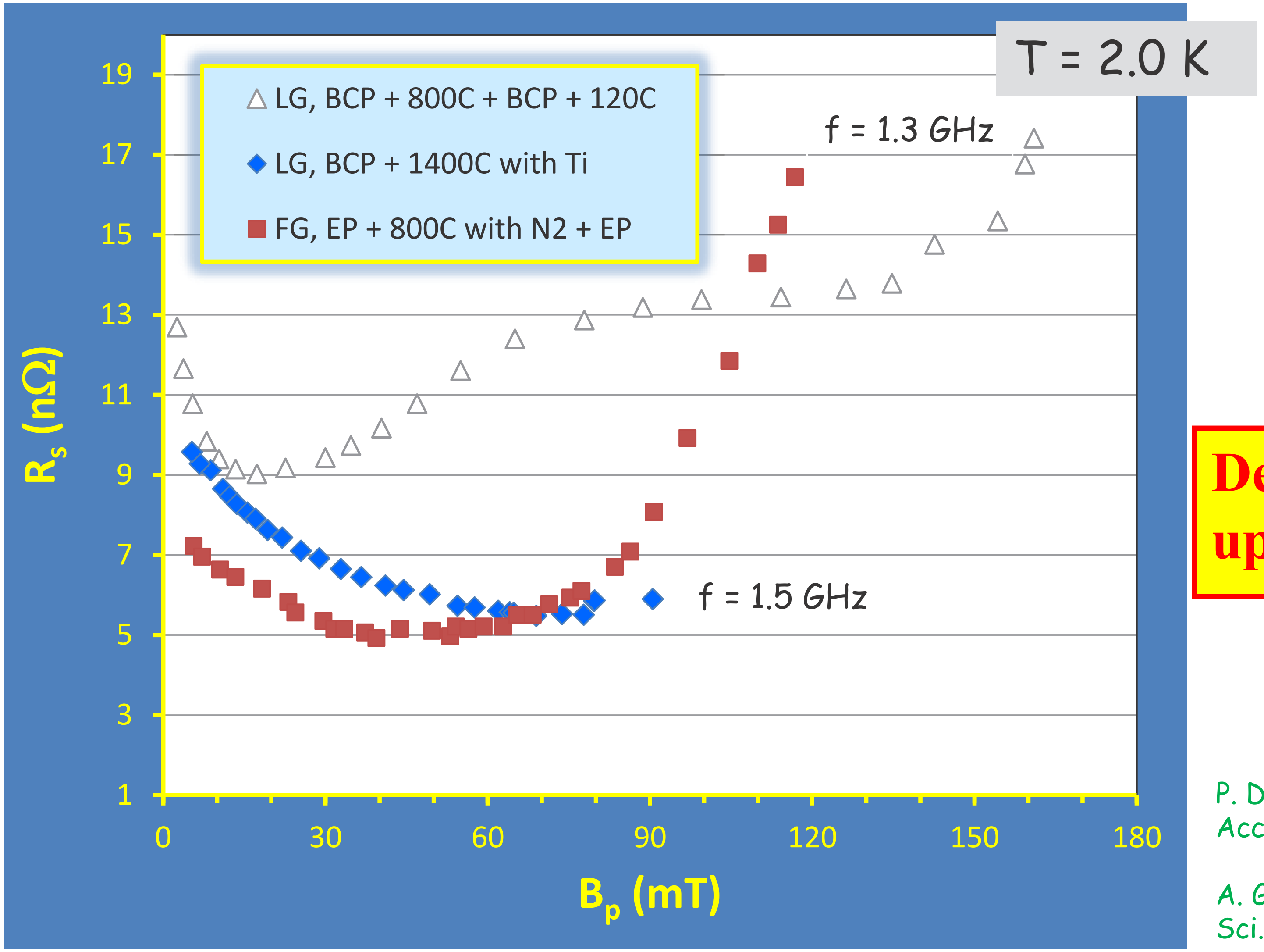
FG: fine grain Nb
LG: large grain Nb

BCP: buffered chemical polishing
EP: electropolishing

800C, 600C, 1350C, 120C: heat treatment temperatures

B. Aune et al., Phys. Rev. STAB **3** (2000) 092001.
R. Geng, SRF'11, p. 74
G. Ciovati, P. Kneisel and G. Myneni, SSTIN10, p. 25.
W. Singer et al., Phys. Rev. ST. Accel. Beams **16** (2013) 012003

Field dependence of R_s: Experimental results



Decreasing $R_s(H)$ up to ~90 mT

P. Dhakal et al., Phys. Rev. ST Accel. Beams **16** (2013) 042001

A. Grassellino et al., Supercond. Sci. Tech. **26** (2013) 102001

Qualitative explanation of Q-rise

- R_s in dirty limit, low *f*, low *T*, $\hbar\omega << kT$ from BCS:

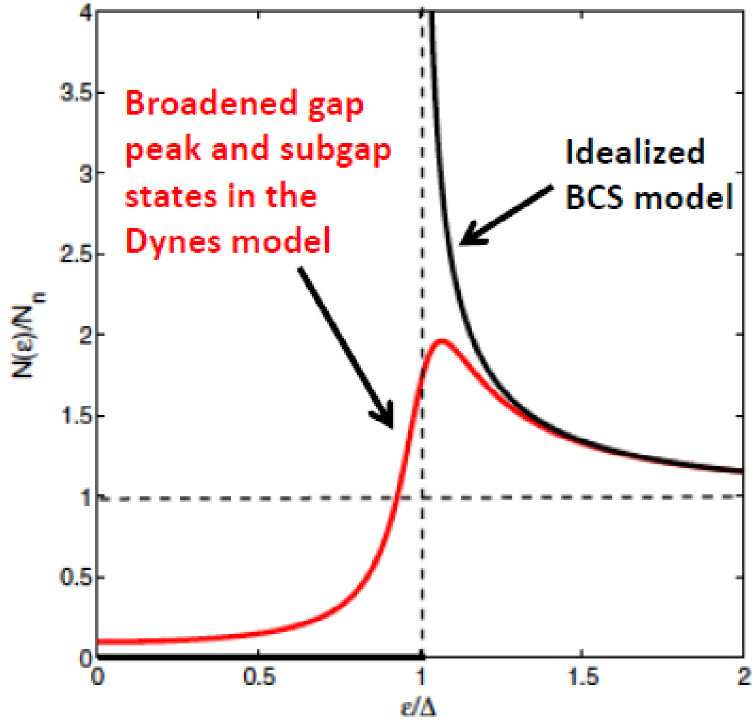
$$R_s = \mu_0^2 \sigma_n \lambda^3 \omega^2 \frac{\Delta}{kT} \ln\left(\frac{9kT}{4\hbar\omega}\right) e^{-\Delta/kT}$$

Comes from singularity in BCS DOS

$$R_s \simeq 2\sigma_n \mu_0^2 \omega^2 \lambda^3 \frac{\Delta}{kT} \int_\Delta^\infty N(\epsilon) N(\epsilon + \hbar\omega) e^{-\epsilon/kT} d\epsilon$$

Density of states with broadened peaks:

$$N(\epsilon) = \mathrm{Re}\frac{\epsilon - i\gamma}{\sqrt{(\epsilon - i\gamma)^2) - \Delta^2}}$$



$$\ln\frac{9kT}{4\hbar\omega} \rightarrow \ln\frac{kT}{\gamma}$$

$$\hbar\omega << \gamma << kT$$

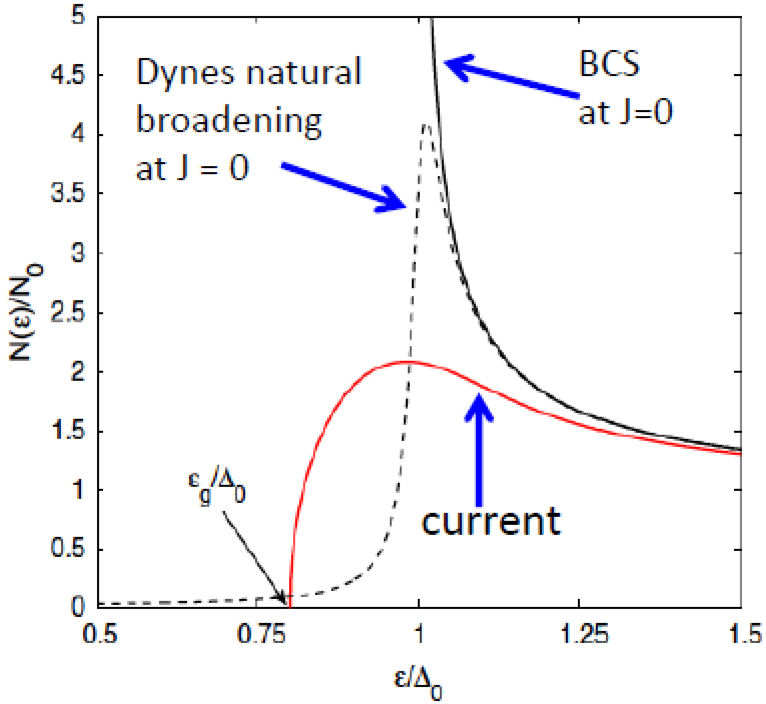
Small broadening of gap peaks in $N(\varepsilon)$ can reduce R_s by 4-5 times

Qualitative explanation of Q-rise

- Current-induced broadening of $N(\varepsilon)$
- Occurs over

$$\delta\varepsilon = \Delta - \varepsilon_g \sim (H/H_c)^{3/4}\Delta \quad \text{for } H \ll H_c$$

- With increasing H, $\delta\varepsilon > \max(\hbar\omega, \gamma)$



$$\ln\frac{kT}{\gamma} \to \ln\frac{kT}{\Delta - \epsilon_g} \sim \ln\left[\frac{T}{T_c}\left(\frac{H_c}{H}\right)^{3/4}\right]$$

- Competition between "natural" and "current-induced" broadening determines the occurrence of Q-rise
- γ depends on surface treatments

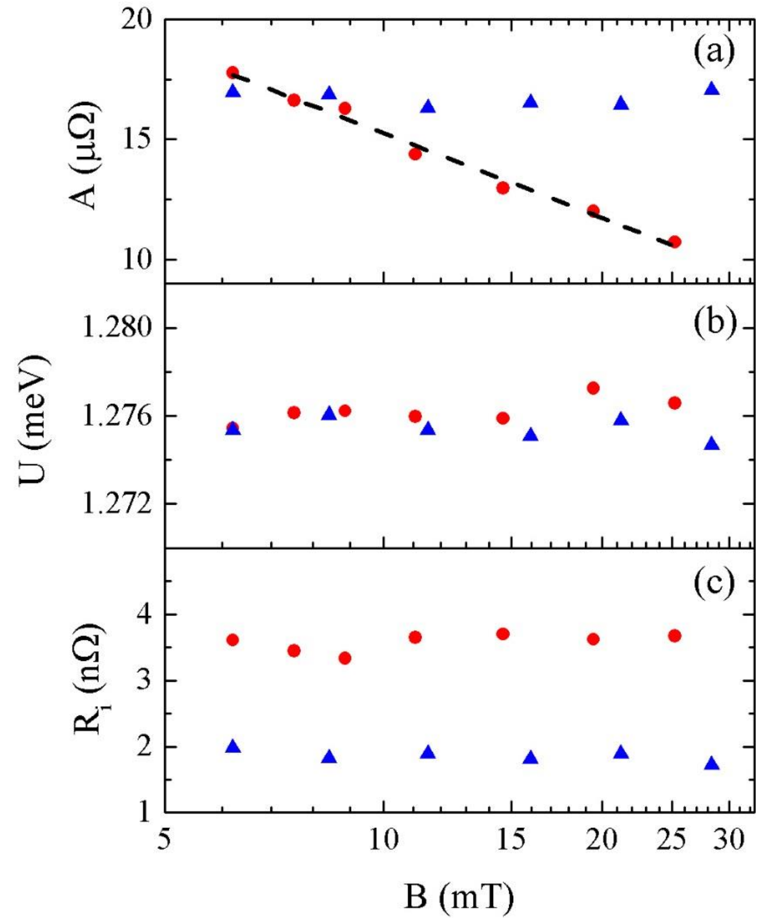
Experimental results

Arrhenius fit of the Ti-doped Jlab data using the generic formula:

$$R_s = A\exp(-U/kT) + R_i$$

where A(H), U(H) and R_i(H) were measured at different fields

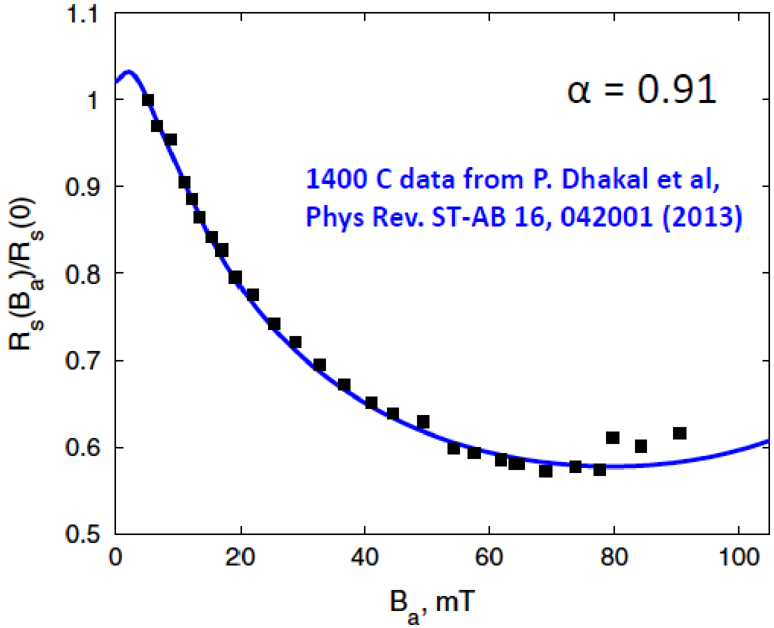
Take into account rf heating to provide stable fit in a wide temperature range



G. Ciovati, P. Dhakal and A. Gurevich, Appl. Phys. Lett. **104**, 092601 (2014)

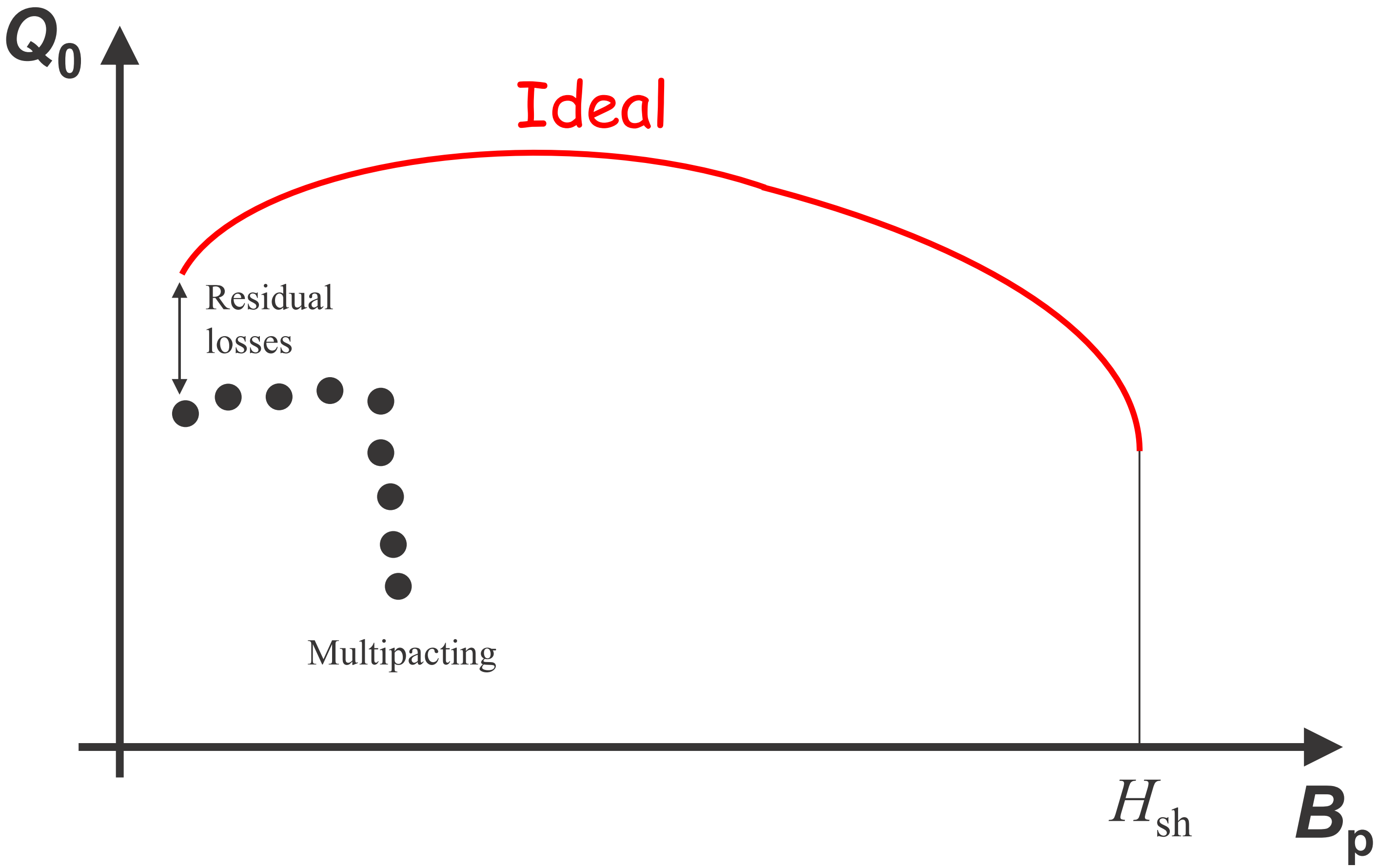
Nonlinear R_s at high-field

- A. Gurevich published a theory of non-linear R_s at high field
- $R_s(H)$ was re-derived from first principles (BCS) taking into account oscillations of $N(\varepsilon, t)$ due to RF current pairbreaking and non-equilibrium distribution function of quasiparticles in the dirty limit



A. Gurevich, *Phys. Rev. Lett.* **113**, 087001 (2014)

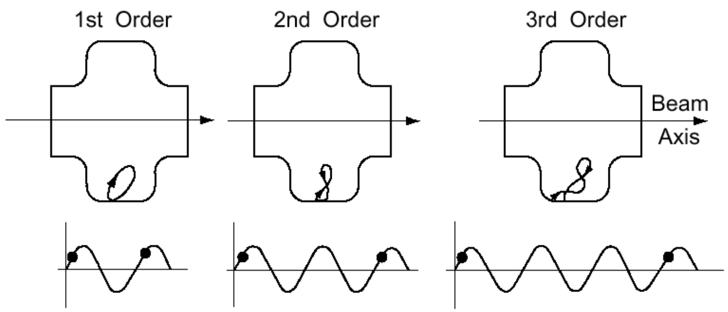
Performance limitations



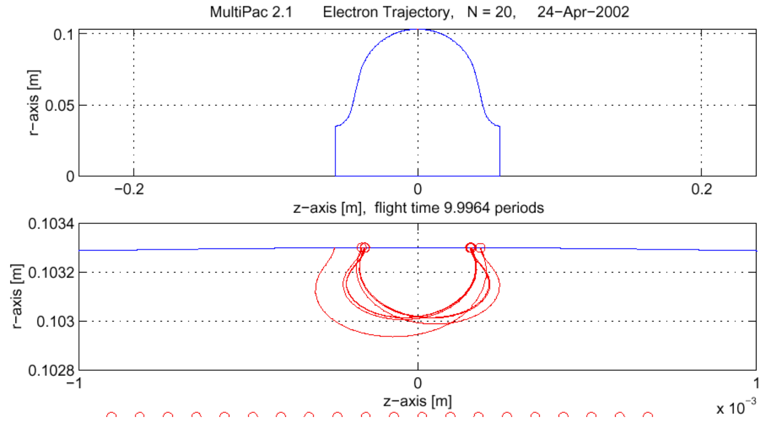
Multipacting

- Multipacting is characterized by an exponential growth in the number of electrons in a cavity
- Multipacting requires 2 conditions:
  - Electron motion is periodic (resonance condition),
    Cavity frequency = $n \times$ cyclotron frequency
  - Impact energy is such that secondary emission coefficient is >1

One-point MP



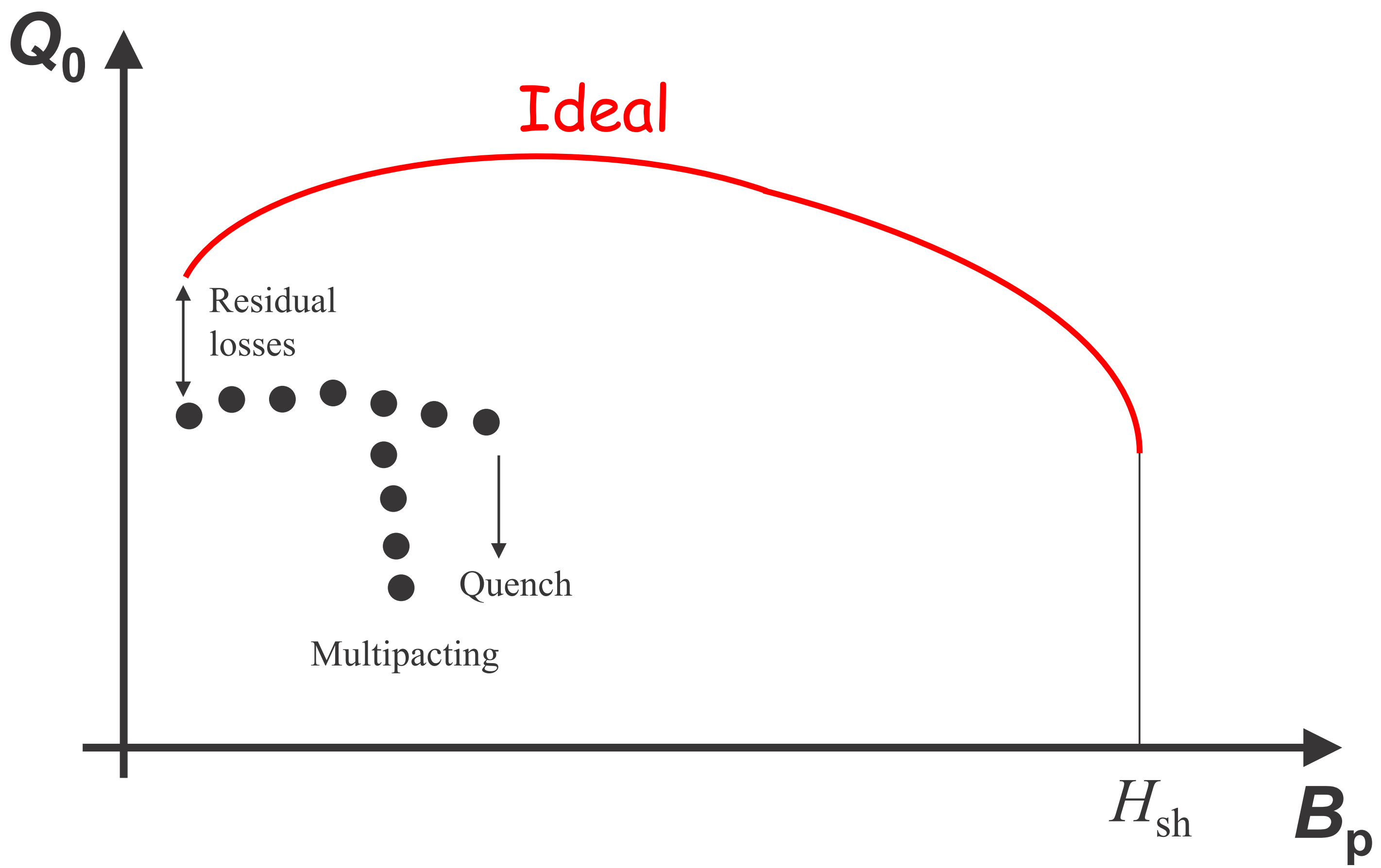
Two-points MP



Cures for Multipacting

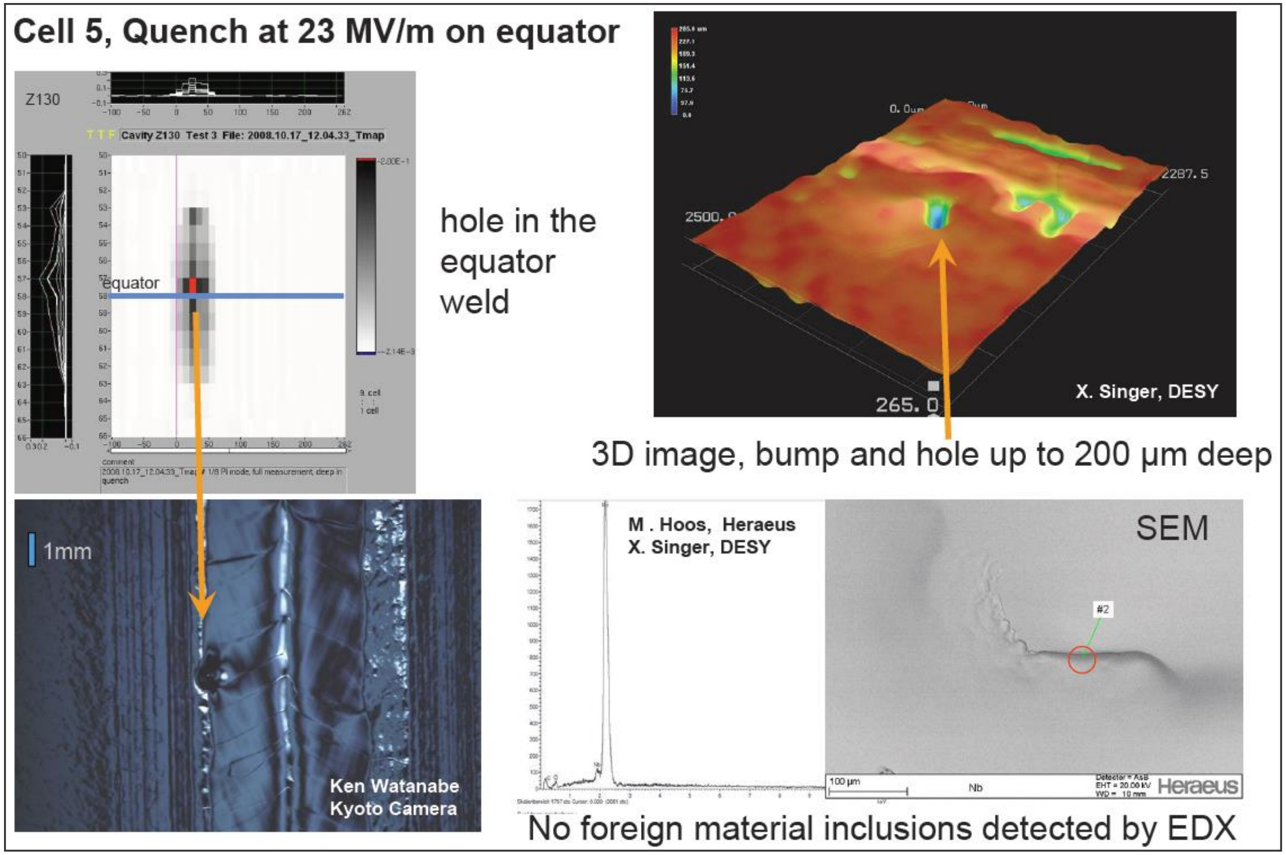
- Cavity design
- Lower SEY: clean vacuum systems (low partial pressure of hydrocarbons, hydrogen and water), Ar discharge
- RF Processing: lower SEY by e^- bombardment (minutes to several hours)

Performance limitations



Quench

- Localized heating at normal-conducting defects
- Local magnetic field enhancement at sharp edges



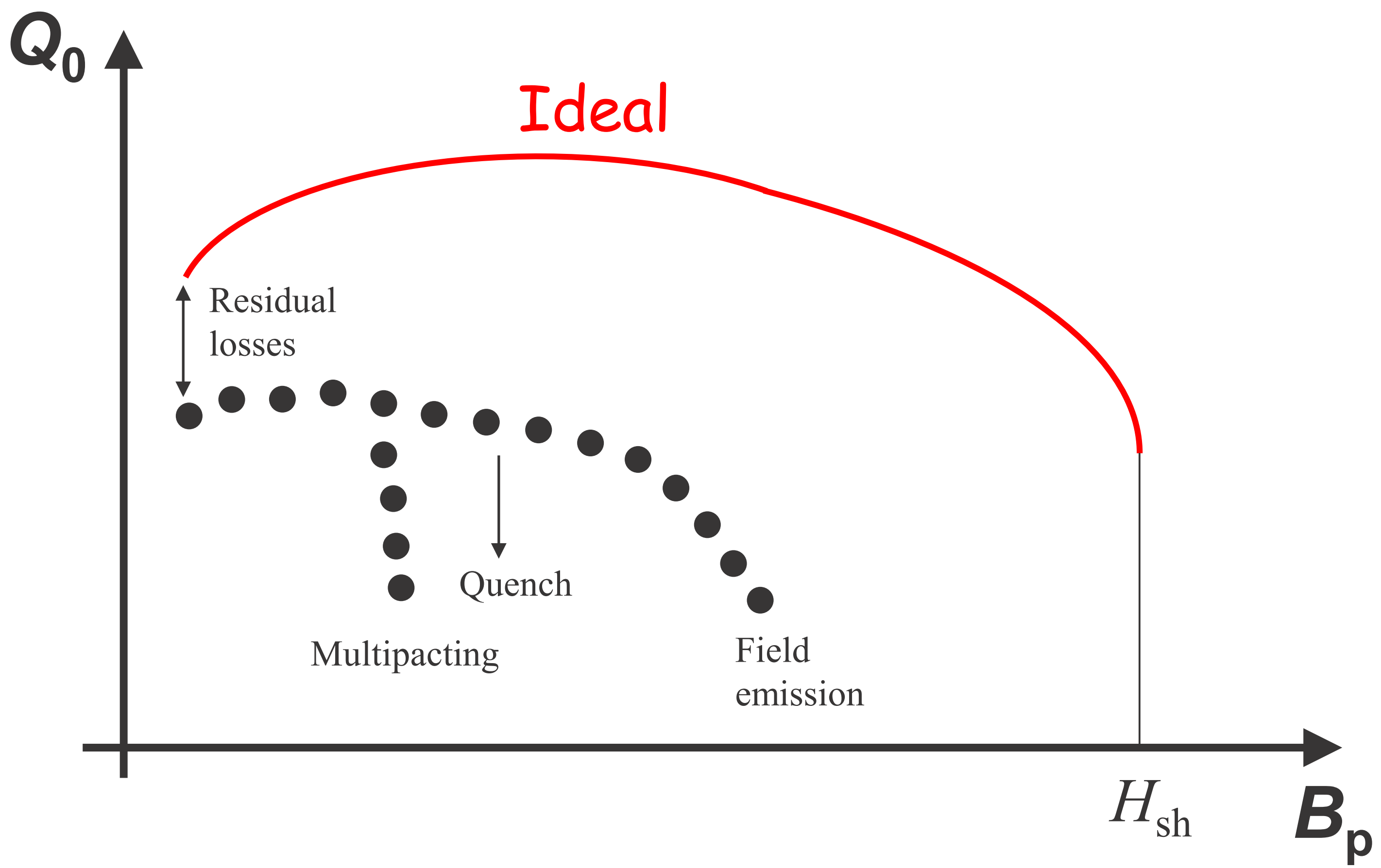
W. Singer, DESY

Cures for Quench

- **Prevention: avoid the defects**
  - High-quality Nb sheets
  - Eddy-current scanning of Nb sheets
  - Great care during cavity fabrication steps

- **Post-treatment:**
  - Thermally stabilize defects by increasing the RRR
  - Remove defects: local grinding

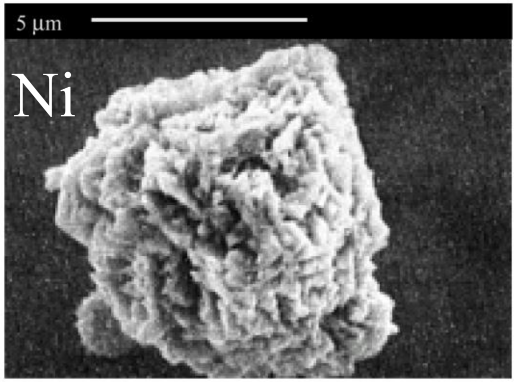
Performance limitations

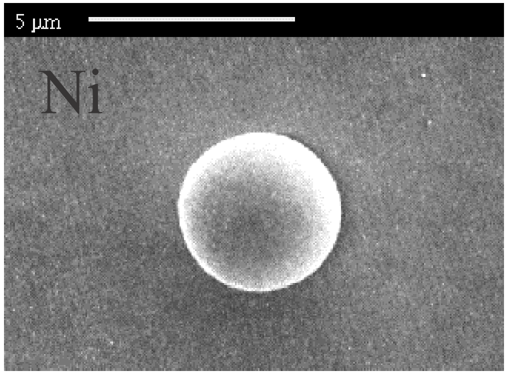


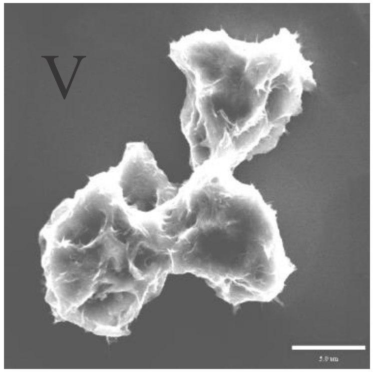
Field emission

- Under high electric field electrons can tunnel out of the surface and accelerated.
- Field-emitted electrons will impact on the cavity wall creating x-rays and heating

- Foreign particulates on the cavity surface





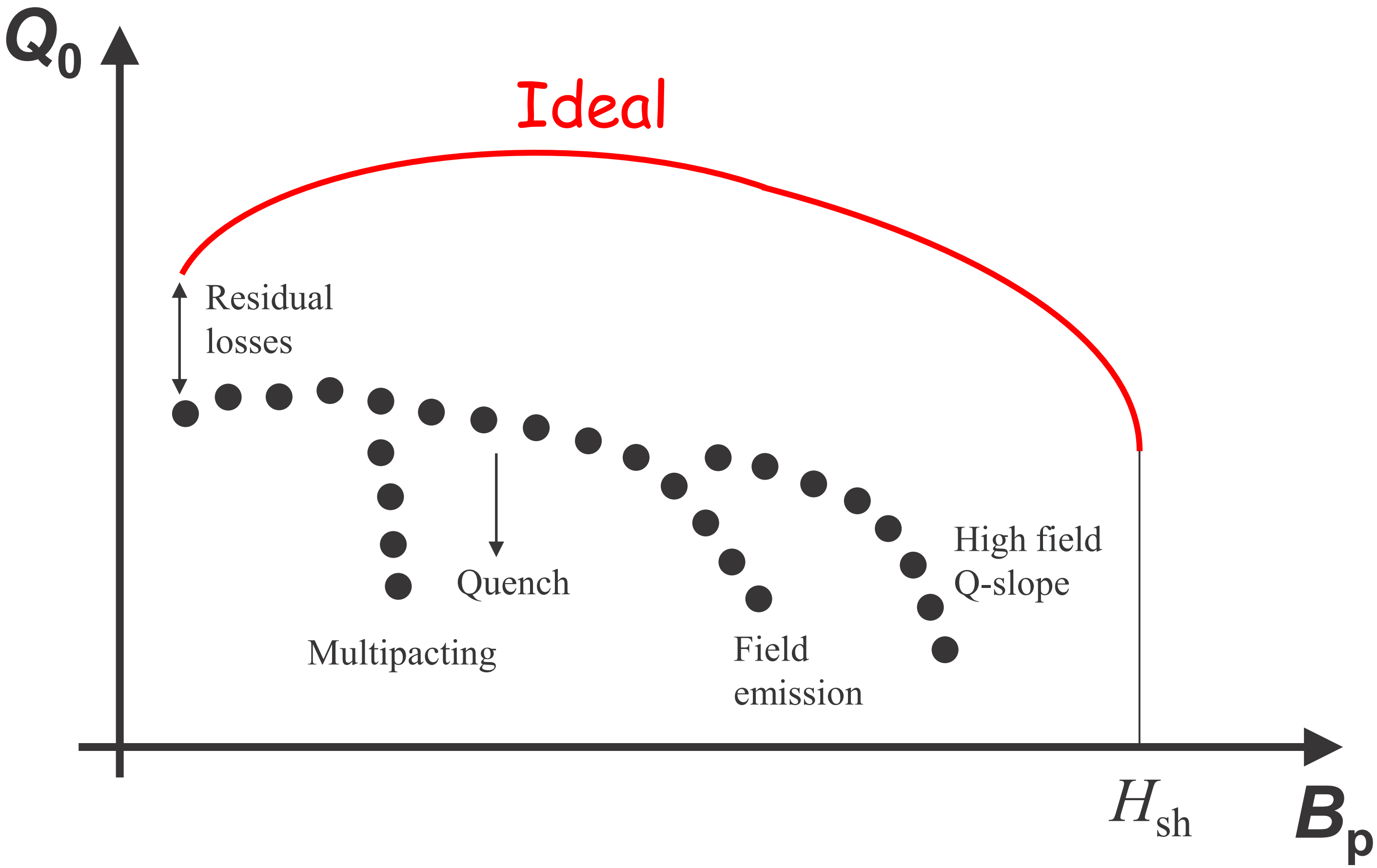


Cures for Field Emission

- **Prevention:**
  - Semiconductor grade acids and solvents
  - High-Pressure Rinsing with ultra-pure water
  - Clean-room assembly
  - Simplified procedures and components for assembly
  - Clean vacuum systems (evacuation and venting without re-contamination)

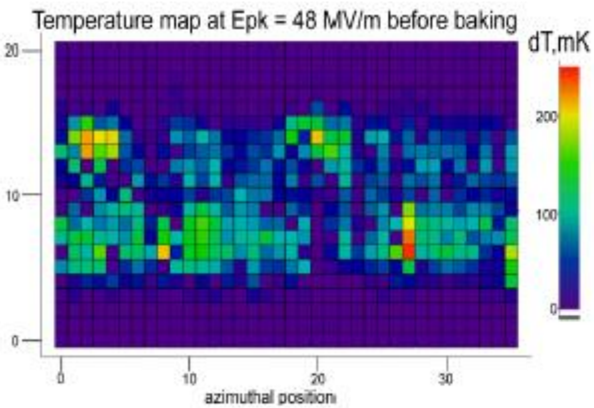
- **Post-processing:**
  - Helium processing
  - High Peak Power (HPP) processing

Performance limitations



High field Q-slope

- Exponential decrease of Q_0 above ~90 mT, without x-rays
- Local heating mostly at the equator (“hot-spots)

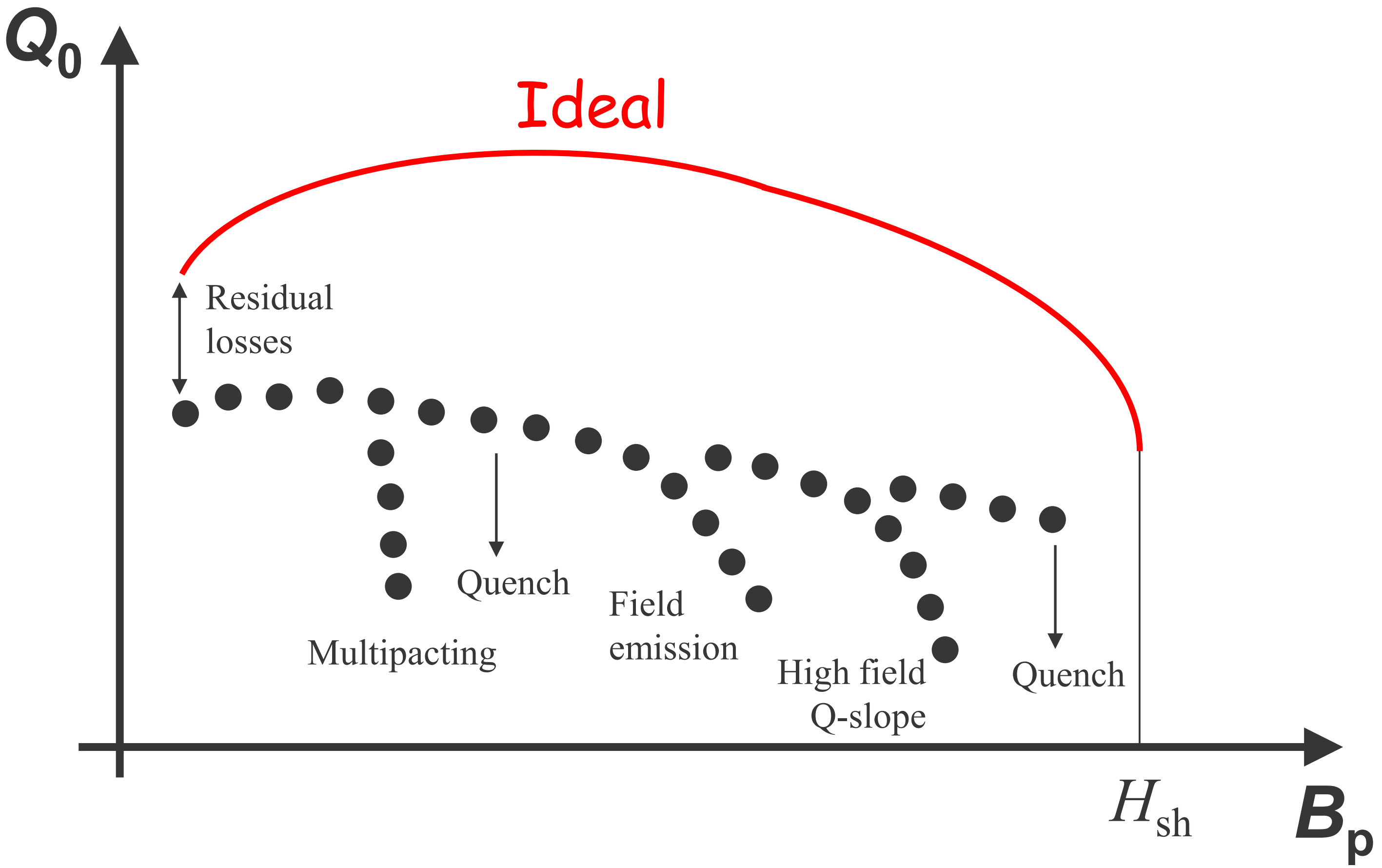


- Many models:
  - Resonant tunneling of electrons between the Nb and localized states in the oxide layer
  - Local magnetic field enhancement
  - Reduced critical field by high concentration of interstitials
  - Hydride nano-precipitates
  - Penetration of vortices

Cure for High Field Q-Slope

- Baking at ~100-140 °C for 24-48 h, typically in high-vacuum but also in air and low-pressure N_2
- This remedy does not work well in fine-grain Nb cavities treated by Buffered Chemical Polishing

Performance limitations



Why is Nb so good for SRF cavities?

- Low surface resistance, including low residual resistance.
- S-wave Cooper pairing with a full superconducting gap on the entire Fermi surface.
- High lower critical magnetic field H_{c1} and superheating magnetic H_s
- High thermal conductivity.
- Grain boundaries transparent to high RF screening currents in polycrystals.
- Minimal degradation of superconducting properties by local chemical non-stoichiometry and precipitation of non-superconducting second phases.
- Good formability

Recommended literature

- M. Tinkham, *Introduction to Superconductivity*, McGraw-Hill, New York, 2nd edition, 1996
- H. Padamsee, J. Knobloch and T. Hays, *RF Superconductivity for Accelerators*, J. Wiley & Sons, New York, 1998
- H. Padamsee, *RF Superconductivity: Science, Technology and Applications*, WILEY-VCH Verlag, Weinheim, 2009
- A. Gurevich, "Superconducting Radio-Frequency Fundamentals for Particle Accelerators", *Rev. Accel. Sci. Tech*. **5**, 119 (2012)
- A. Gurevich, "Theory of RF superconductivity for resonant cavities", *Supercond. Sci. Technol.* **30**, 034004 (2017)

Thank you for your attention!