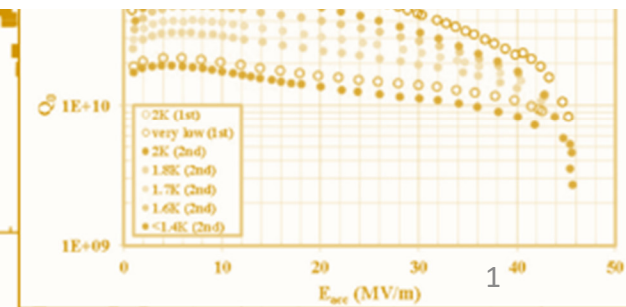
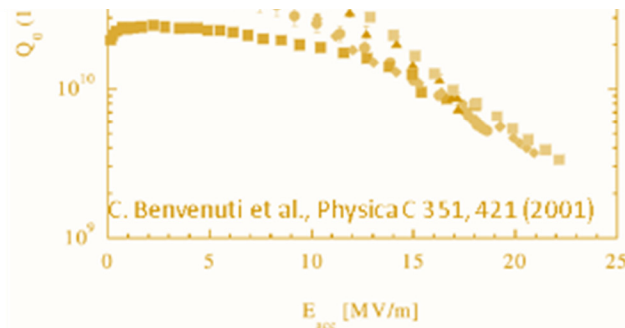


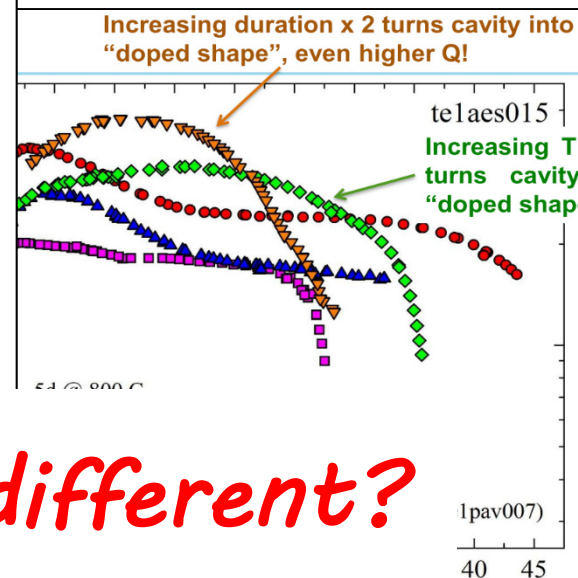
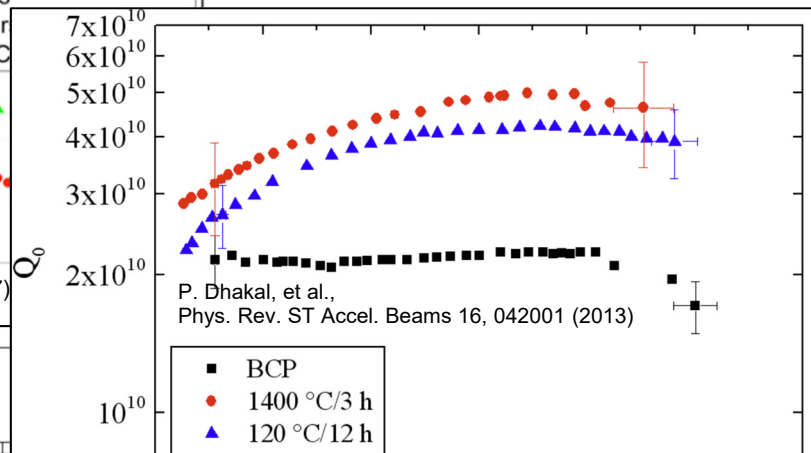
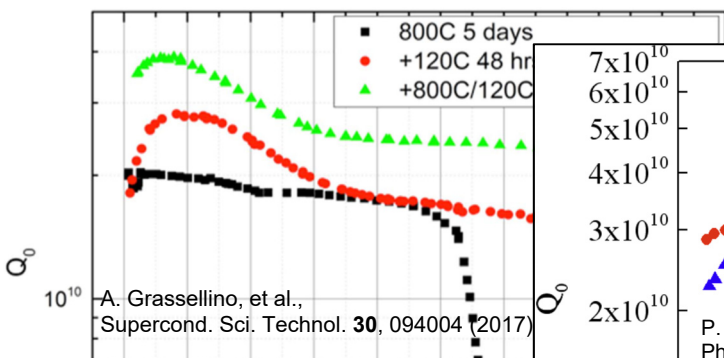
Field-dependent nonlinear surface resistance and its optimization by surface nano-structuring of SRF cavities

T. Kubo (KEK, ODU) and A. Gurevich (ODU)

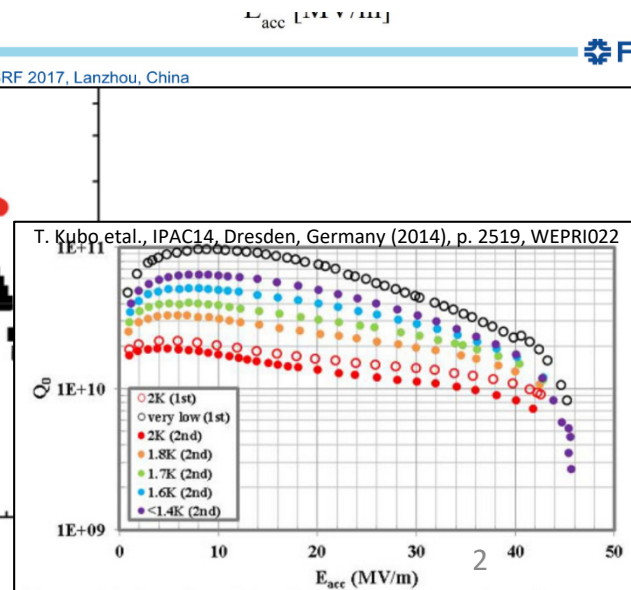
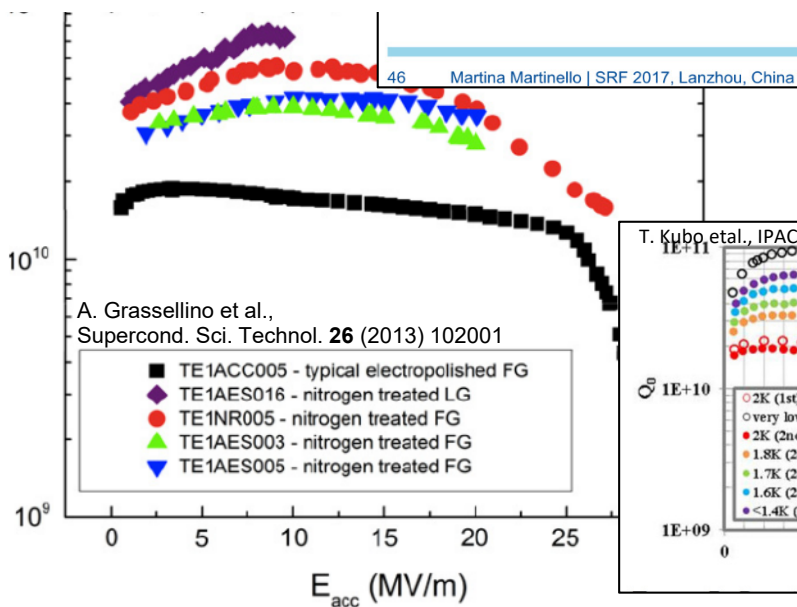
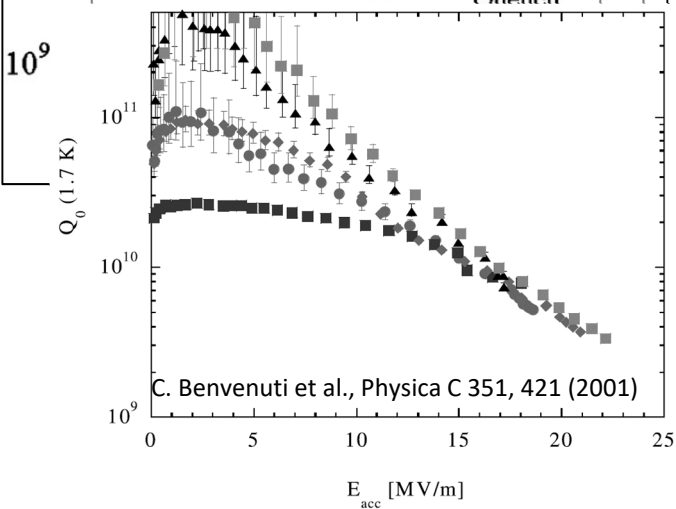
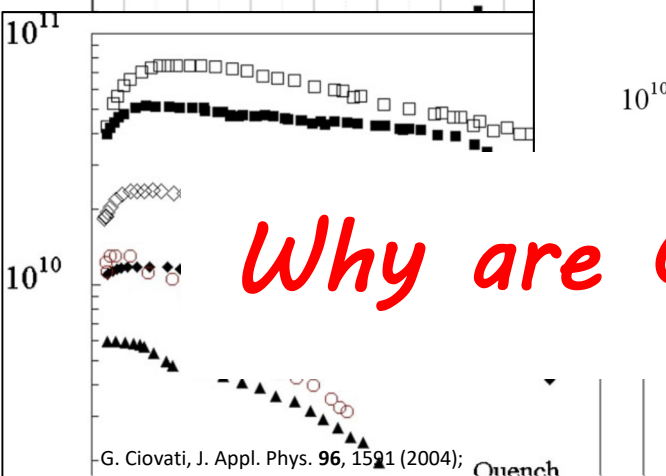
Supported by JSPS KAKENHI Grant No.17H04839, and No.17KK0100

Supported by NSF under Grant No.PHY-1416051

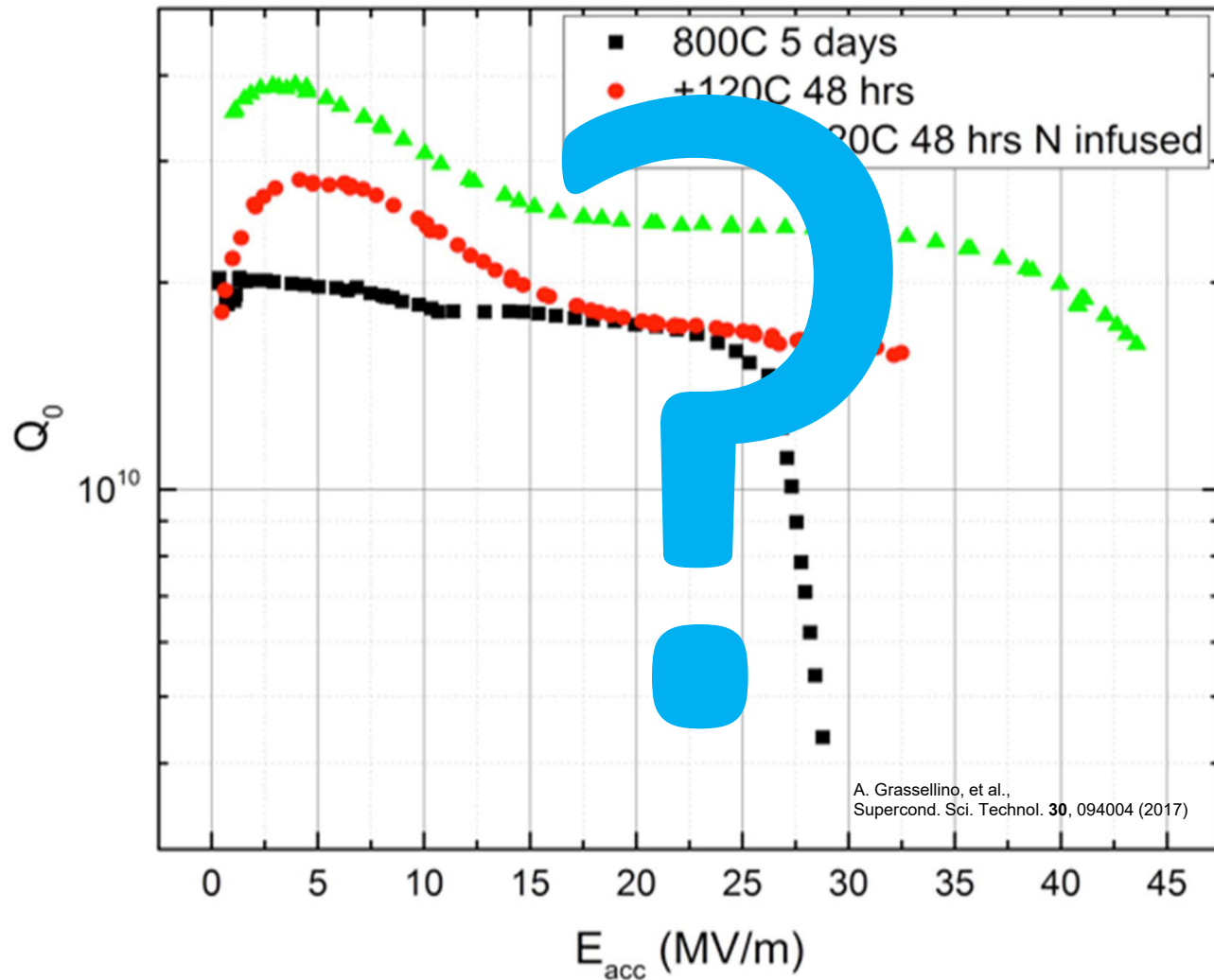




Why are Q-E curves so different?

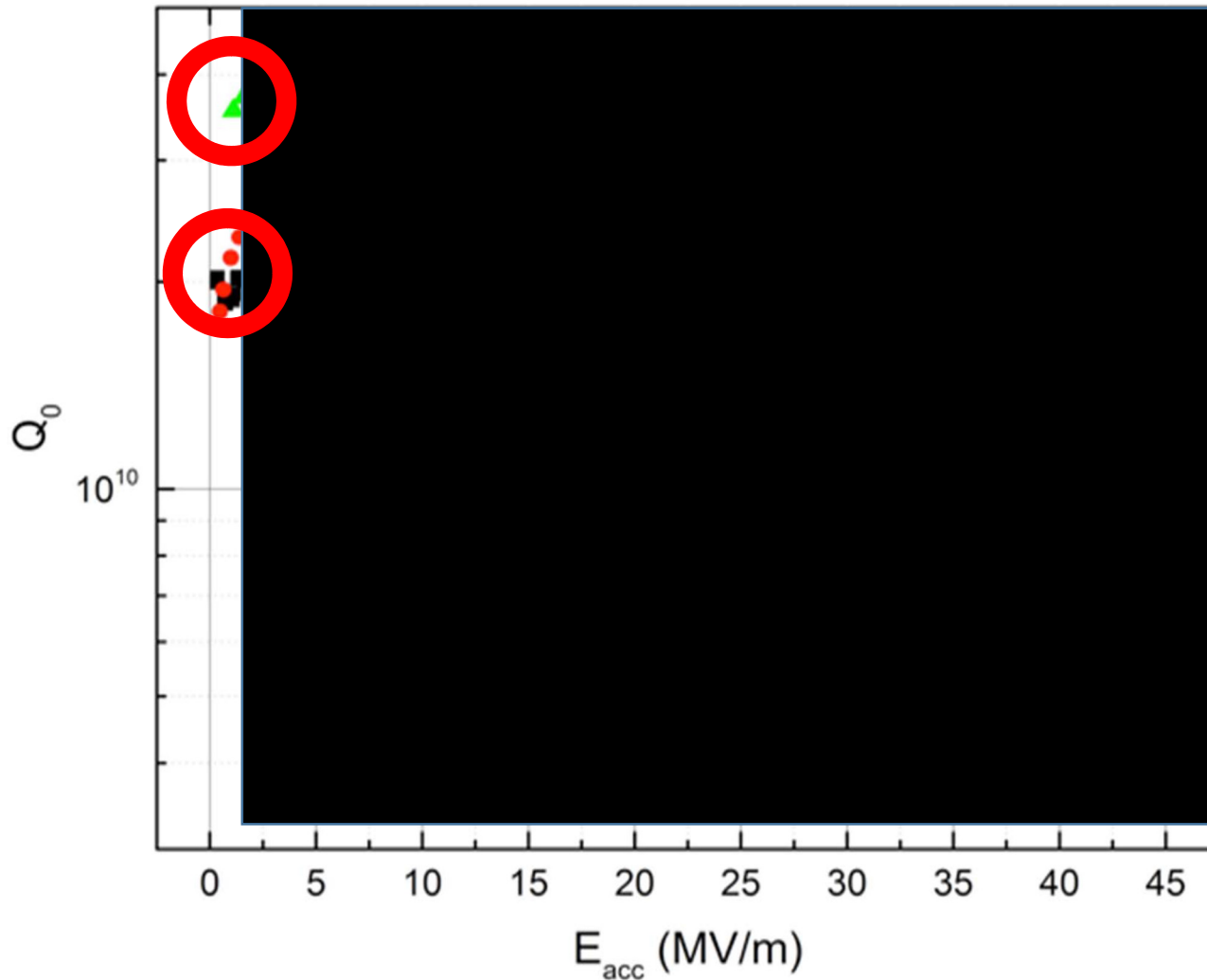


Unfortunately, the Mattis-Bardeen's theory for the weak-field R_s tells you nothing about the shape of Q - E curve.



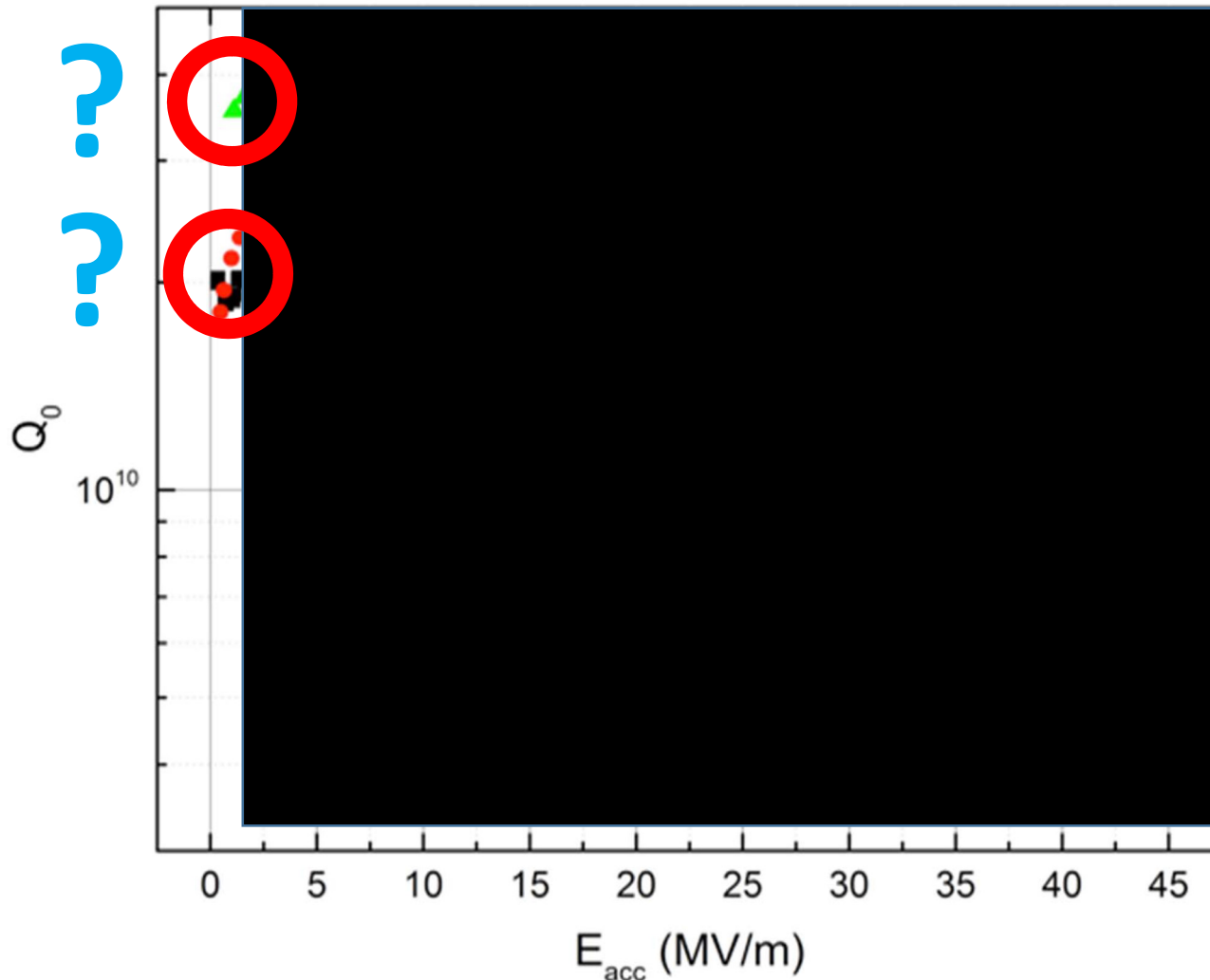
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- It is valid only at the weak-field limit: the left end of Q-E curve.



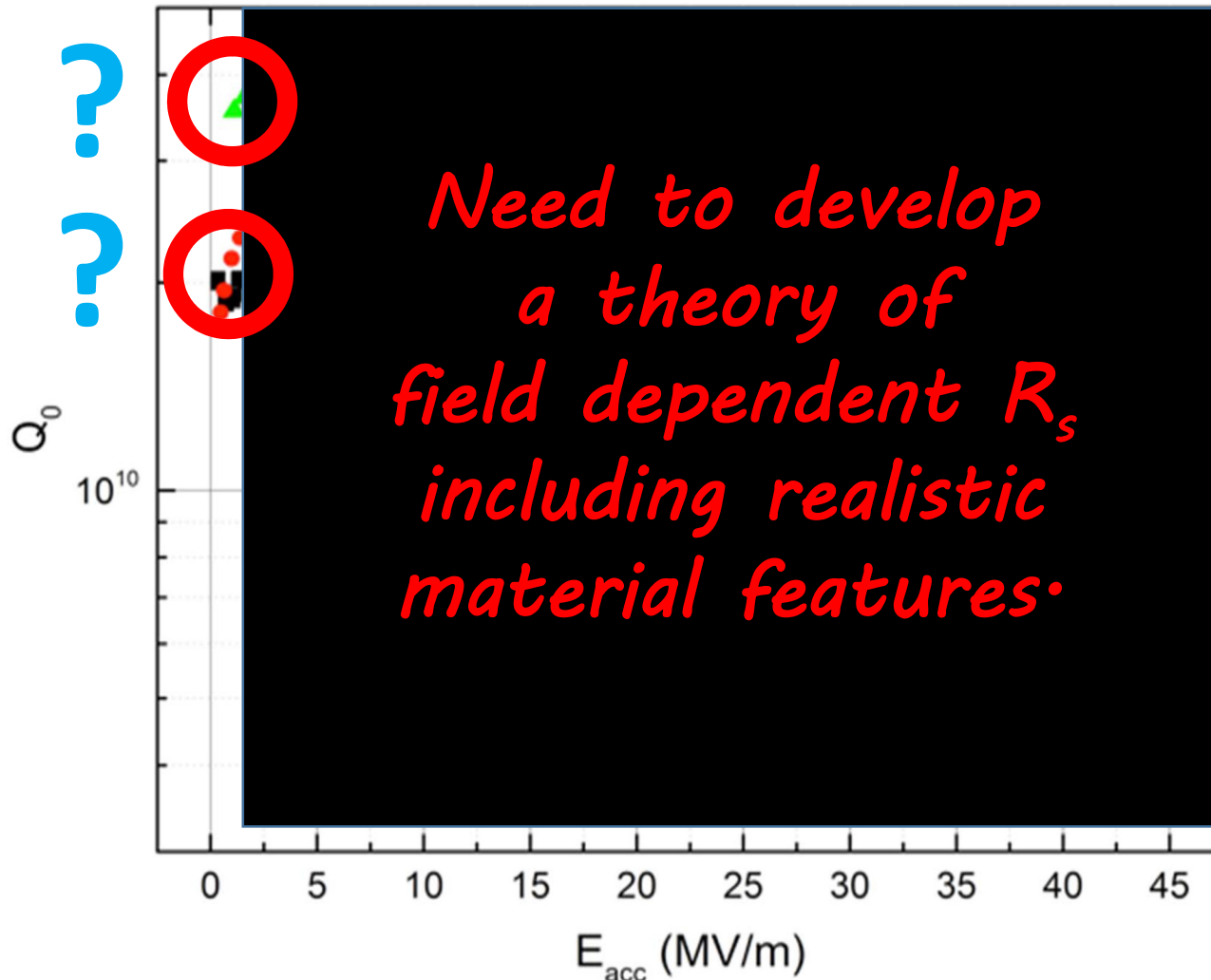
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Part 1: Review

Effects of density of states (DOS) broadening

- A. Gurevich,
Phys. Rev. Lett. **113**, 087001 (2014)
- A. Gurevich and T. Kubo,
Phys. Rev. B **96**, 184515 (2017)

Review (1)

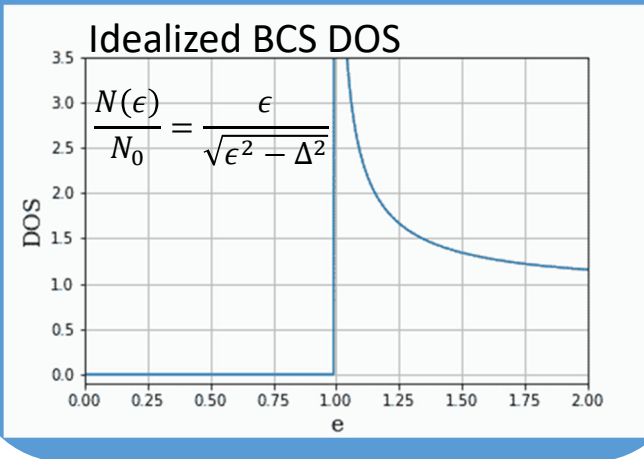
The surface resistance is given by

$$R_s = \frac{1}{2} \mu_0^2 \omega^2 \lambda^3 \sigma_1$$

Here σ_1 is roughly (when $T \ll T_c$ and $\omega \ll T$)

$$\sigma_1 \sim \sigma_n \int_{\Delta}^{\infty} \underbrace{N(\epsilon)}_{\text{DOS}} N(\epsilon + \hbar\omega) e^{-\frac{\Delta}{kT}} d\epsilon$$

Weak-field limit



Mattis Bardeen's formula

$$\sigma_1 = \sigma_n \frac{2\Delta}{kT} \ln \frac{CkT}{\hbar\omega} e^{-\frac{\Delta}{kT}}$$

comes from this DOS

J. Bardeen, Rev. Mod. Phys. 34, 667 (1962).

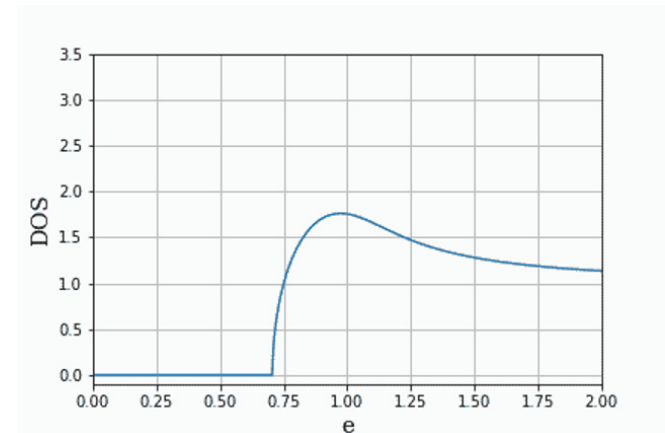
K. Maki, Prog. Theor. Phys. 29, 333 (1963)

P. Fulde, Phys. Rev. 137, A783 (1965).

A. Anthore, H. Pothier, D. Esteve, Phys. Rev. Lett. 90, 127001 (2003).

However, it is well known that the DOS is affected by the pair-breaking current.

DOS under a dc current



Review (1)

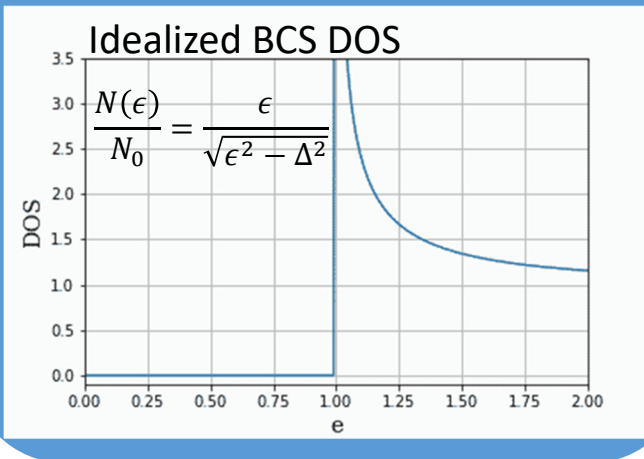
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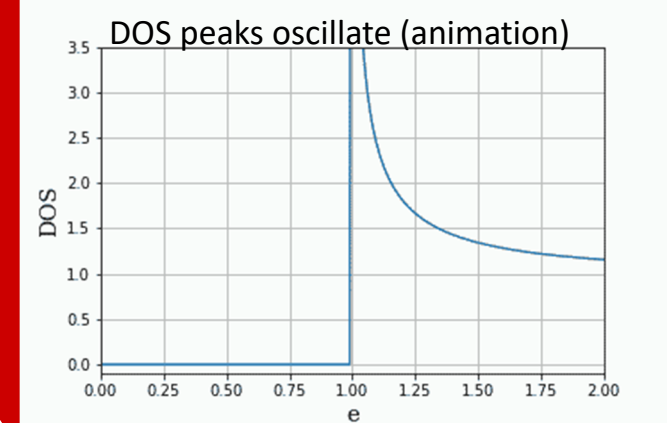
K. Maki, Prog. Theor. Phys. 29, 333 (1963)

P. Fulde, Phys. Rev. 137, A783 (1965).

A. Anthore, H. Pothier, D. Esteve, Phys. Rev. Lett. 90, 127001 (2003).

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Under a strong rf current



Review (1)

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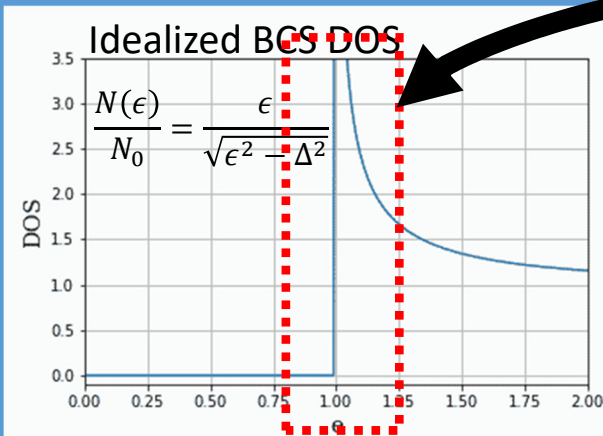
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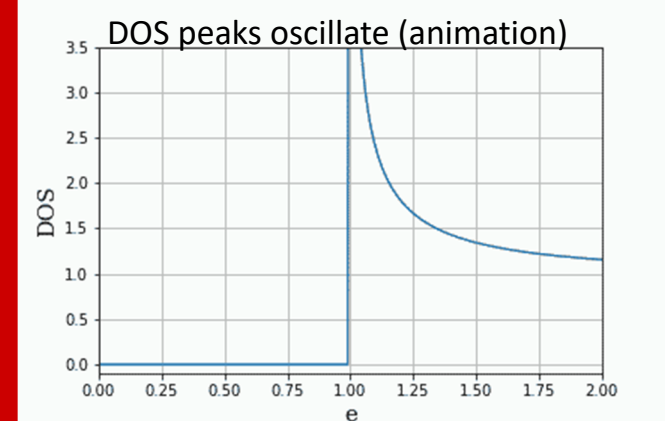
Weak-field limit



$$\sigma_1 = \sigma_n \frac{2\Delta}{kT} \ln \frac{CkT}{\hbar\omega} e^{-\frac{\Delta}{kT}}$$

This logarithmic factor in MB's formula comes from the sharp peak of the idealized BCS DOS

Under a strong rf current



Mattis Bardeen's formula

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Review (1)

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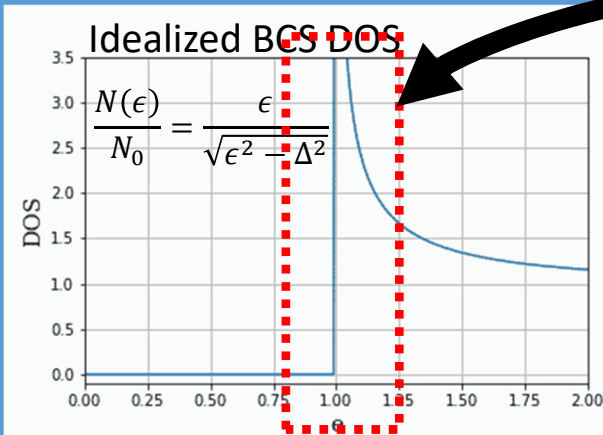
K. Maki, Prog. Theor. Phys. 29, 333 (1963)

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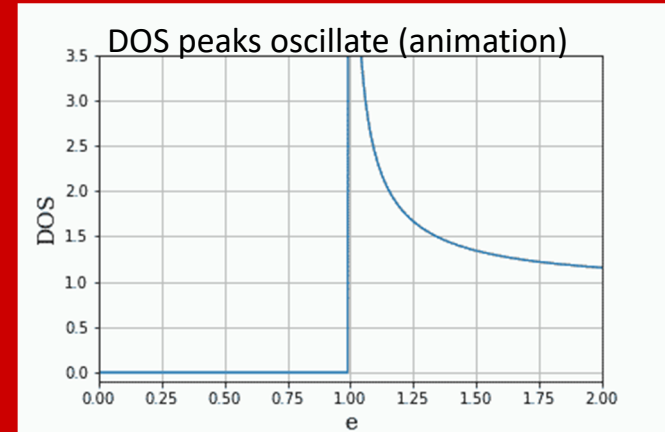
Weak-field limit



$$\sigma_1 = \sigma_n \frac{2\Delta}{kT} \ln \frac{CkT}{\hbar\omega} e^{-\frac{\Delta}{kT}}$$

decreases $\downarrow$ CkT peak width

Under a strong rf current

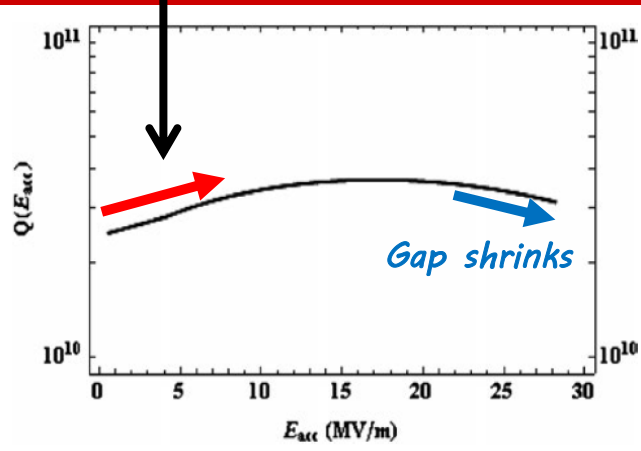


Mattis Bardeen's formula

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comes from this DOS

Broadening of DOS peaks causes the Q rise

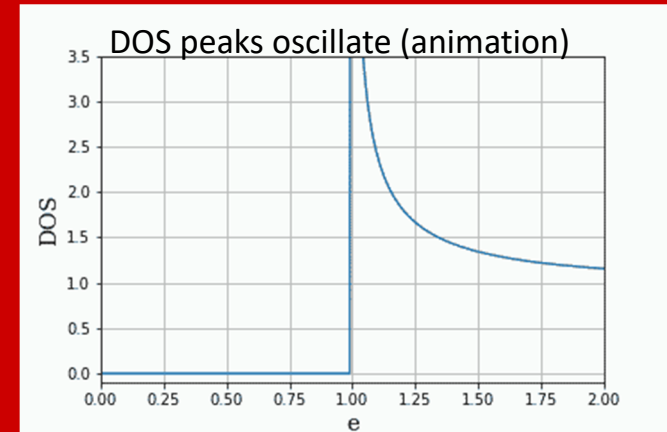


$$\sigma_1 = \sigma_n \frac{2\Delta}{kT} \ln \frac{CkT}{\hbar\omega} e^{-\frac{\Delta}{kT}}$$

decreases

$\ln \frac{CkT}{\text{peak width}}$

Under a strong rf current



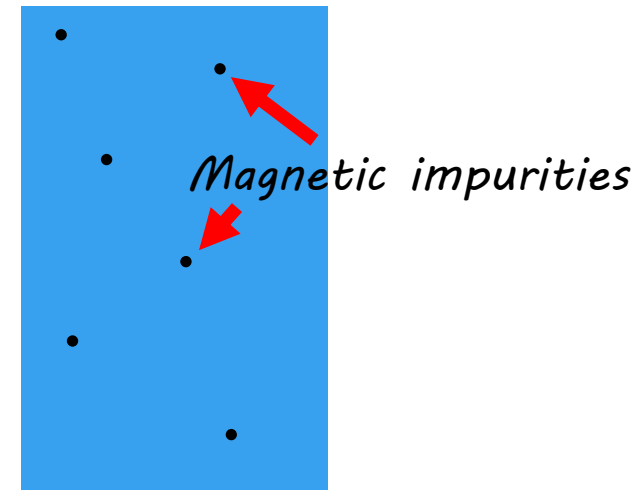
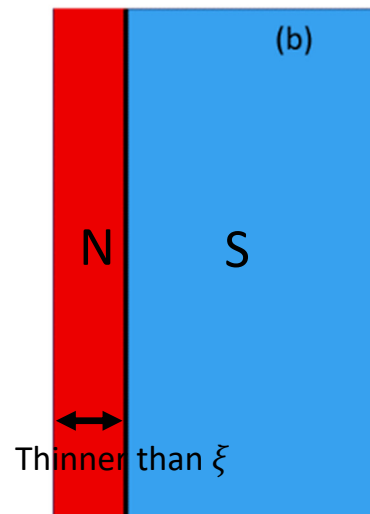
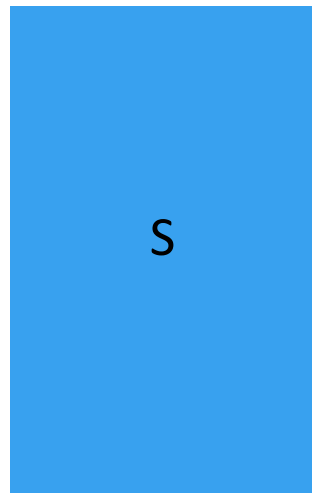
The extended Q-rise is not an exotic but the behavior which follows from the BCS model with the idealized DOS!

Review (2)

Other mechanisms which broaden DOS also affect R_s .

→ We Incorporated effects of pair-breaking mechanisms originating from realistic material features into R_s at the weak-field limit.

- Subgap states originating from a finite quasiparticle lifetime.
- Proximity coupled thin Normal layer on the surface
- Small density of magnetic impurities

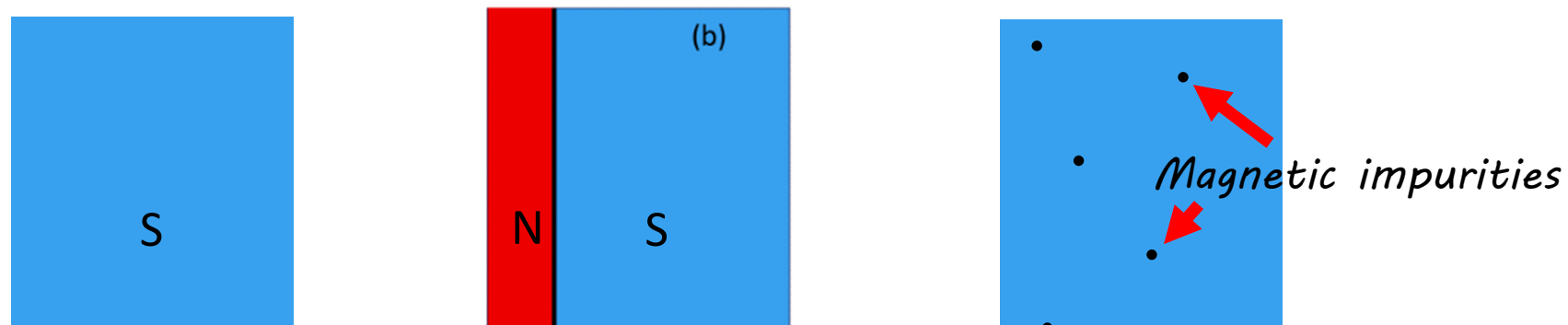


Review (2)

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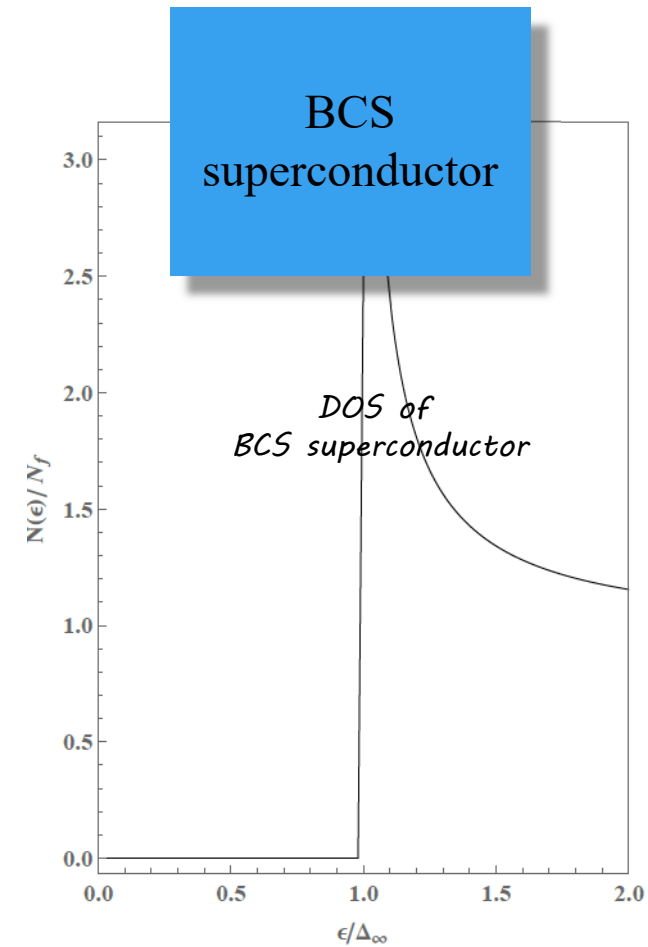
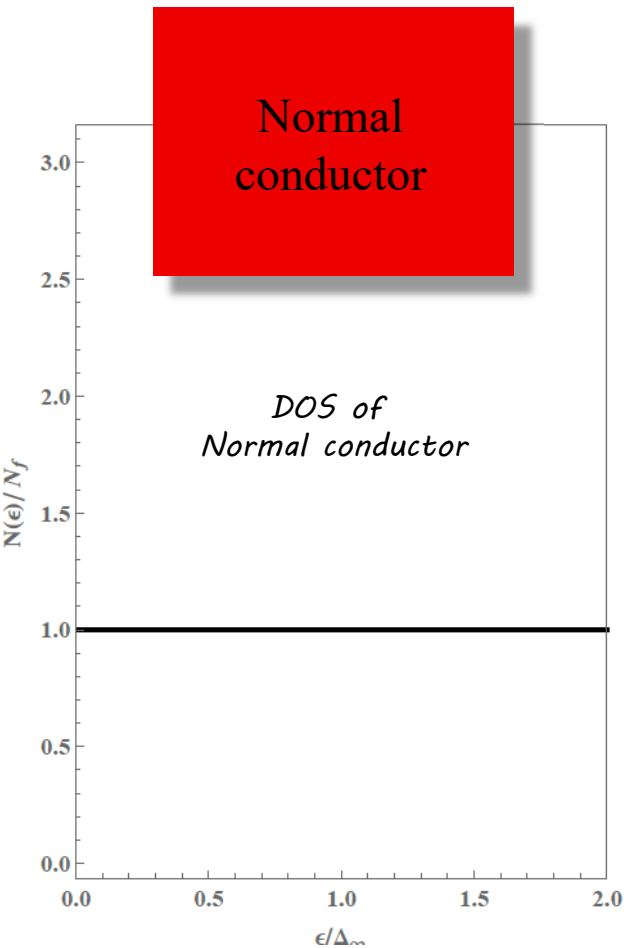
- Subgap states originating from a finite quasiparticle lifetime.
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- Small density of magnetic impurities



These structures model realistic surfaces of superconducting materials which can contain **oxide layers, hydrides, absorbed impurities or nonstoichiometric composition.**

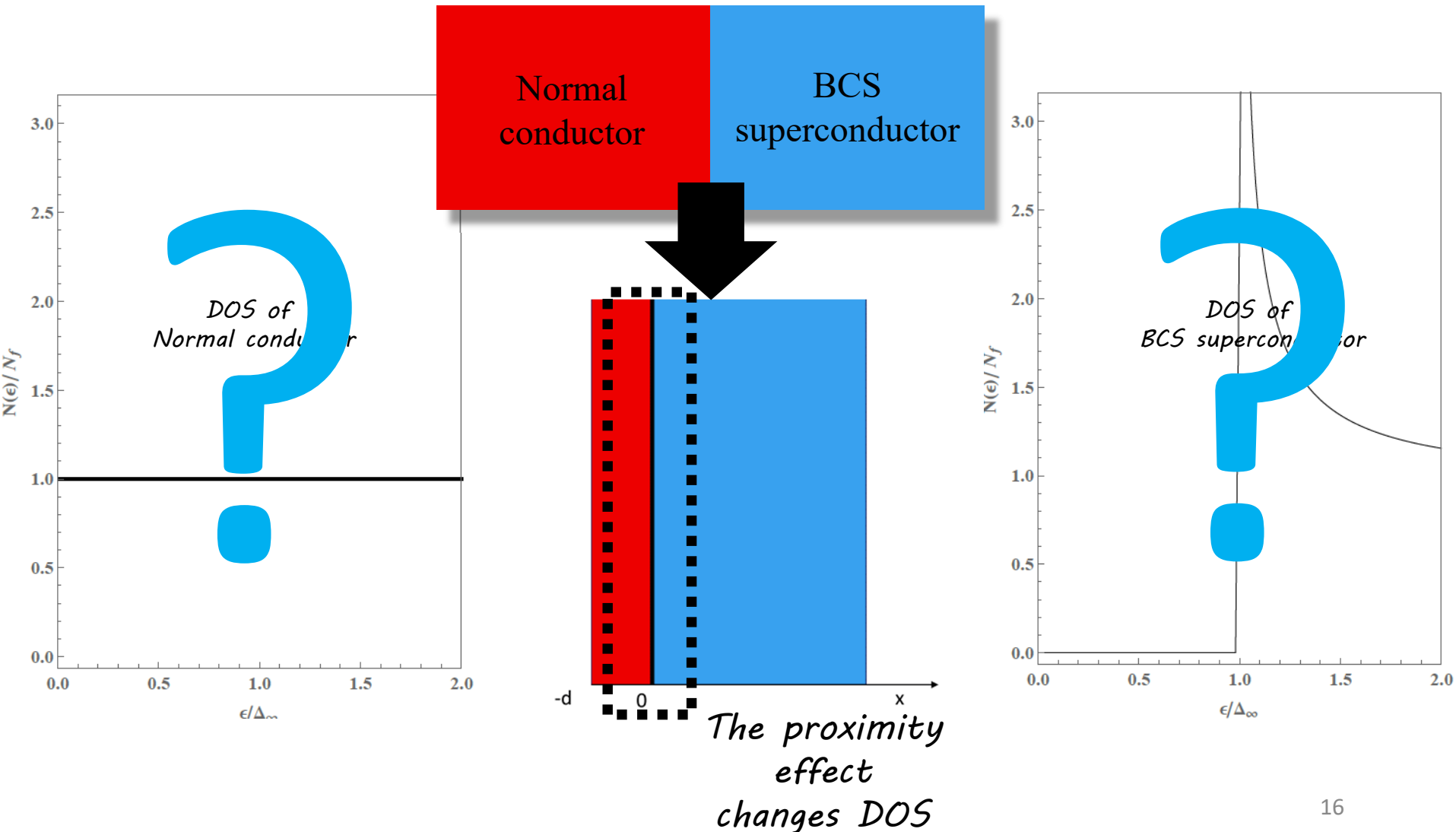
Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)



Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)



Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)

Parameters are sensitive to material processing!

d is an N layer thickness.

(e.g., thickness of suboxide on the Nb surface)

$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

~thickness

$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

~barrier
between N&S

R_B is an interface resistance

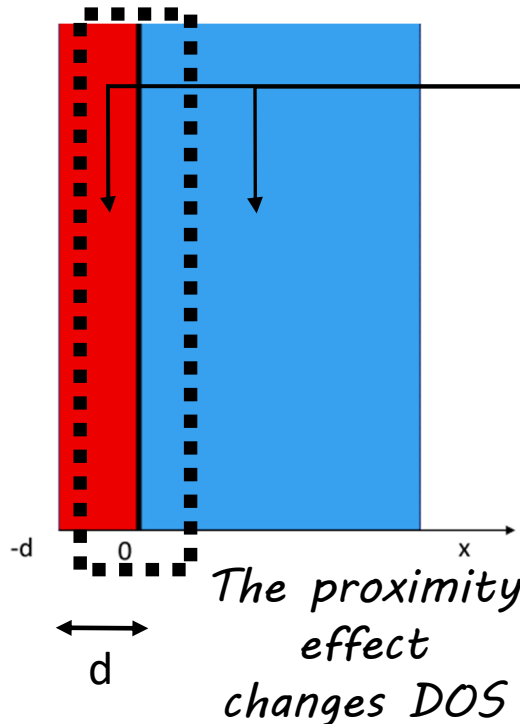
Sensitive to heat treatment

(e.g., between Nb suboxide and Nb)

Ref: The lowest contact resistance of YBCO/Ag is $R_B \sim 10^{-13} \Omega \cdot m^2$

J. W. Ekin et al., Appl. Phys. Lett. **62**, 369 (1993)

We can calculate DOS by using the well-established method: Quasiclassical Green's function formalism of the BCS theory.

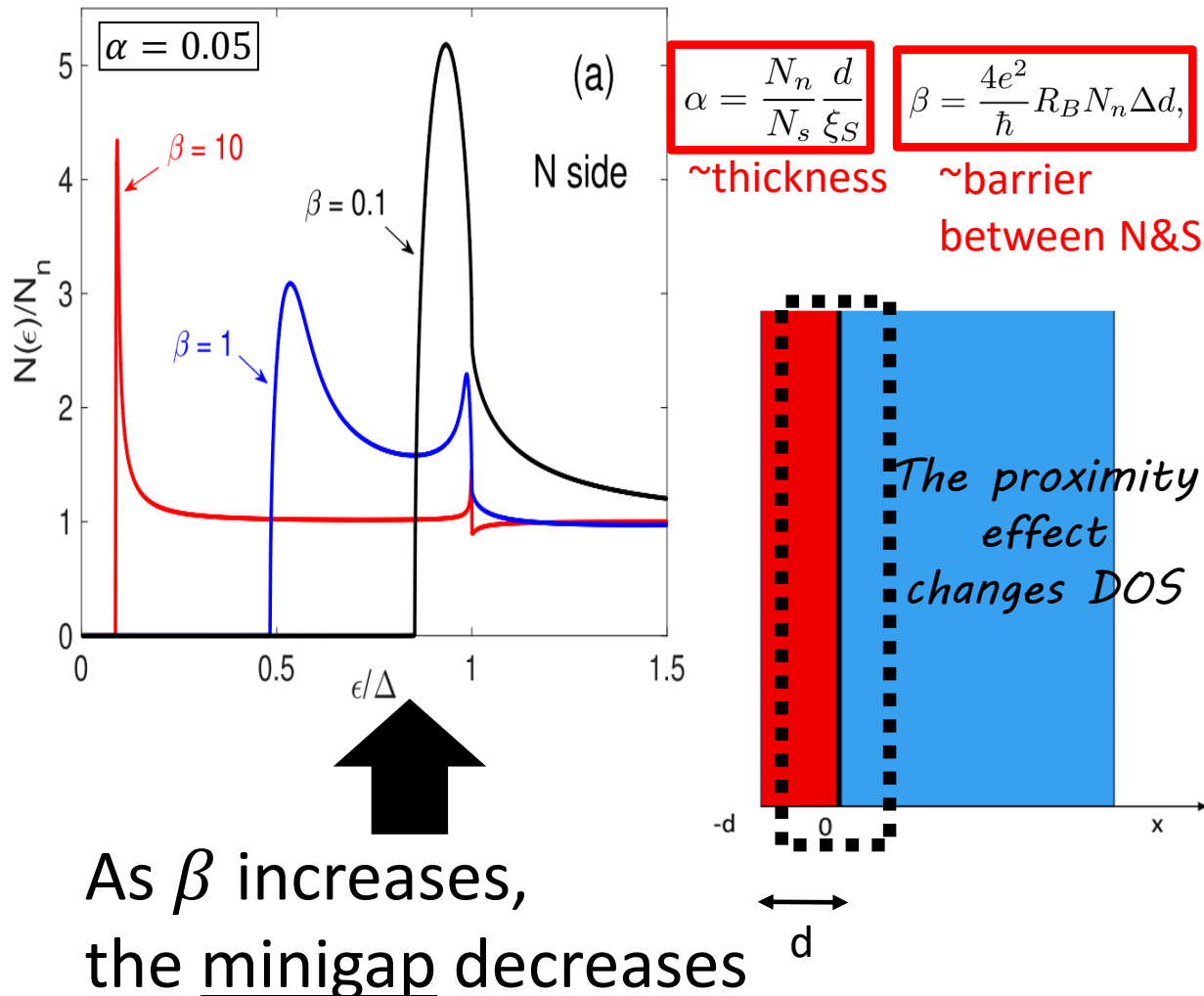


Conductance
 σ_n σ_s
N region S region

Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)

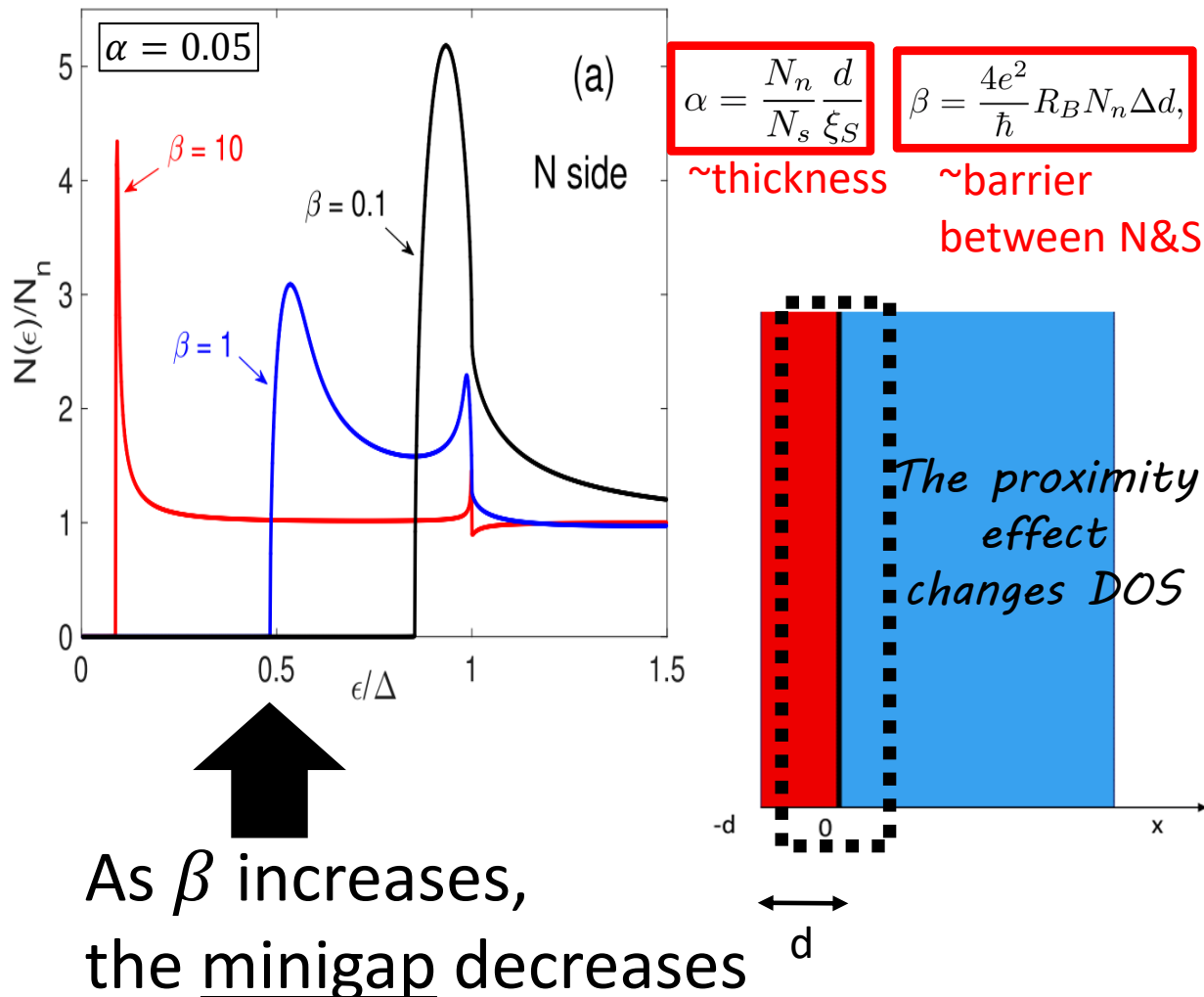
N-side DOS



Review (2)

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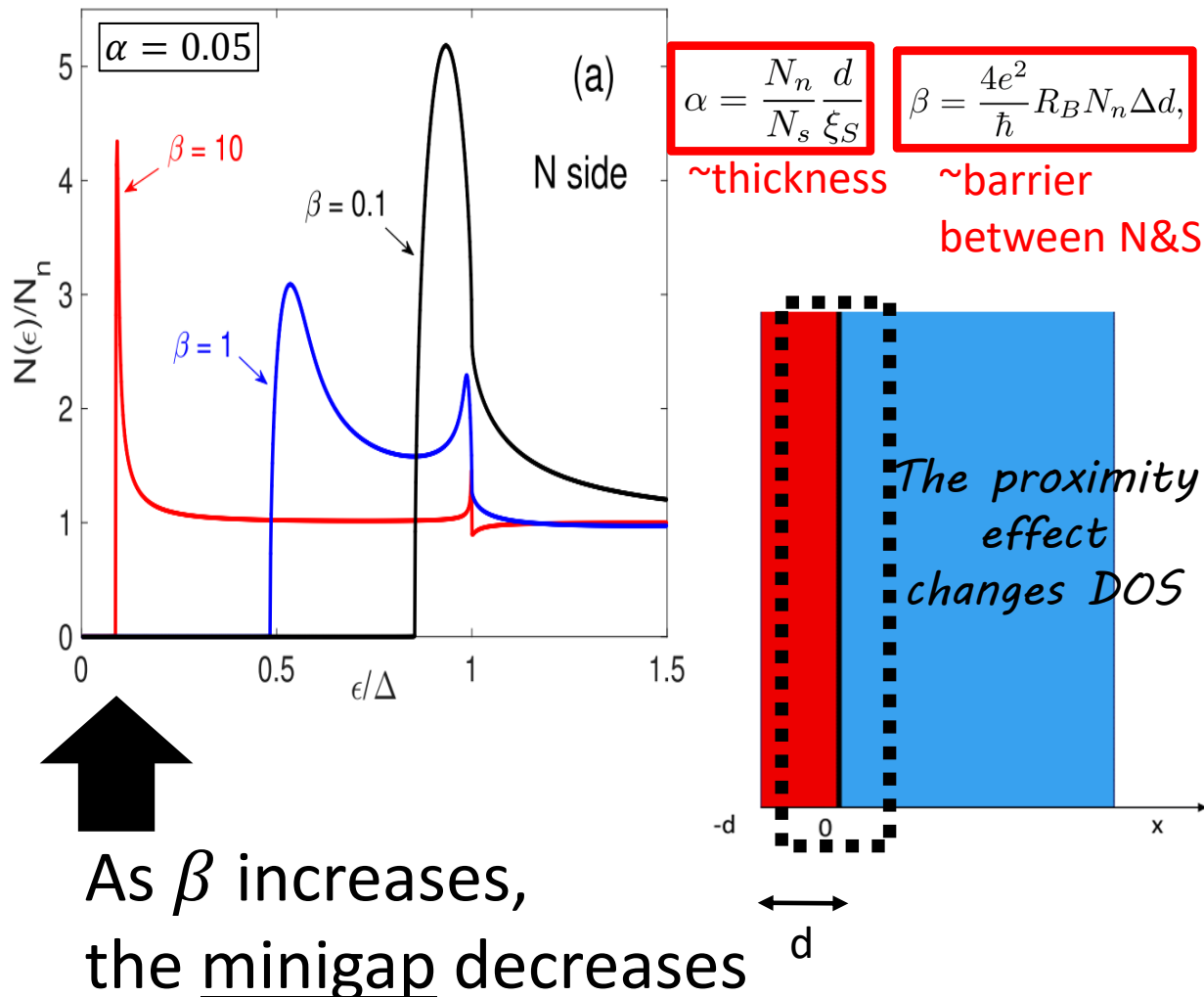
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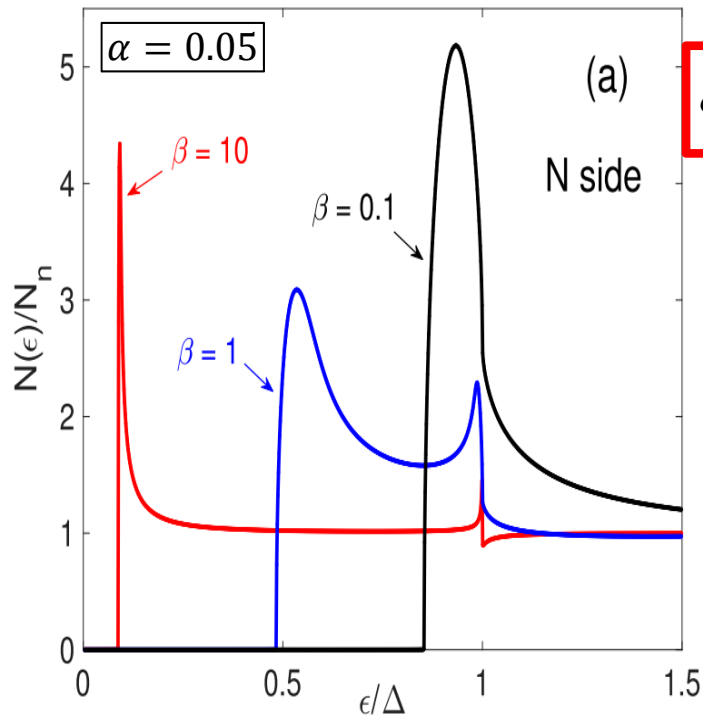
N-side DOS



Review (2)

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N-side DOS

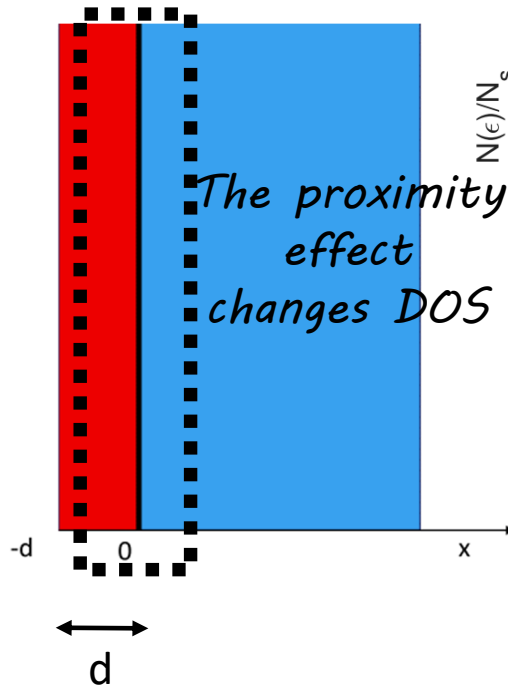


$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

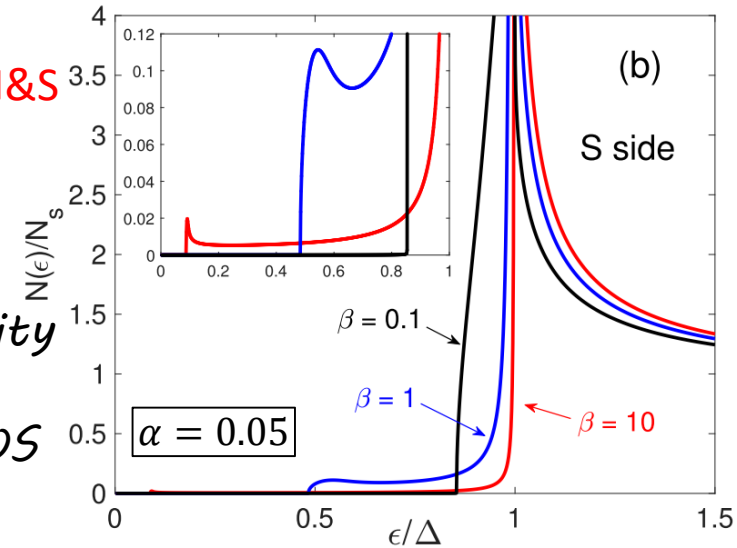
~thickness

$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

~barrier
between N&S



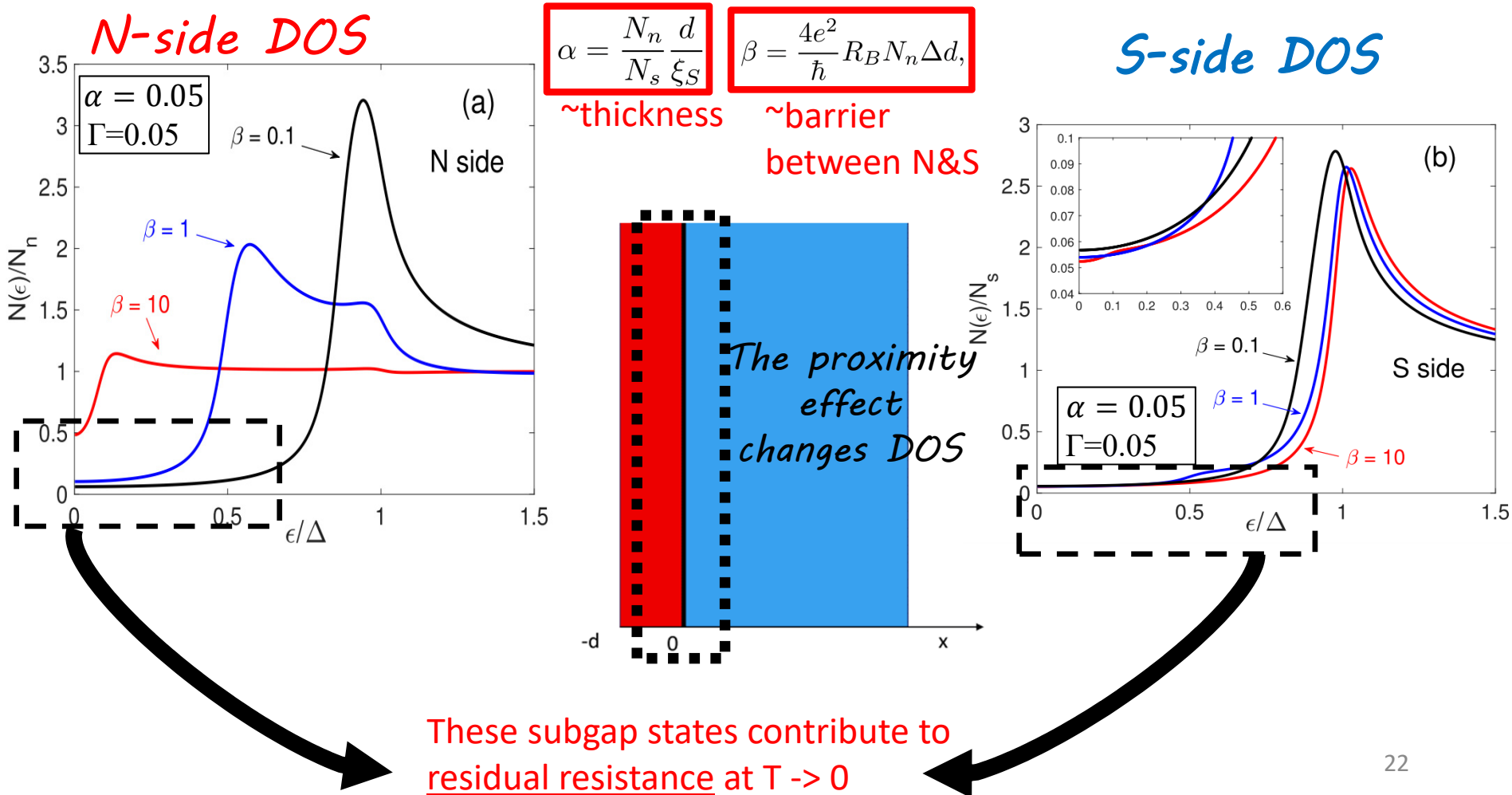
S-side DOS



Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)

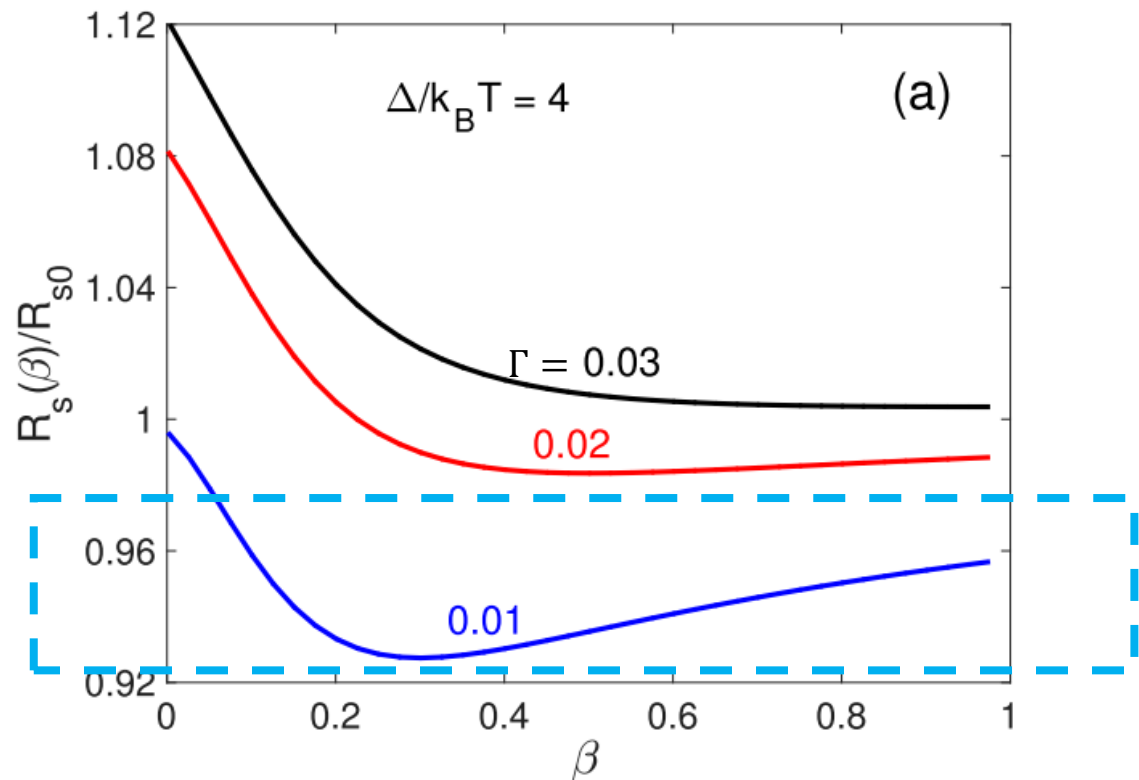
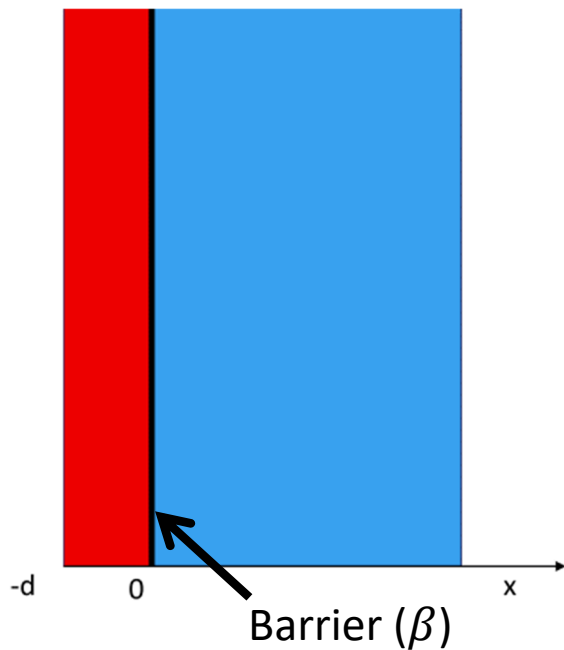
Taking a finite quasi particle lifetime into account ($\varepsilon \rightarrow \varepsilon + i\Gamma$), the cusps are smeared out.



Review (2)

Example: Proximity-coupled thin N layer (metallic suboxides)

- R_s depends on the N -layer parameters.
- R_s can be optimized by tuning them.
- R_s can be smaller than R_s for the ideal surface without N layer



$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d, \quad (\sim \text{barrier between N\&S}) \text{ can be changed by } \underline{\text{heat treatments}}$$

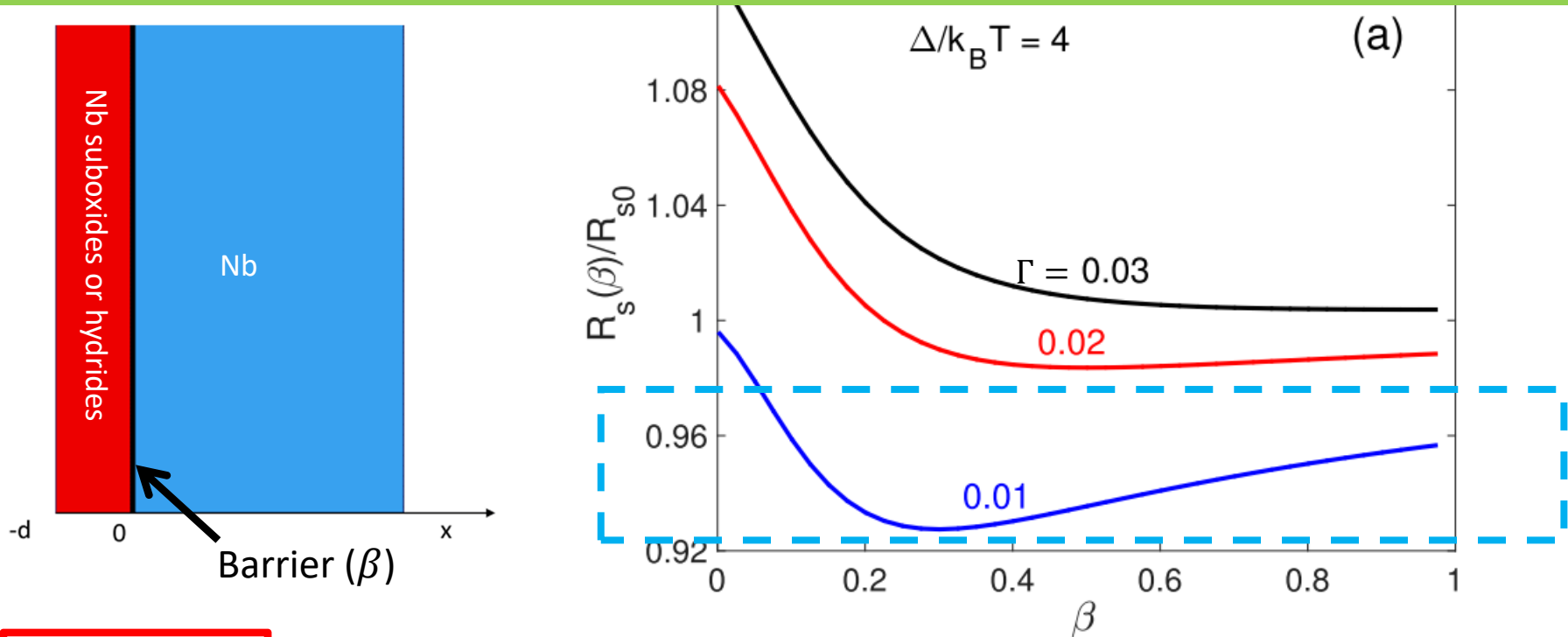
Review (2)

Taking Nb for example,

- $\alpha \propto$ thickness of Nb-suboxides or hydrides on the surface
- $\beta \propto$ the interface resistance between Nb suboxide and Nb

These can be easily affected by material processing recipes.

→ Link to the dependence of R_s on various recipes?



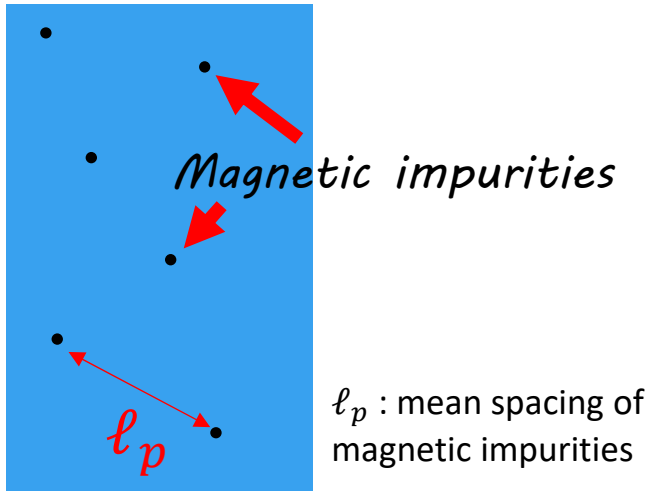
$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

(~ barrier between N&S) can be changed by heat treatments

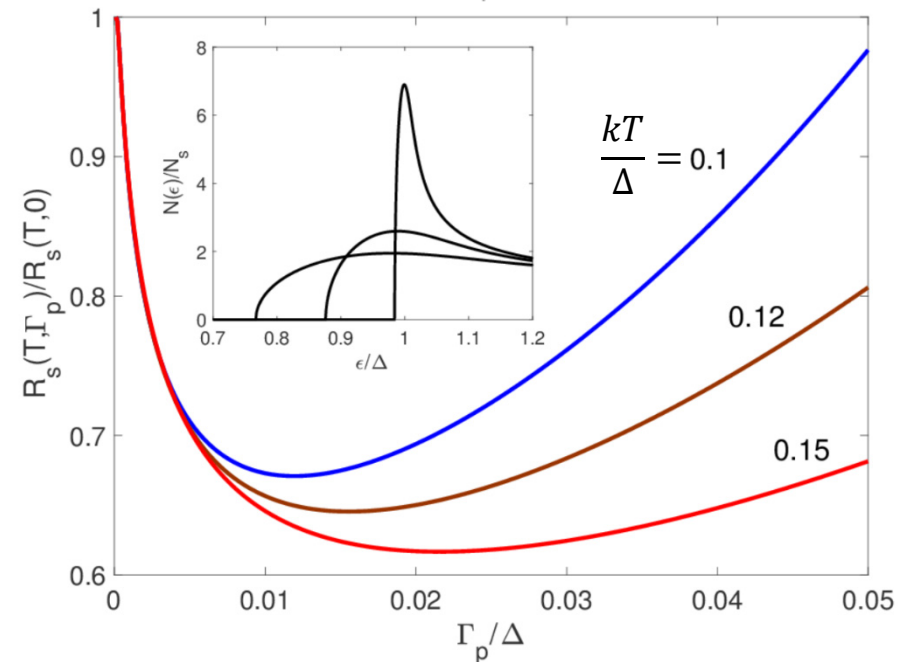
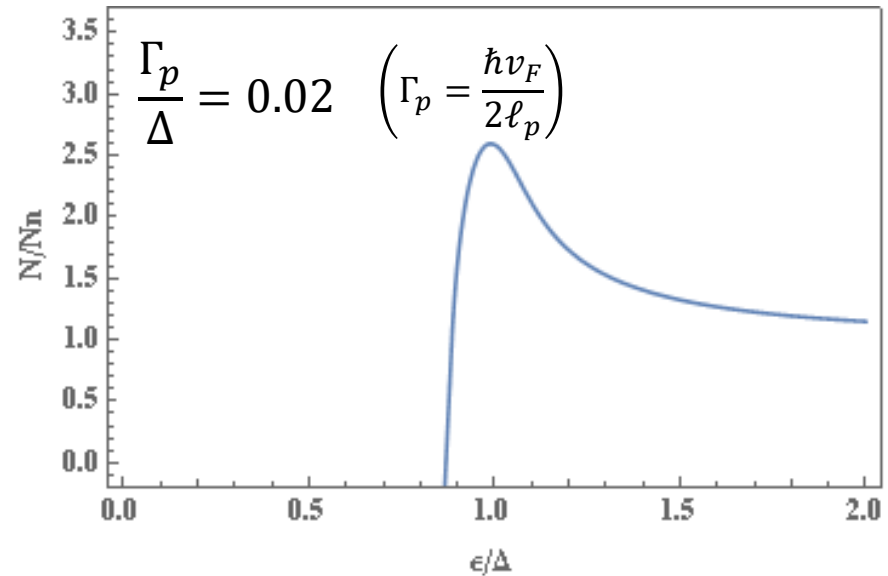
Changing the temperature of heat treatment, β can change and then R_s can change.

Review (2)

Example2: Magnetic impurities can also broaden DOS peaks



ℓ_p : mean spacing of magnetic impurities



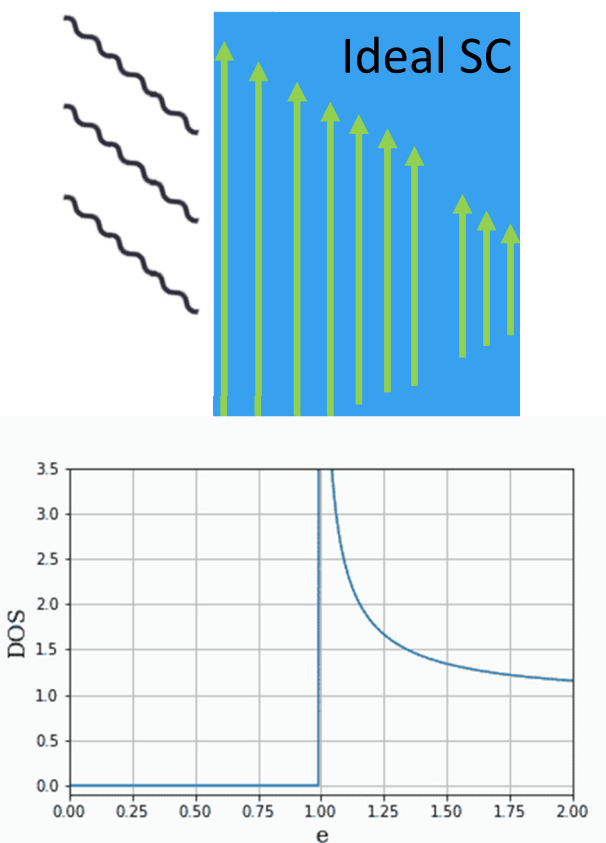
An appropriate density of magnetic impurities significantly reduce R_s !

$\frac{\Gamma_p}{\Delta} \sim 0.01$ corresponds to the mean spacing of magnetic impurities

$$\ell_p \sim \frac{\xi_0}{0.01} \sim 4\mu m$$

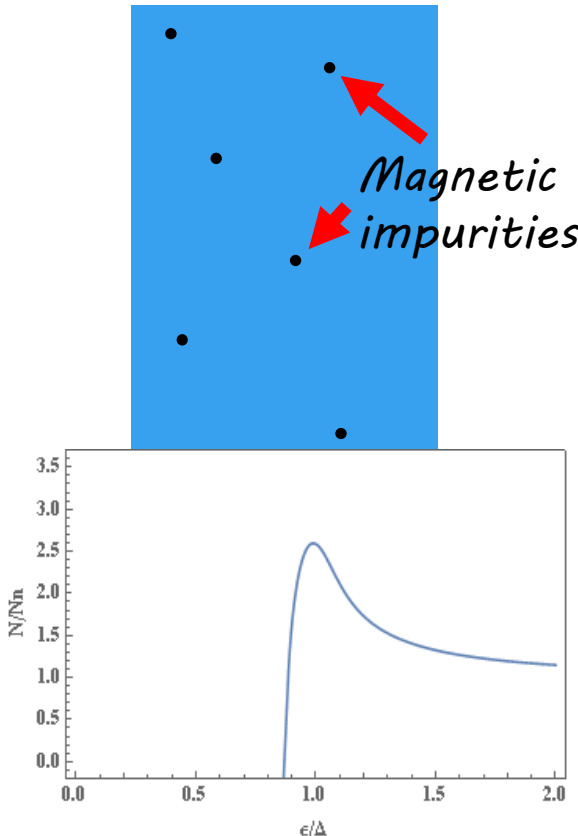
Summary of the review part

Pair-breaking current

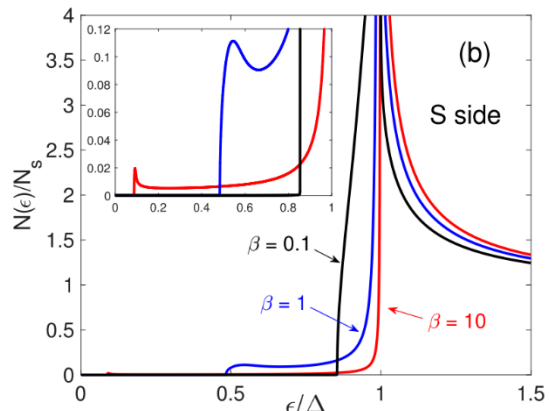
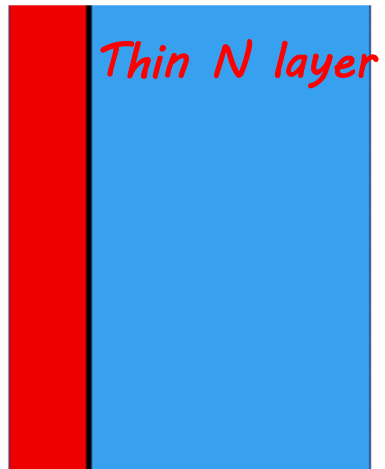


Current broadens DOS and affects R_s → field dependence

Pair-breaking mechanism originating from realistic material features

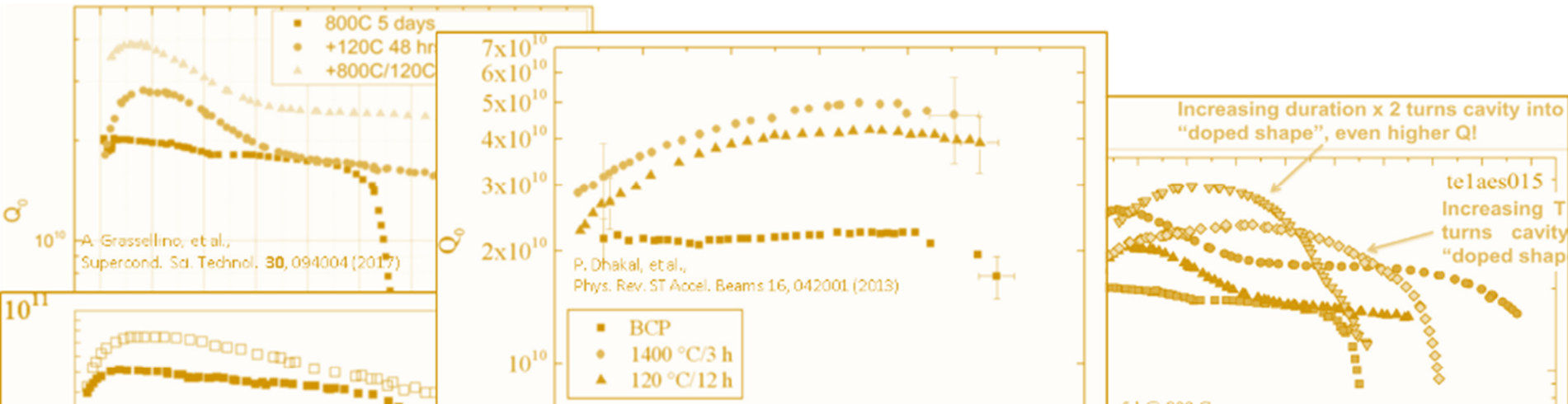


Magnetic impurities broaden DOS and affect R_s

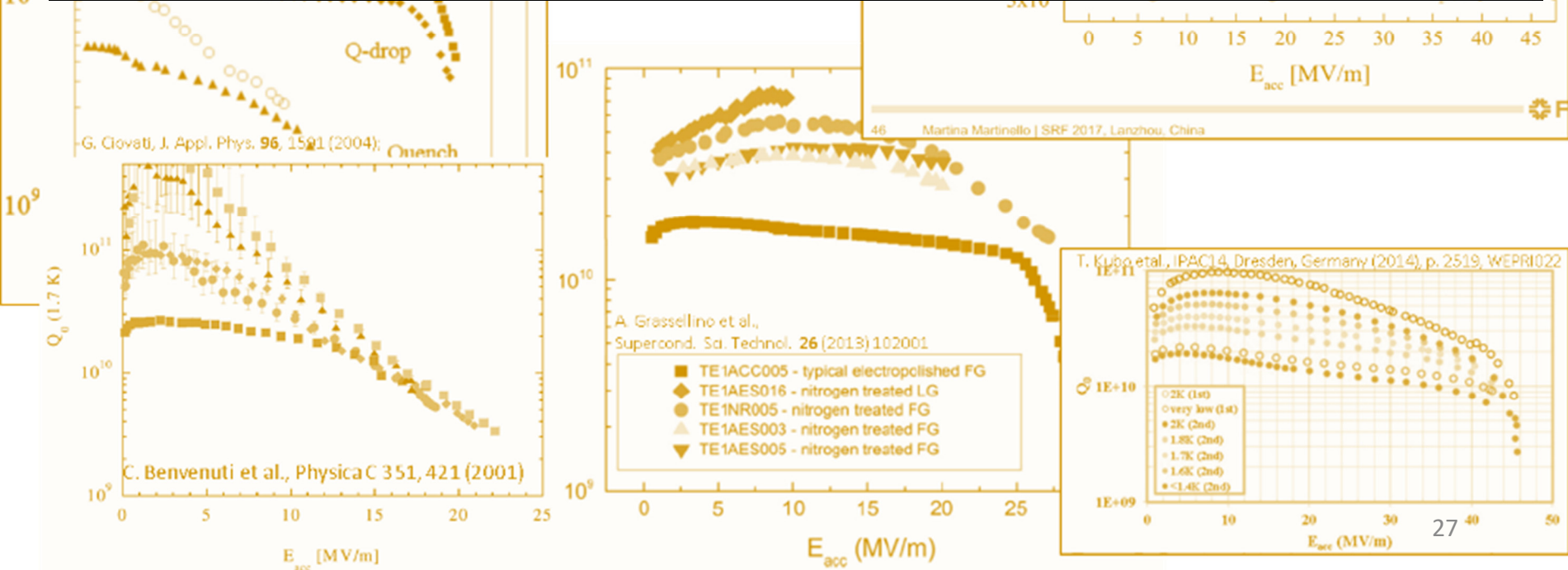


Proximity effect broadens DOS and affects R_s . The N layer properties are sensitive to material processing.

What is the origin of many different Q-E curves?



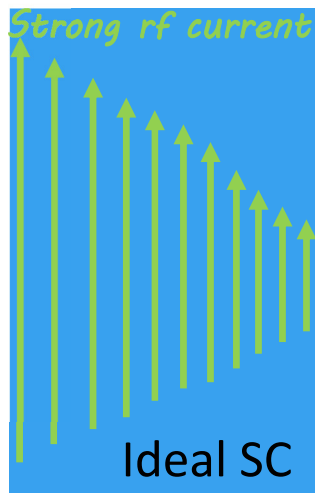
Now we address the nonlinear $R_s(H)$



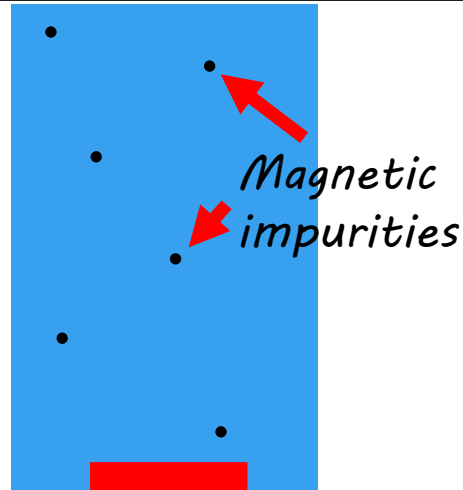
Part 2

*Effects of realistic material features
on the field dependent $R_s(H)$*

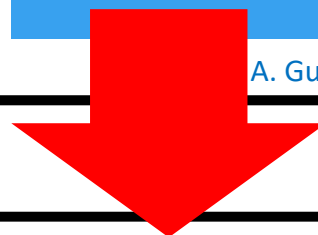
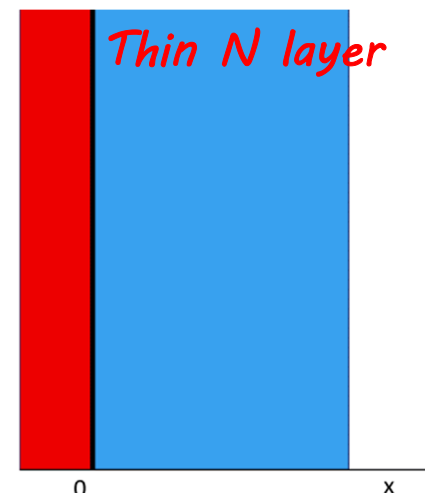
T. Kubo and A. Gurevich, to be published



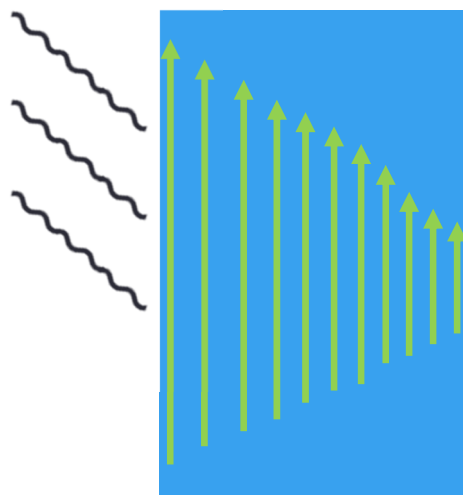
A. Gurevich, Phys. Rev. Lett. **113**, 087001 (2014)



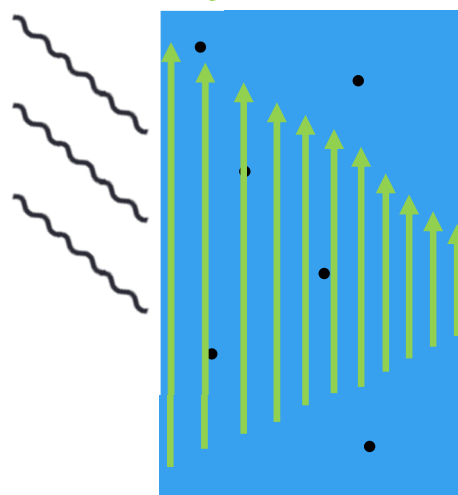
A. Gurevich and T. Kubo, Phys. Rev. B **96**, 184515 (2017)



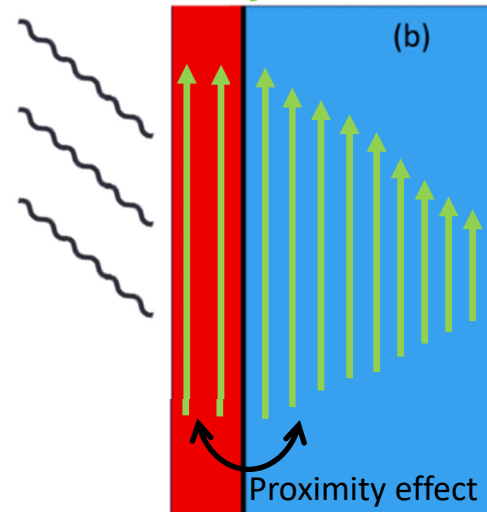
Strong rf current



Strong rf current



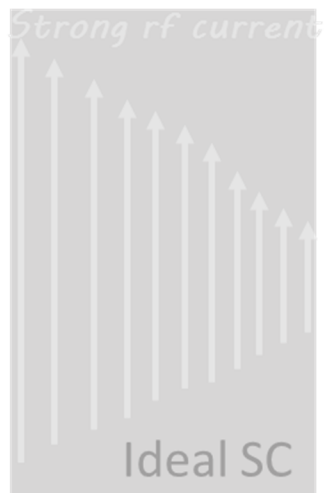
Strong rf current



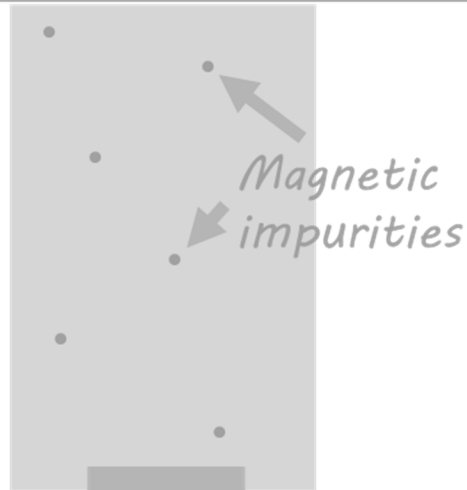
Incorporate a finite quasiparticle lifetime

Magnetic impurities

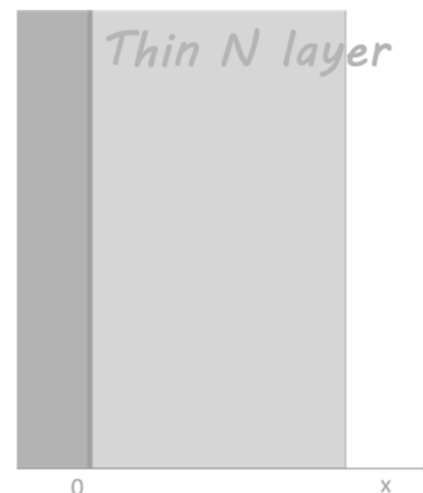
proximity coupled N layer at the surface



A. Gurevich, Phys. Rev. Lett. 113, 087001 (2014)



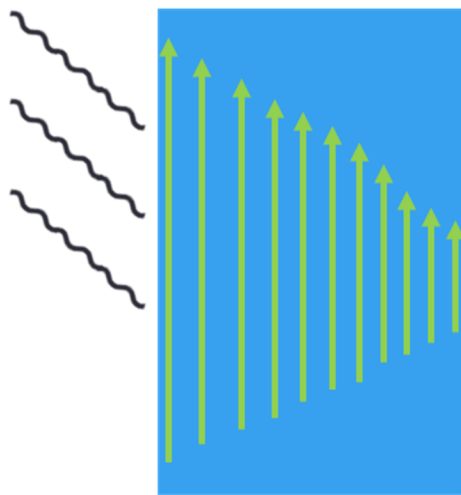
A. Gurevich and T. Kubo, Phys. Rev. B 96, 184515 (2017)



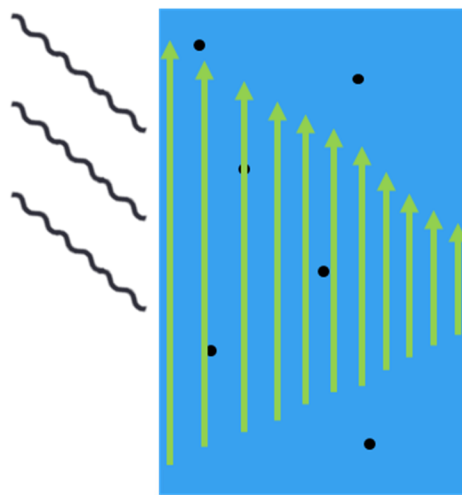
Strong rf current

Strong rf current

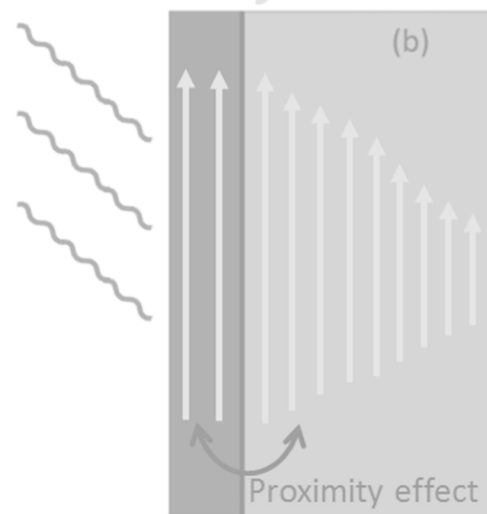
Strong rf current



Incorporate a finite quasiparticle lifetime



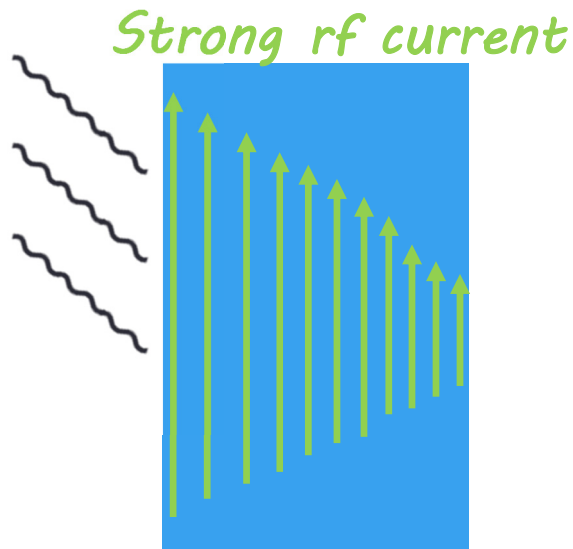
Magnetic impurities



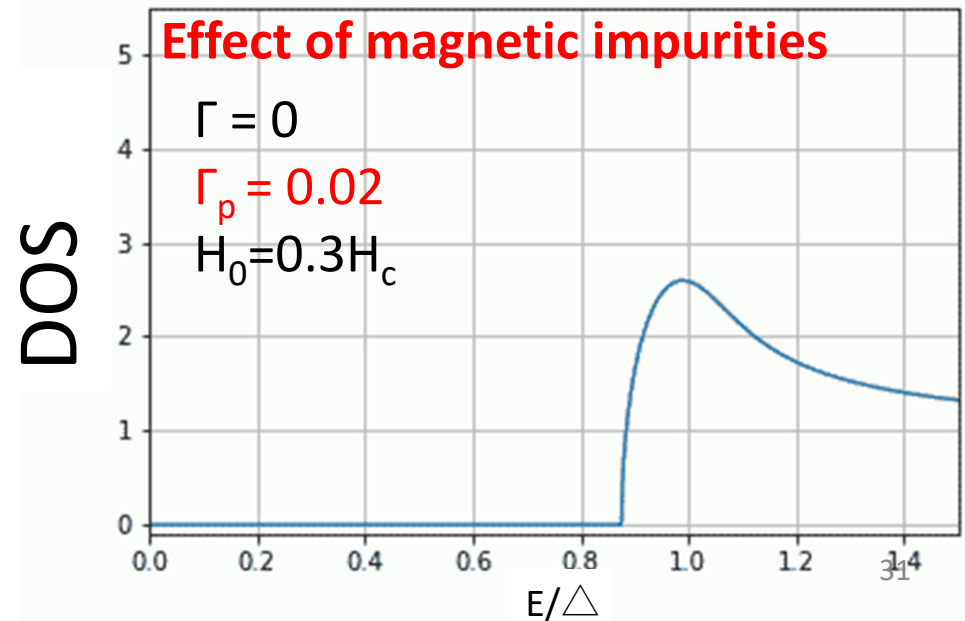
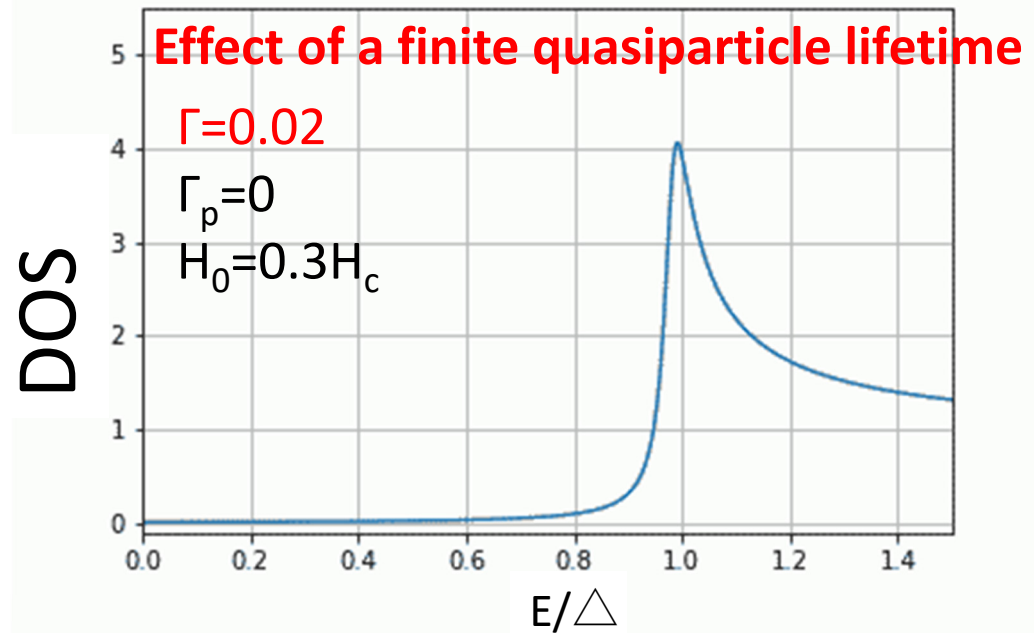
*proximity coupled N layer at the surface*³⁰

DOS under a strong rf field

Contain animations. See in [Slide Show Mode](#).

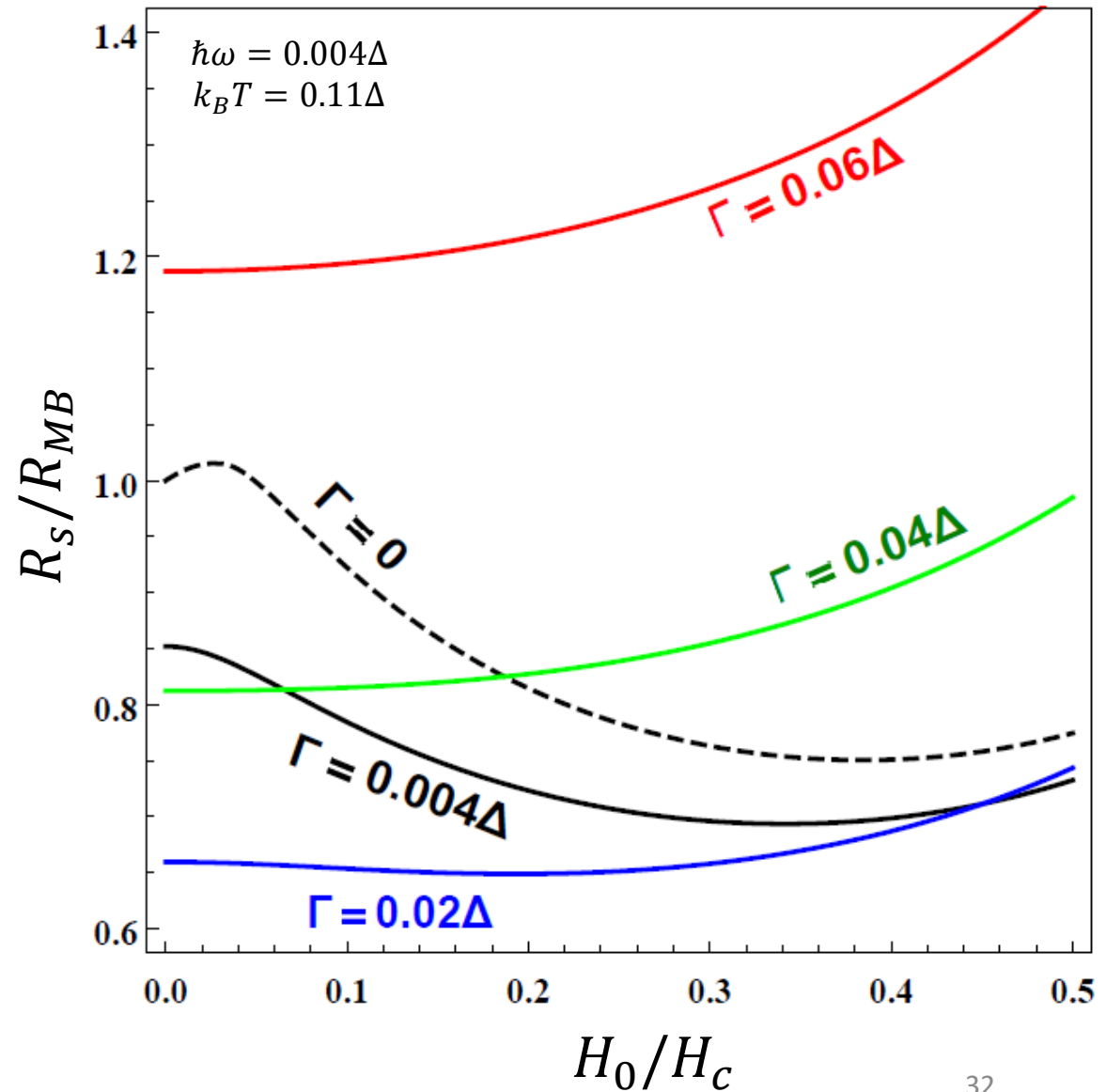
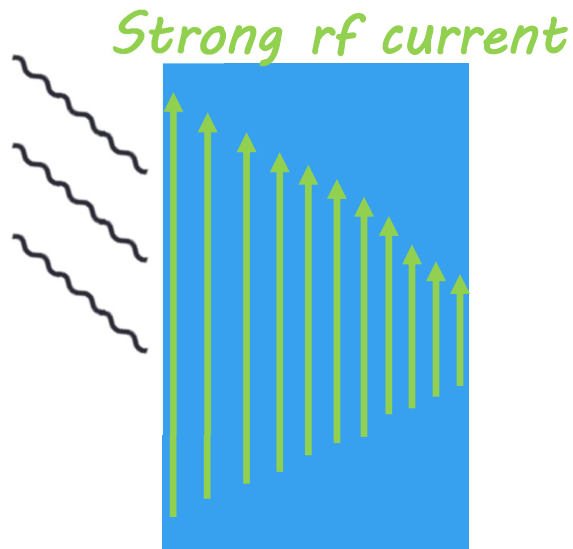


Dirty SC with nonmagnetic impurities;
Can incorporate a finite quasiparticle
lifetime and can include magnetic impurities



Field dependent Surface resistance $R_s(H_0)$

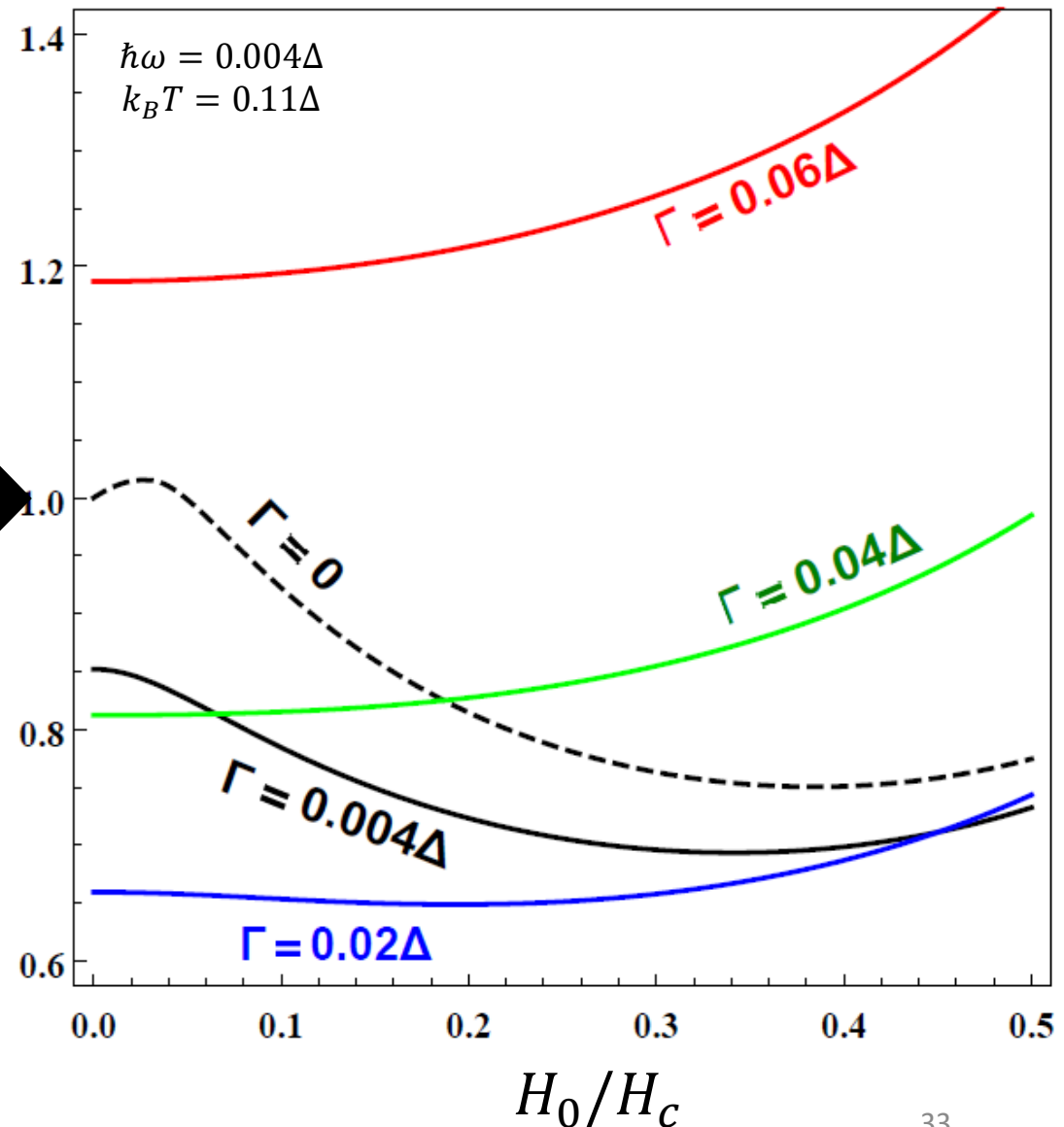
(1) Effect of a finite quasiparticle lifetime (Γ parameter)



Field dependent Surface resistance $R_s(H_0)$

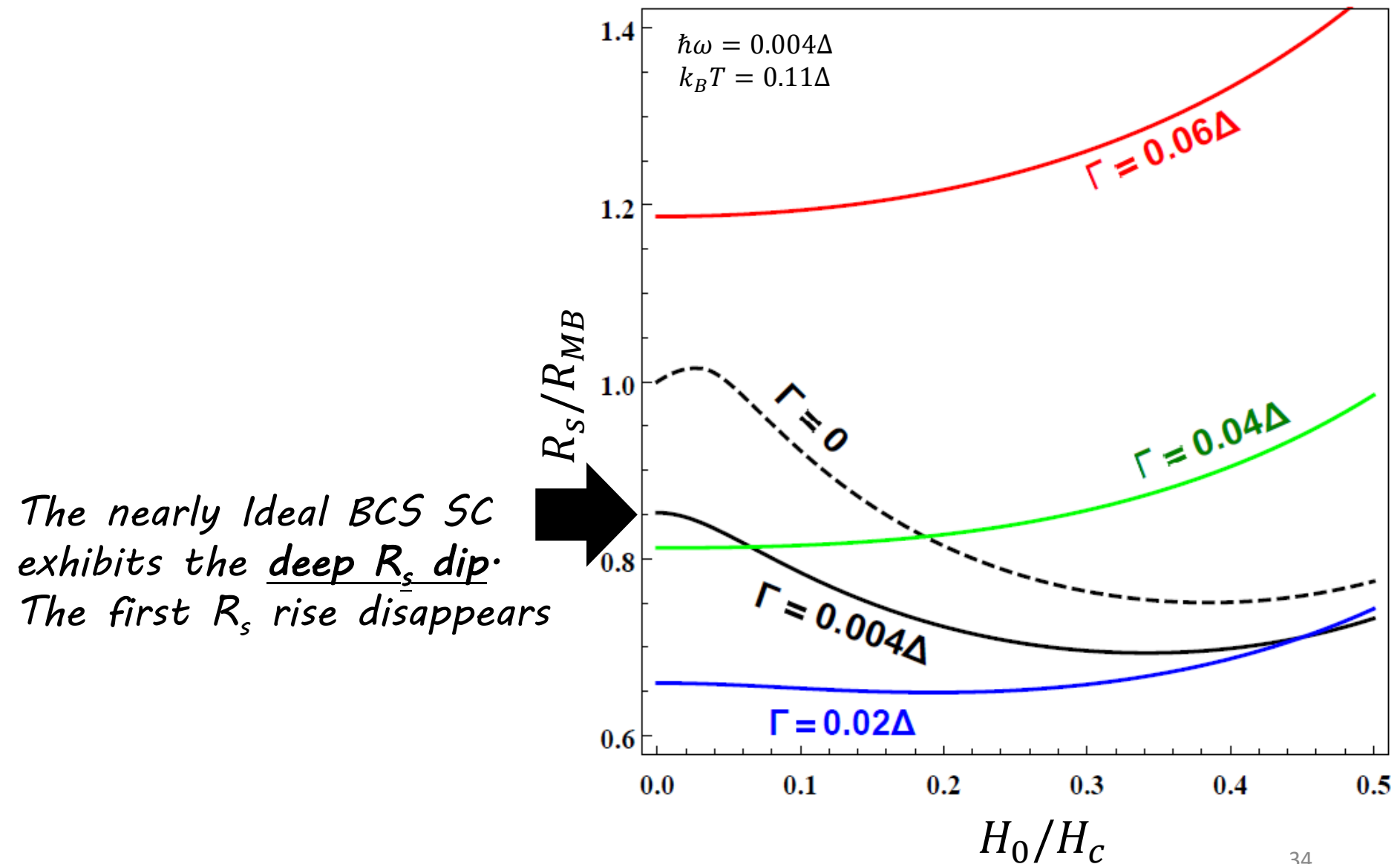
(1) Effect of a finite quasiparticle lifetime (Γ parameter)

The Ideal BCS SC exhibits
a deep R_s dip



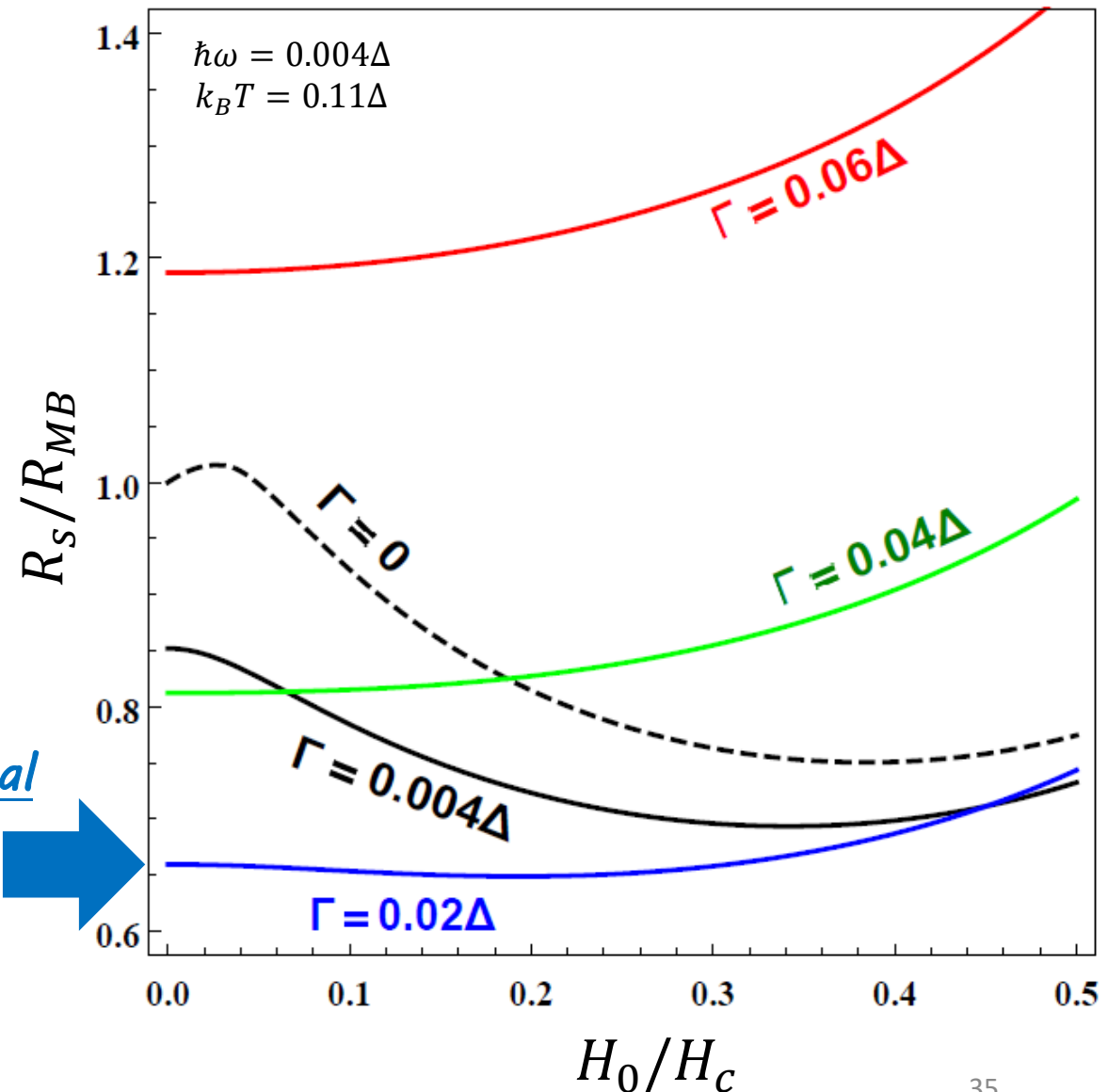
Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime (Γ parameter)



Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime (Γ parameter)

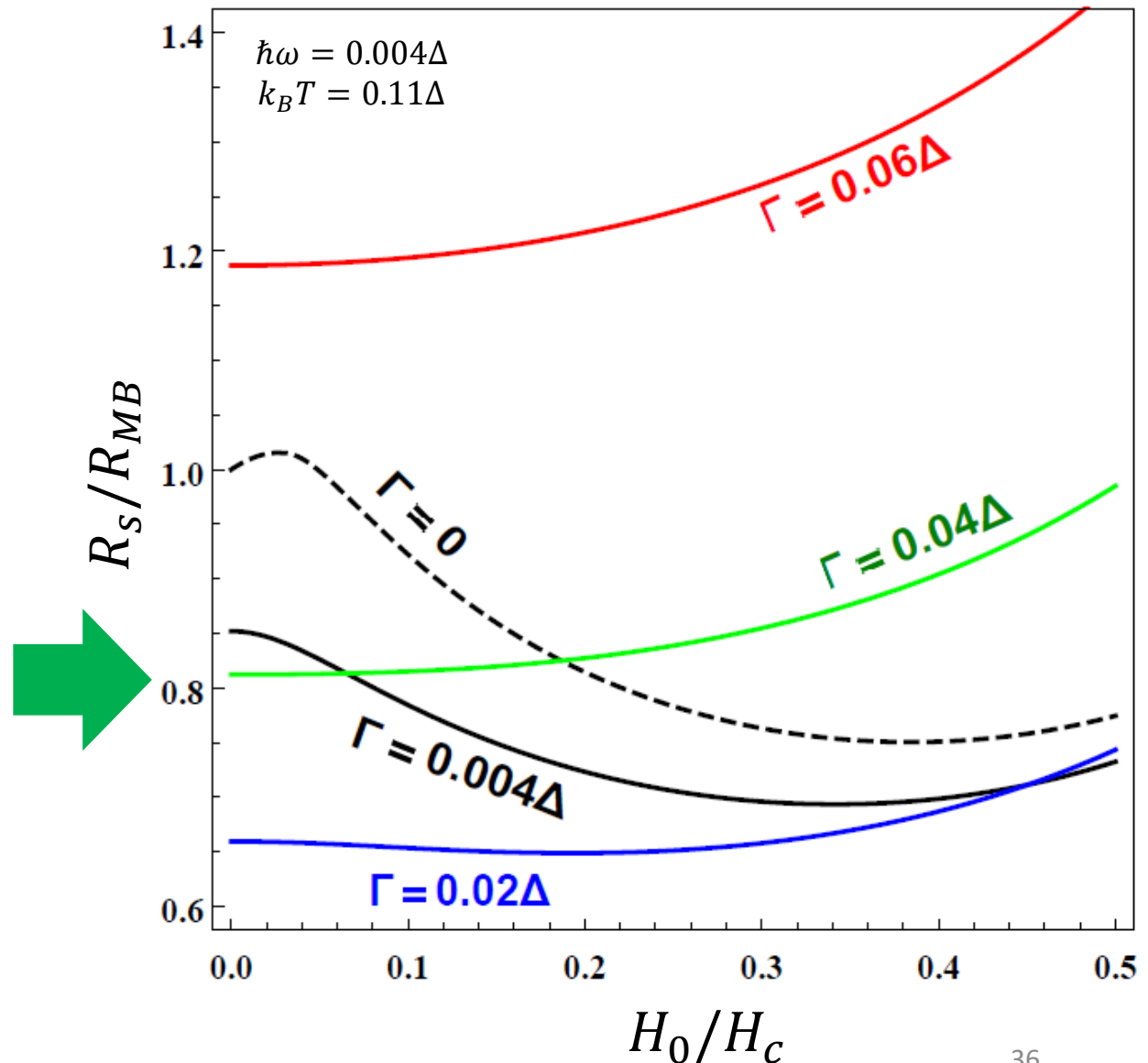


The R_s dip almost disappears, but the low-field R_s is better than ideal BCS SC due to the DOS broadening effect.

Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime (Γ parameter)

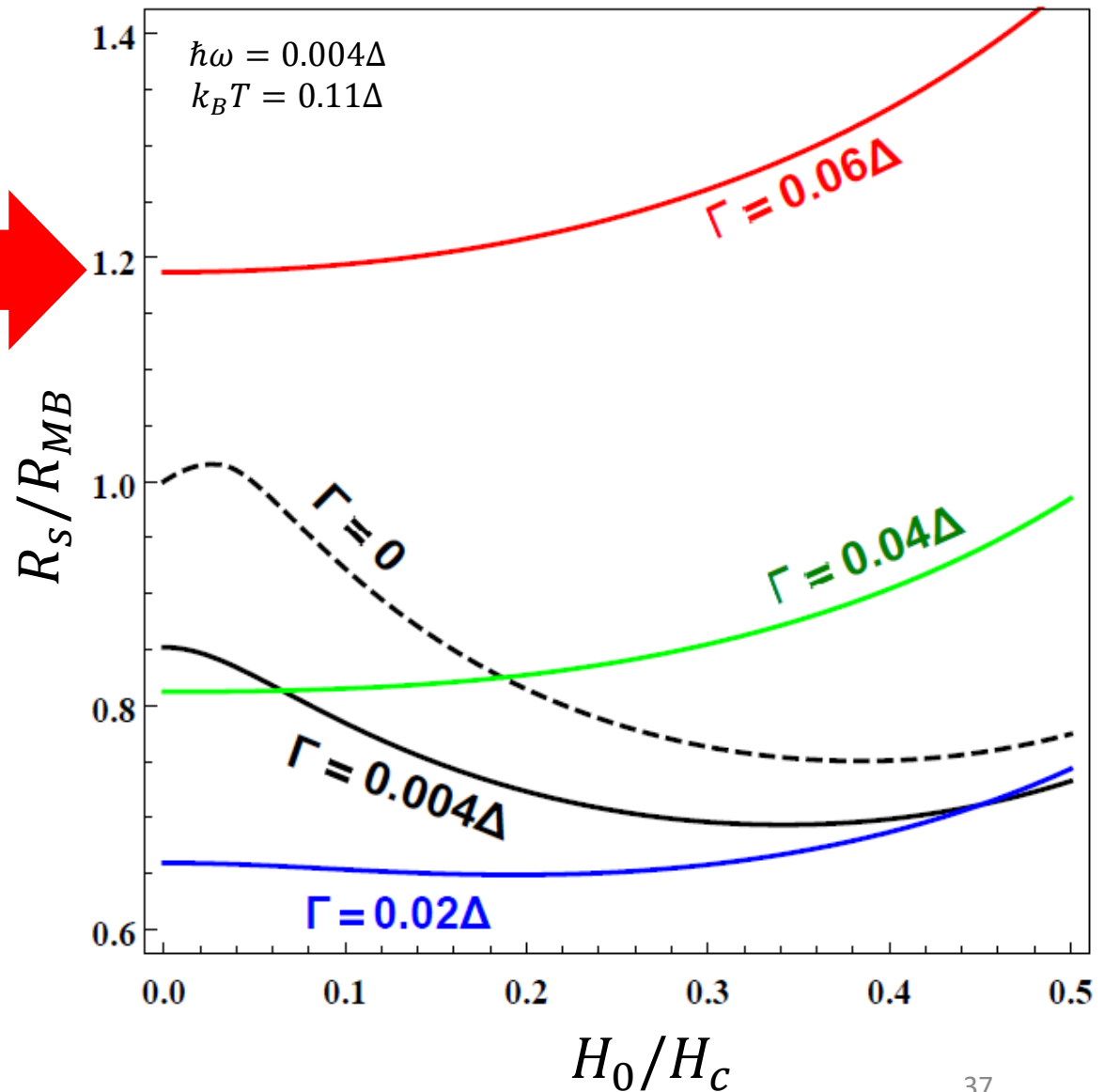
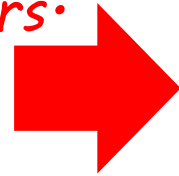
The R_s dip disappears, but the low-field R_s is better than ideal BCS SC due to the DOS broadening effect.



Field dependent Surface resistance $R_s(H_0)$

(1) Effect of a finite quasiparticle lifetime (Γ parameter)

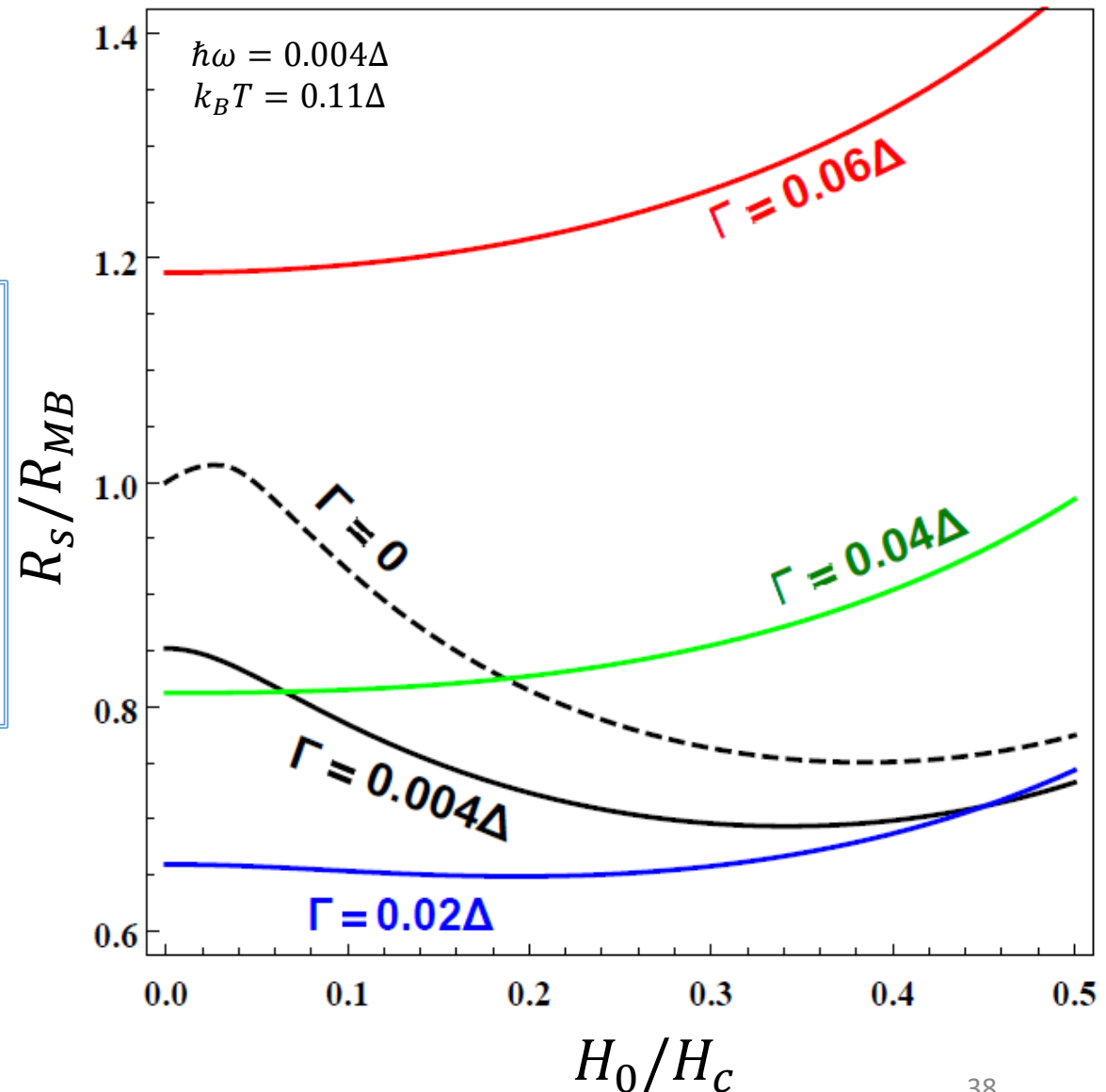
The R_s dip disappears.
 R_s becomes larger.



Field dependent Surface resistance $R_s(H_0)$

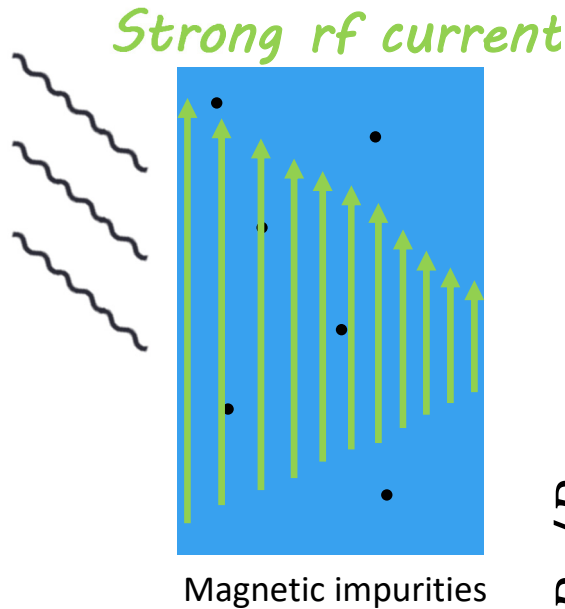
(1) Effect of a finite quasiparticle lifetime (Γ parameter)

The broadening parameter (Γ) has a significant effect on the field dependent R_s .



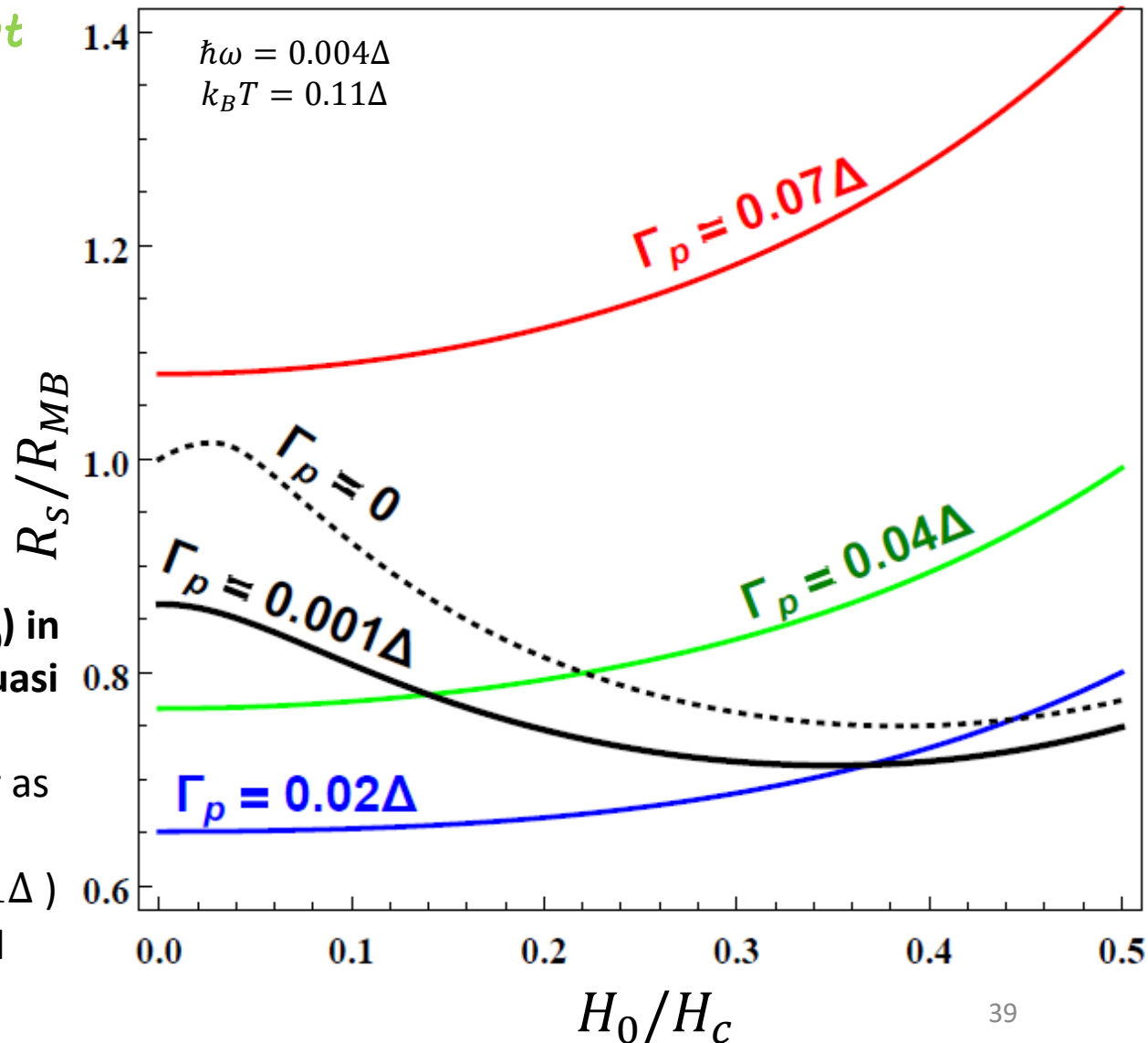
Field dependent Surface resistance $R_s(H_0)$

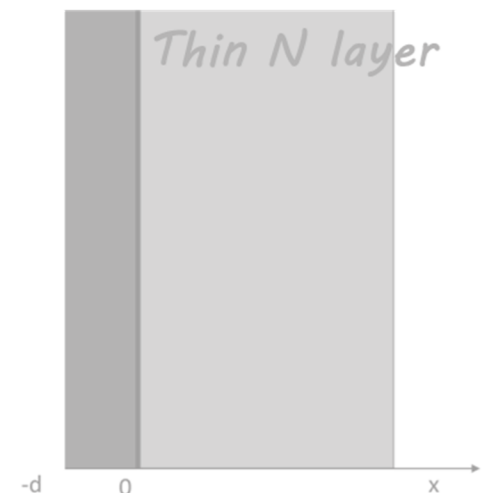
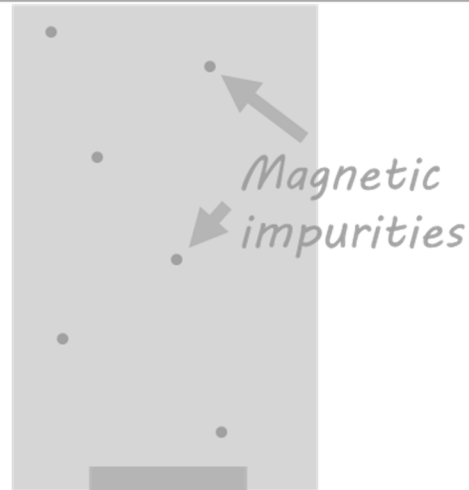
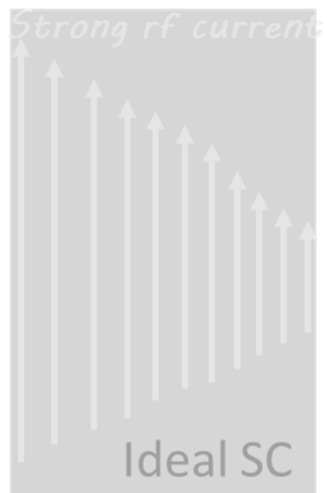
(2) Effect of magnetic impurities (Γ_p parameter)



Magnetic impurities affect $R_s(H_0)$ in the similar manner as a finite quasi particle lifetime Γ .

- The R_s dip becomes shallower as Γ_p increases.
- The low field R_s for ($\Gamma_p \sim 0.01\Delta$) is much smaller than the ideal BCS superconductor ($\Gamma_p = 0$).



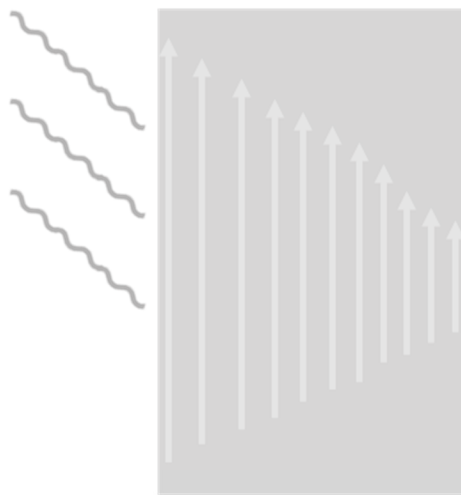


A. Gurevich, Phys. Rev. Lett. 113, 087001 (2014)

A. Gurevich and T. Kubo, Phys. Rev. B 96, 184515 (2017)

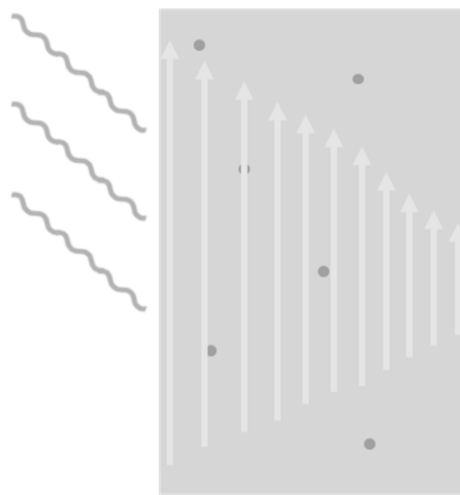


Strong rf current



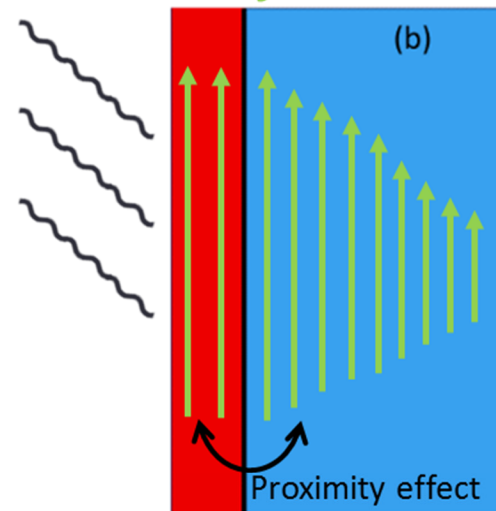
Incorporate a finite quasiparticle lifetime

Strong rf current



Magnetic impurities

Strong rf current



proximity coupled N layer at the surface

DOS under a strong rf field

Contain animations. See in [Slide Show Mode](#).

d is an N layer thickness.

(e.g., thickness of suboxide on the Nb surface)

R_B is an interface resistance

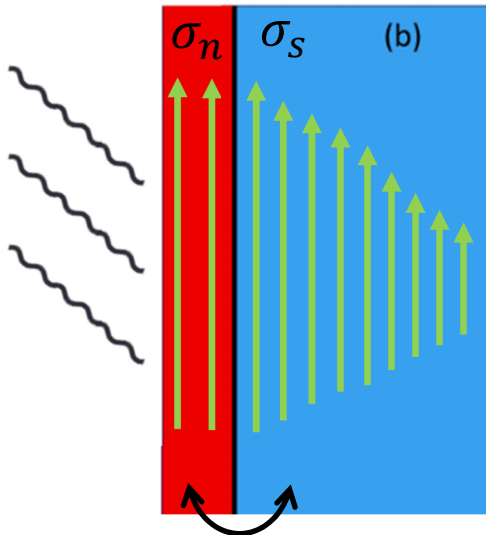
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

~thickness

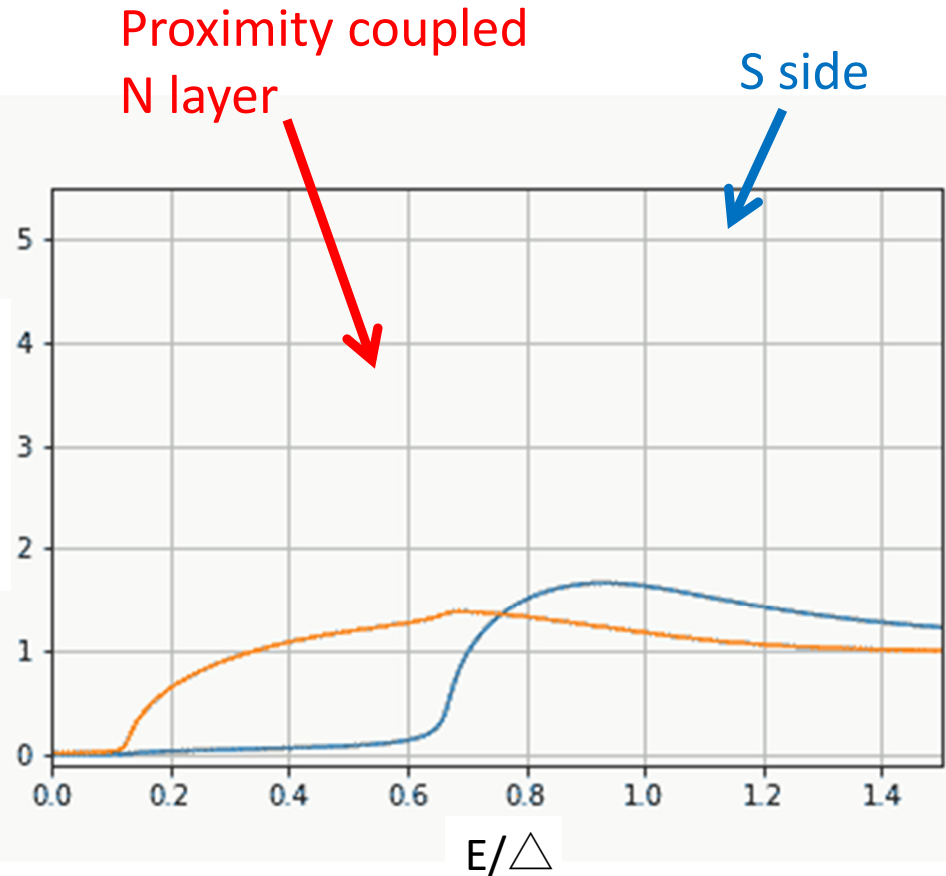
$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

~barrier
between N&S



Proximity effect

DOS



$$\alpha = 0.05, \beta = 1, \Gamma = 0.005$$

Field dependent Surface resistance $R_s(H_0)$

for different N-layer thickness

d is an N layer thickness.

(e.g., thickness of suboxide on the Nb surface)

R_B is an interface resistance

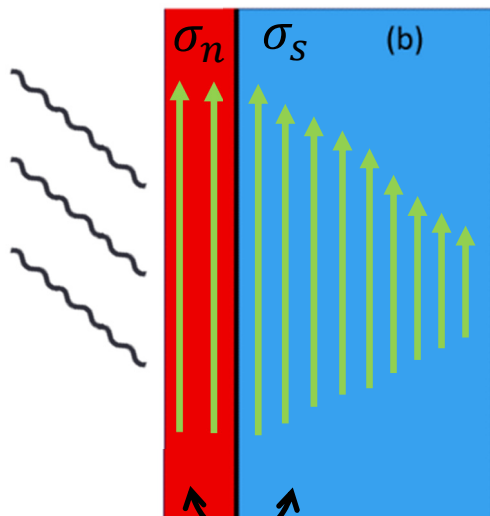
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

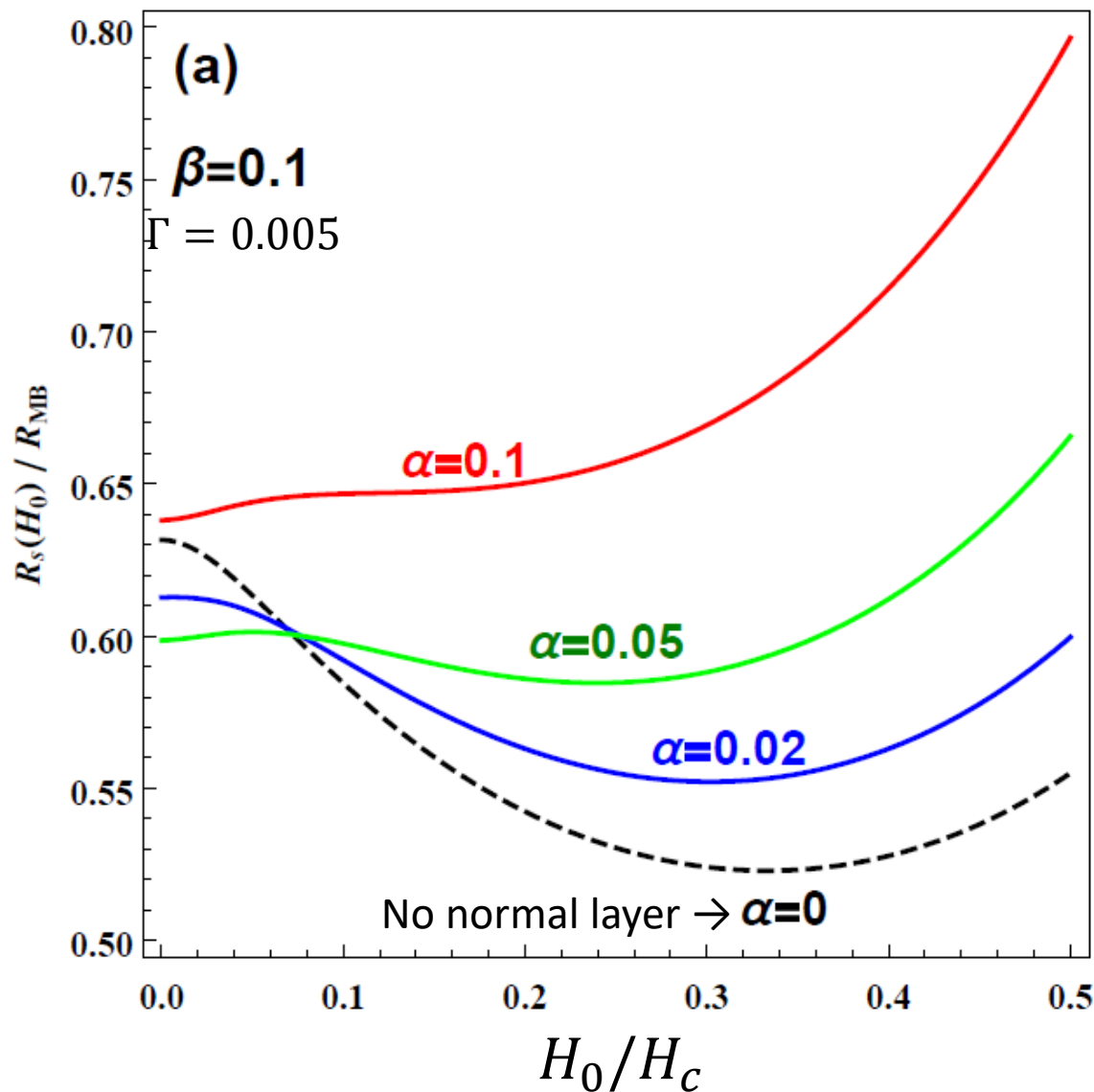
~thickness

$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

~barrier
between N&S



Proximity effect



**As the N-layer thickness increases,
the dip becomes shallower and finally disappears:**
Continuously changes from “N-doping-like” to “EP-like” shape.

R_B is an interface resistance

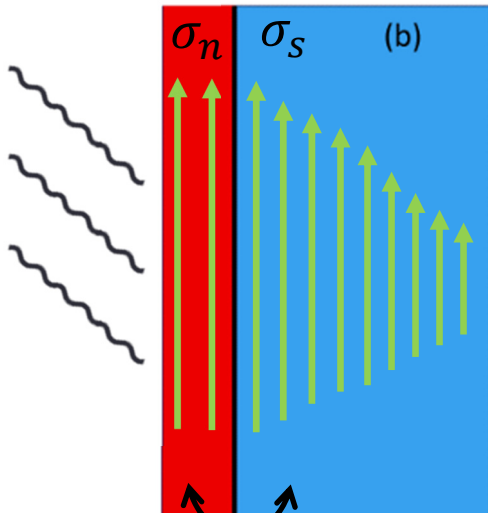
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

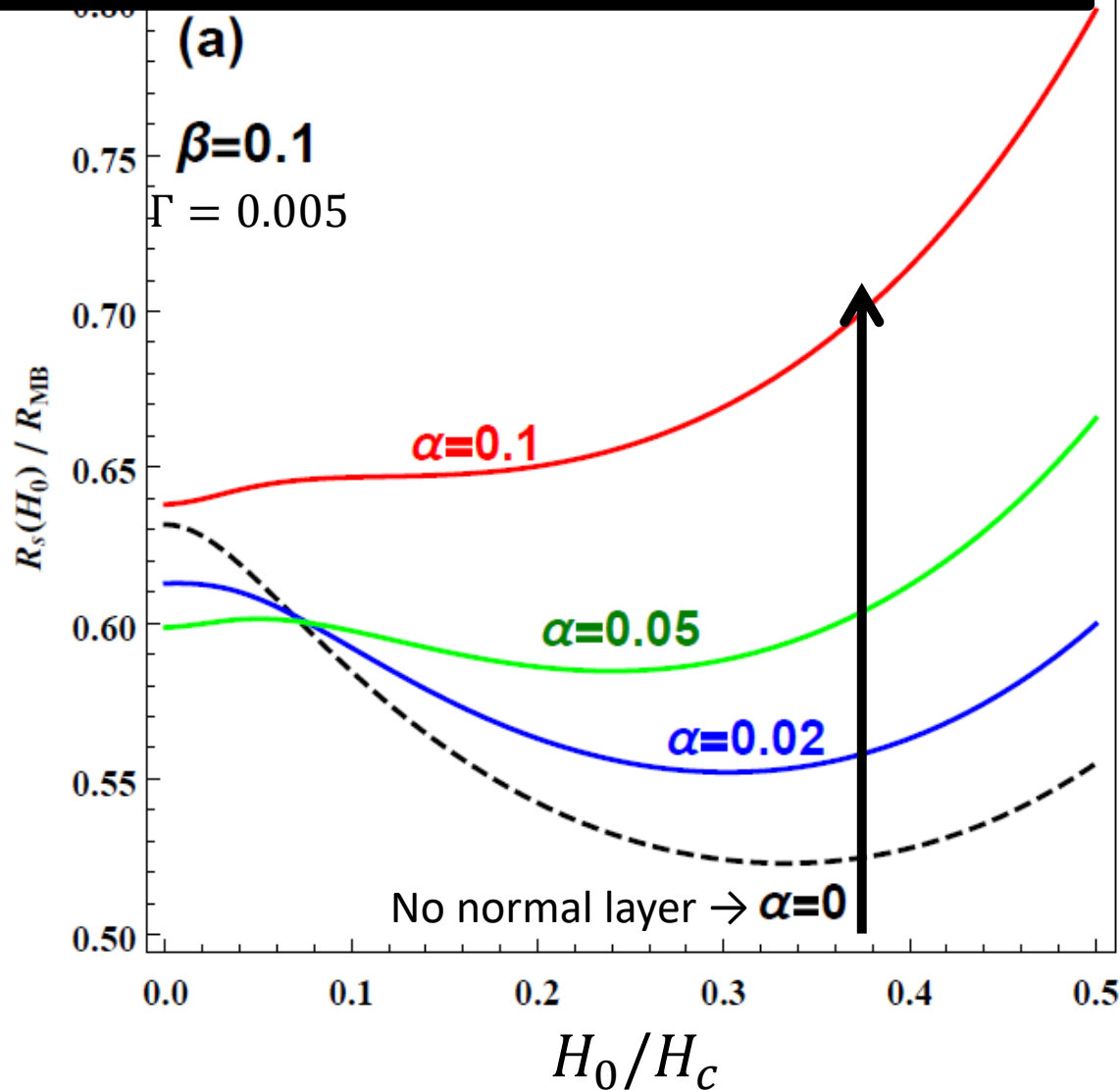
~thickness

$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

~barrier
between N&S



Proximity effect



Field dependent Surface resistance $R_s(H_0)$ for different N-layer conductivity

d is an N layer thickness.

(e.g., thickness of suboxide on the Nb surface)

R_B is an interface resistance

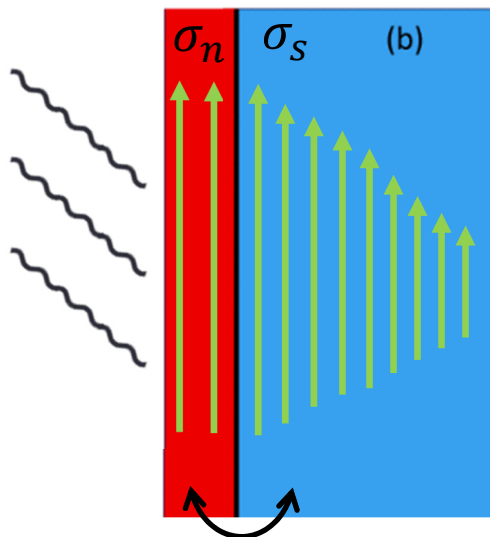
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

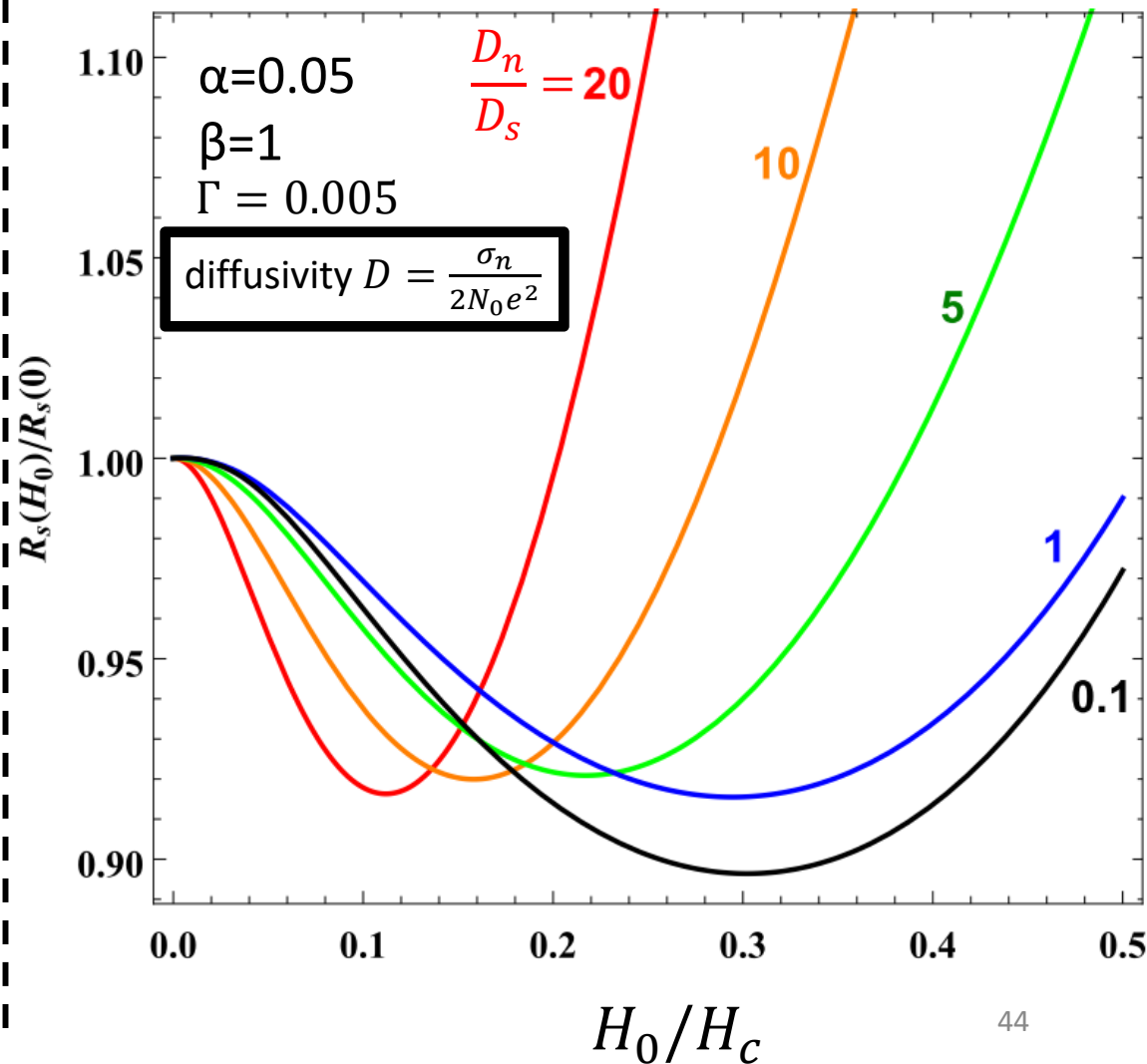
~thickness

$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

~barrier
between N&S



Proximity effect



The position of minimum shifts from medium fields to lower fields.

(e.g., thickness of suboxide on the Nb surface)

R_B is an interface resistance

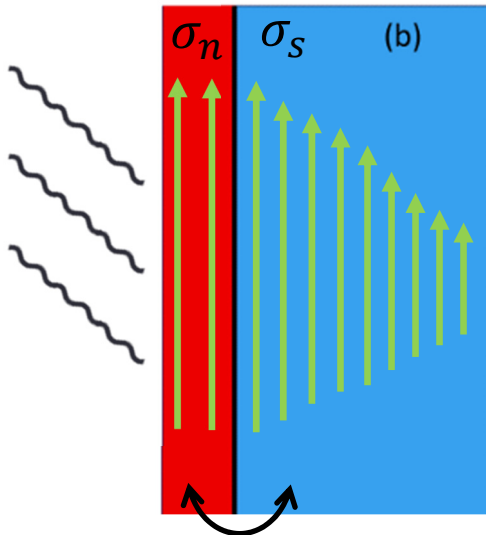
Sensitive to heat treatment
(e.g., between Nb suboxide and Nb)

$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

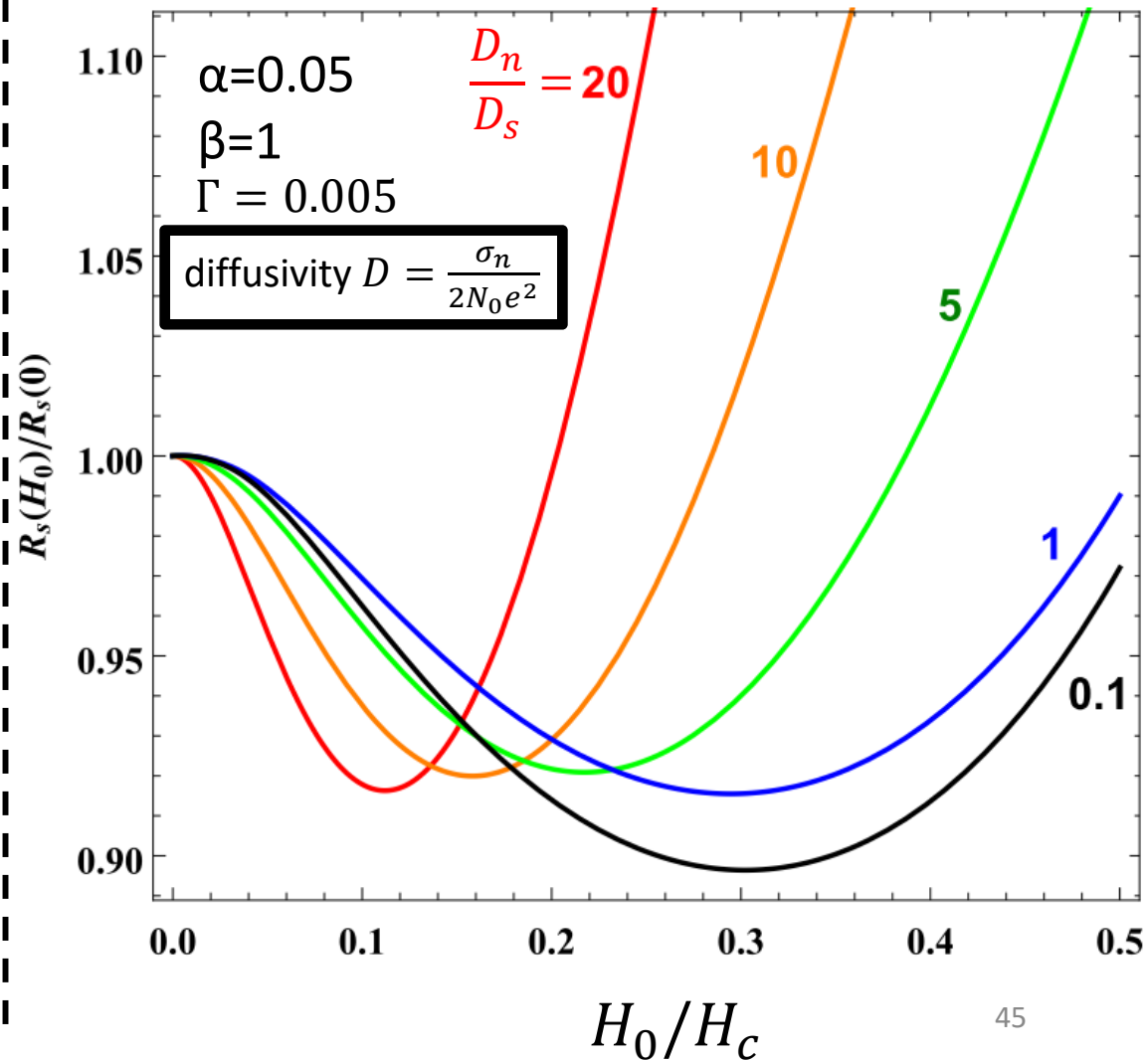
~thickness

$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

~barrier
between N&S



Proximity effect



Field dependent Surface resistance $R_s(H_0)$ for different temperatures

d is an N layer thickness

R_B is an interface resistance

Sensitive to heat treatment (e.g., between Nb suboxide and Nb)

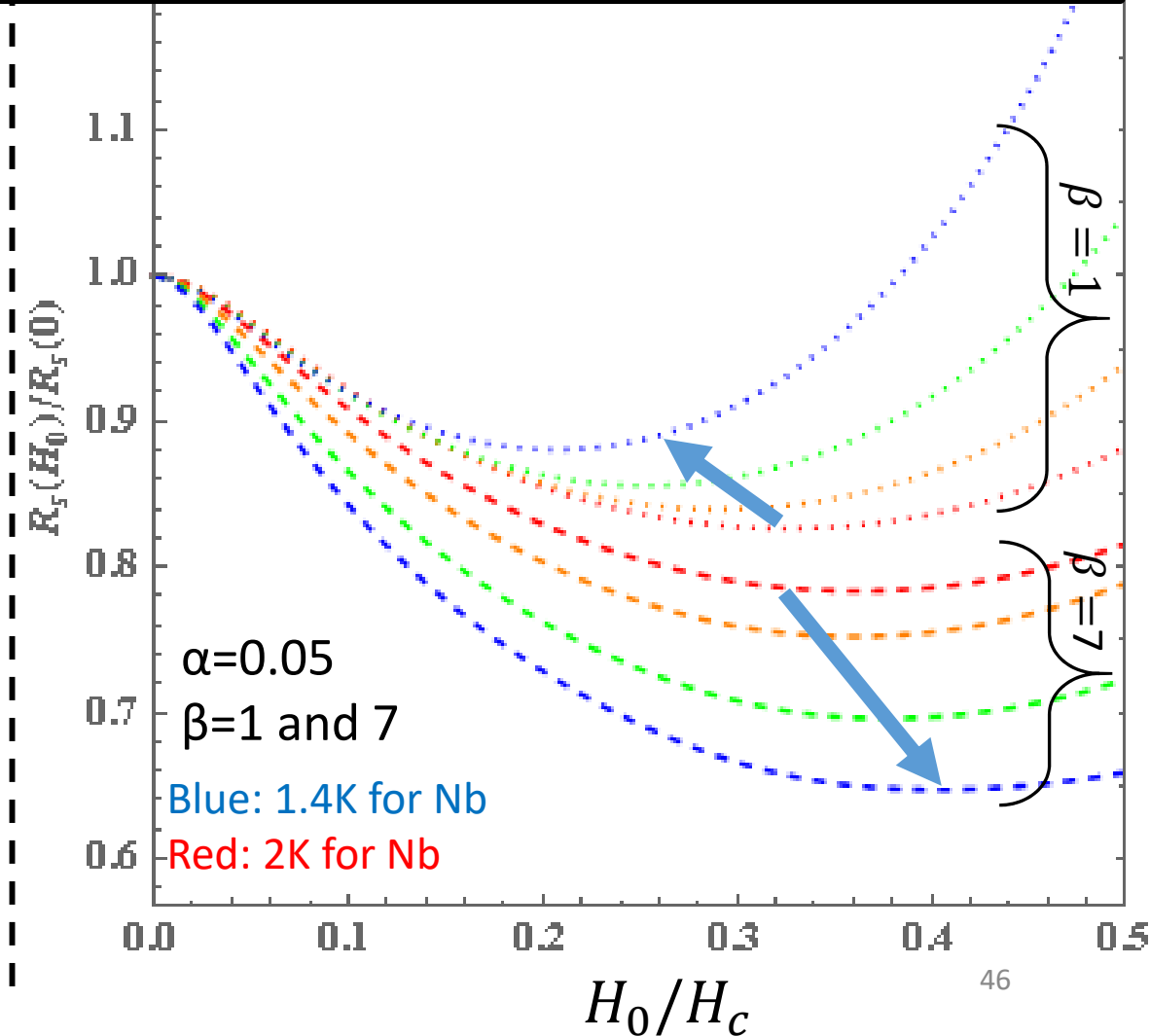
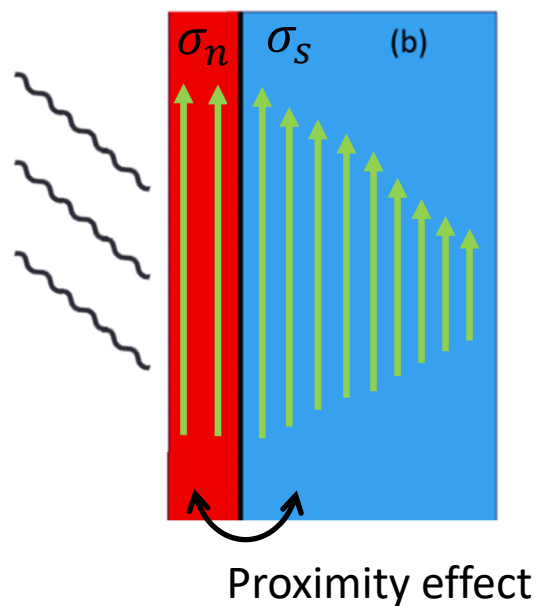
Different types of temperature dependence appear.

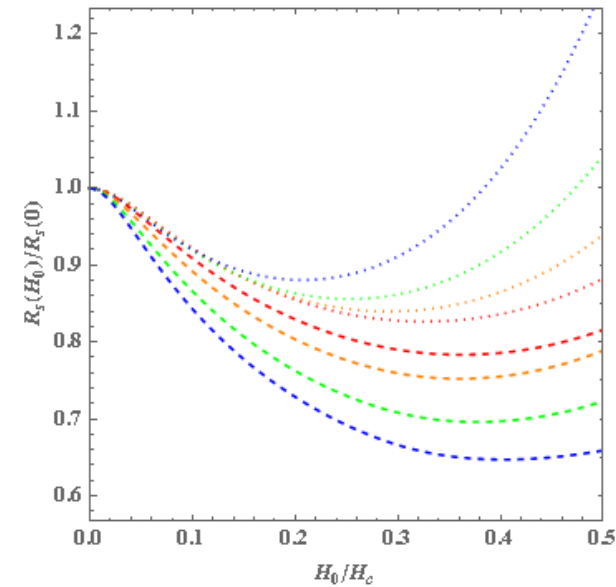
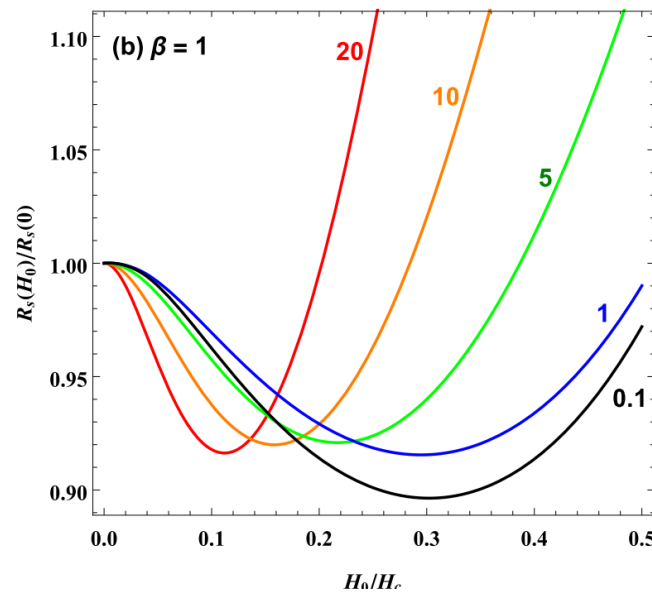
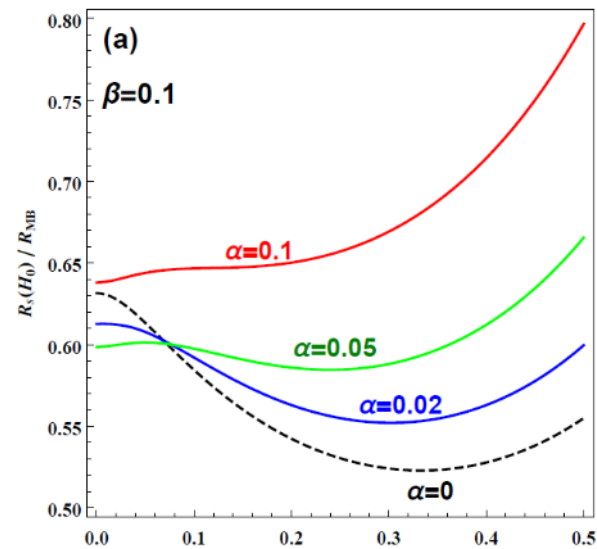
$$\alpha = \frac{N_n}{N_s} \frac{d}{\xi_S}$$

~thickness

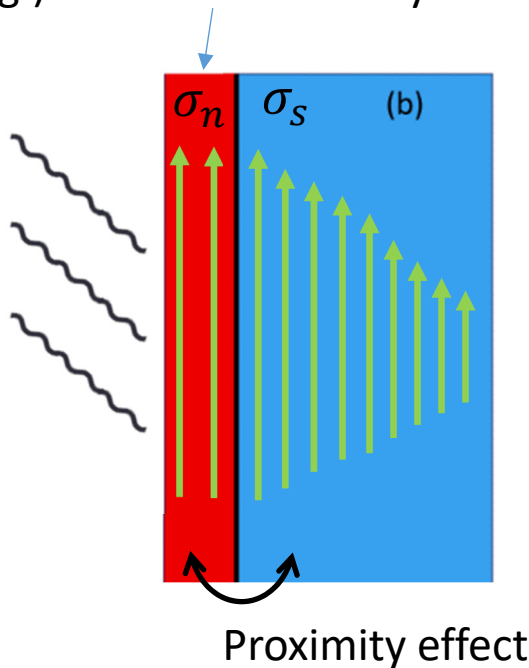
$$\beta = \frac{4e^2}{\hbar} R_B N_n \Delta d,$$

~barrier between N&S





e.g.) Nb suboxides or hydrides

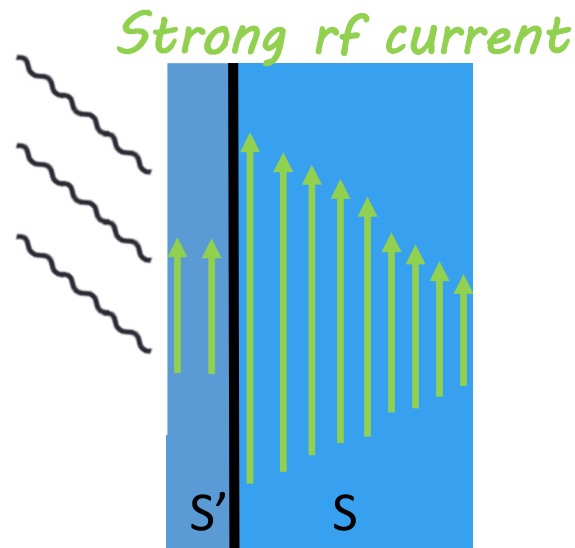


Many different shapes of $R_s(H)$ naturally result from the proximity coupled N-S model.

Field dependent surface resistance of

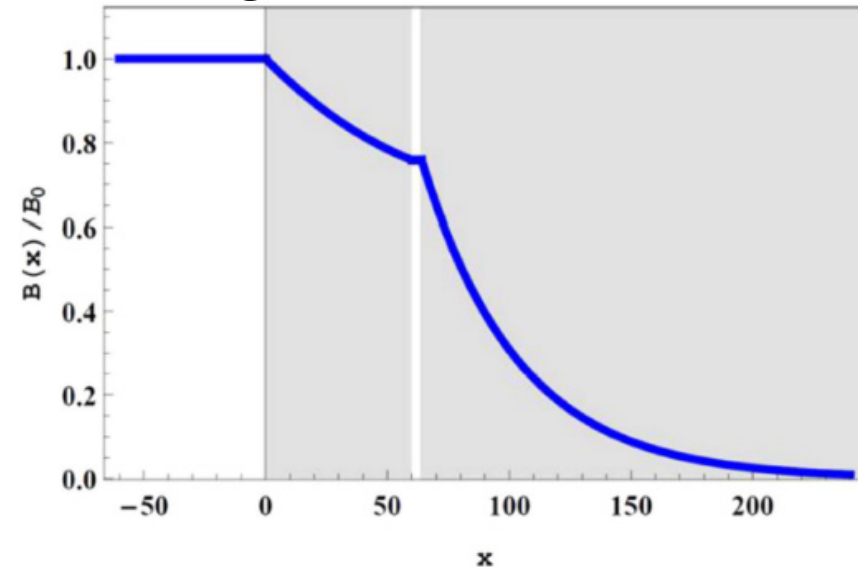
- *Nb-Nb structure*
- *Nb₃Sn-Nb structure*

T. Kubo and A. Gurevich, to be published

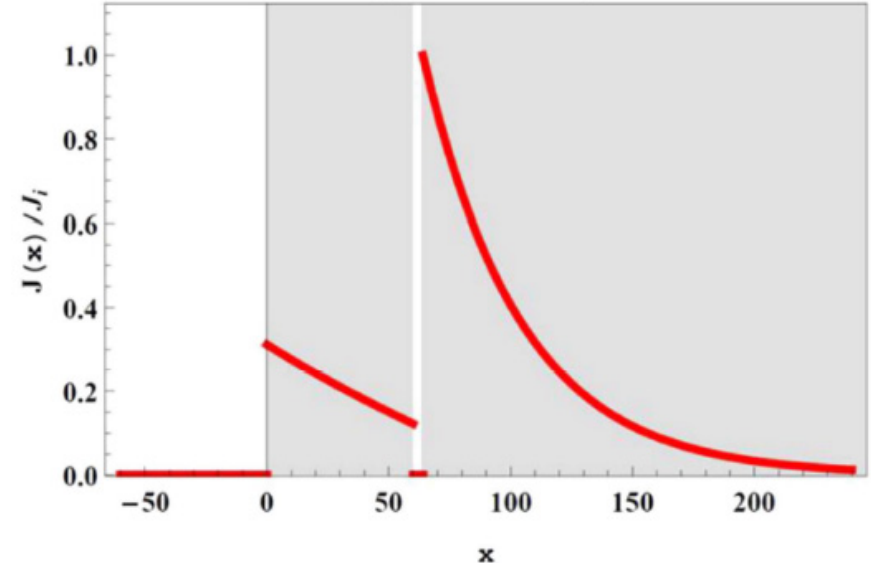


One of the most striking effects is *the reduction of the current density at the top layer with $\lambda'(>\lambda)$*

Magnetic field distribution



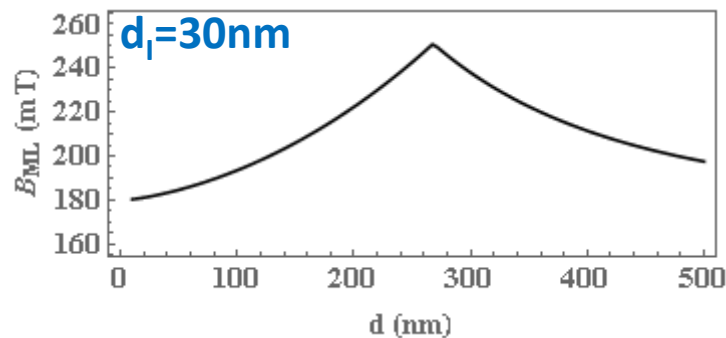
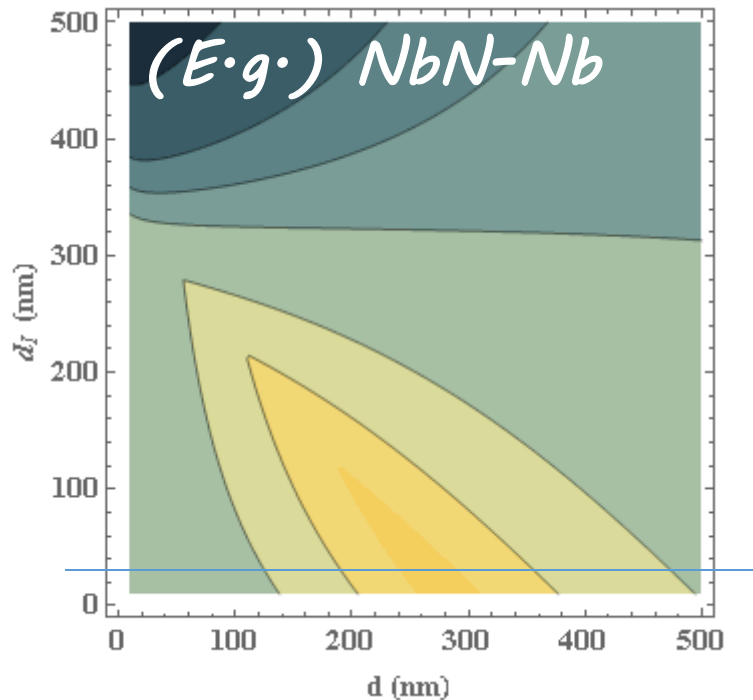
Current density distribution



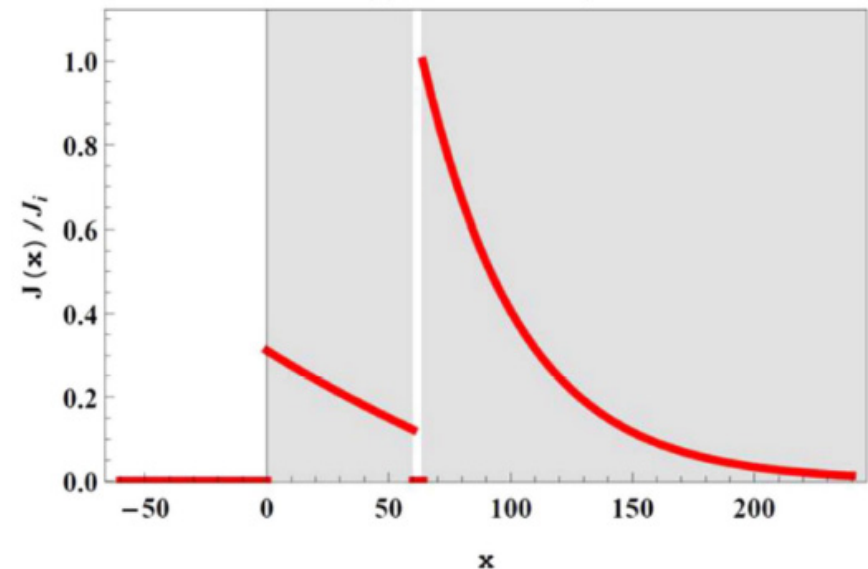
T. Kubo, Y. Iwashita, and T. Saeki, Appl. Phys. Lett. 104, 032603 (2014).
A. Gurevich, AIP Adv. 5, 017112 (2015).
T. Kubo, Supercond. Sci. Technol. 30, 023001 (2017).

One of the most striking effects is *the reduction of the current density at the top layer with $\lambda'(>\lambda)$*

This enhances the field limit



Current density distribution



T. Kubo, Y. Iwashita, and T. Saeki, Appl. Phys. Lett. 104, 032603 (2014).

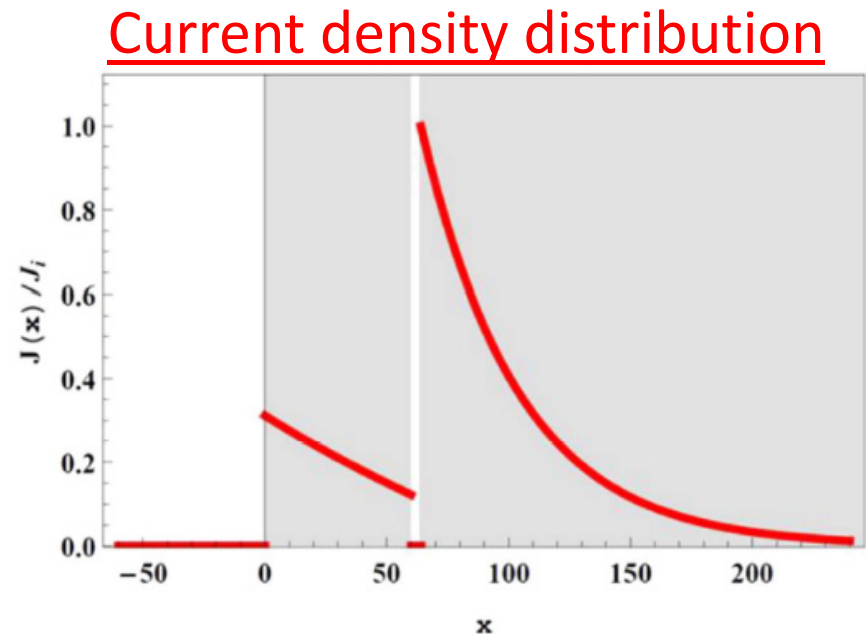
A. Gurevich, AIP Adv. 5, 017112 (2015).

T. Kubo, Supercond. Sci. Technol. 30, 023001 (2017).

One of the most striking effects is *the reduction of the current density* at the top layer with $\lambda'(>\lambda)$

*This enhances the field limit
and
should affects $R_s(H)$ too.*

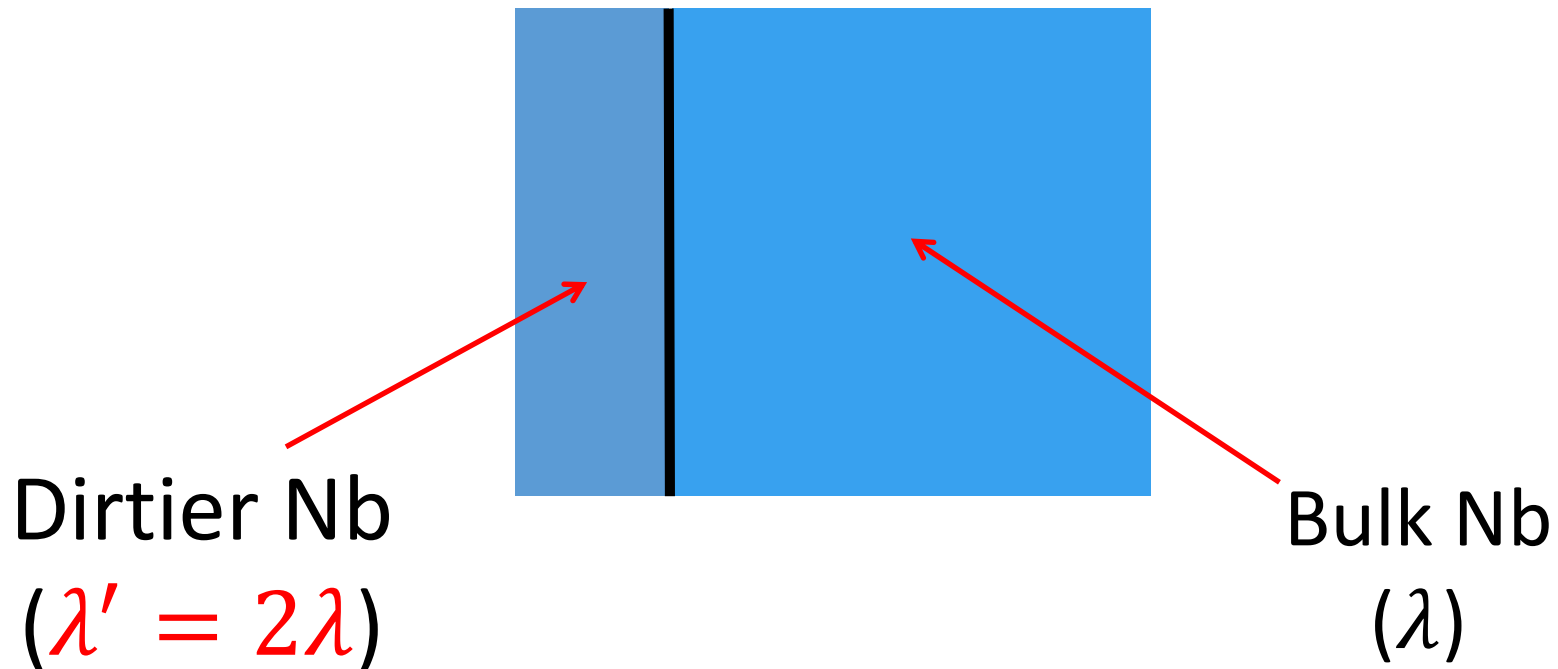
*Let's see what
happens to $R_s(H)$*



T. Kubo, Y. Iwashita, and T. Saeki, Appl. Phys. Lett. 104, 032603 (2014).
A. Gurevich, AIP Adv. 5, 017112 (2015).
T. Kubo, Supercond. Sci. Technol. 30, 023001 (2017).

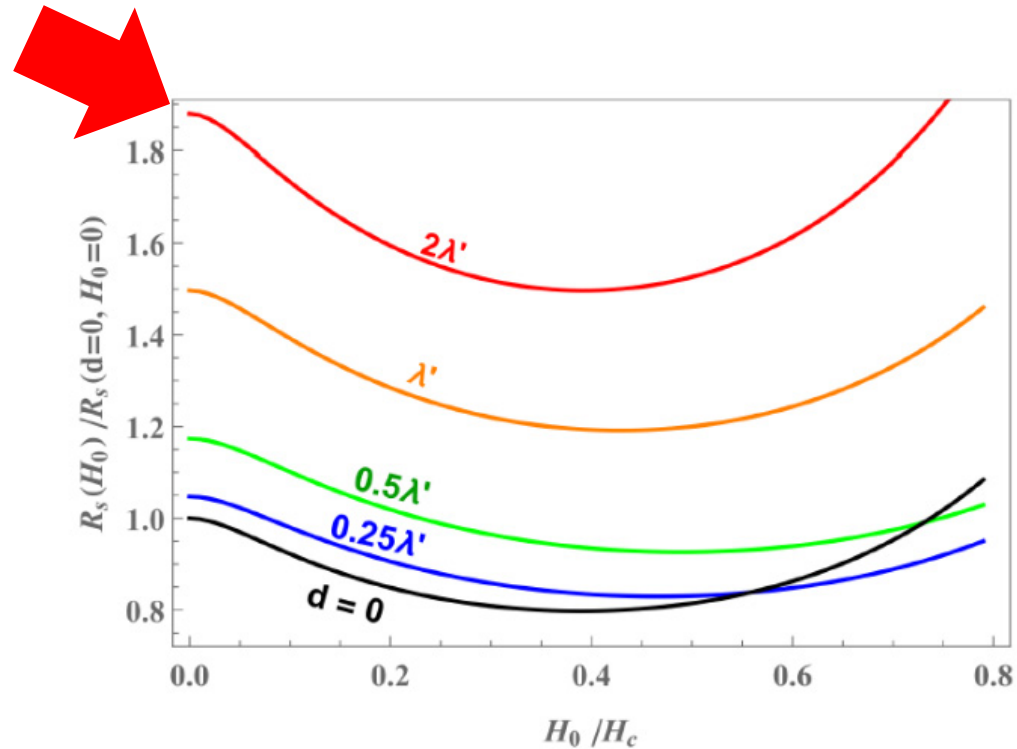
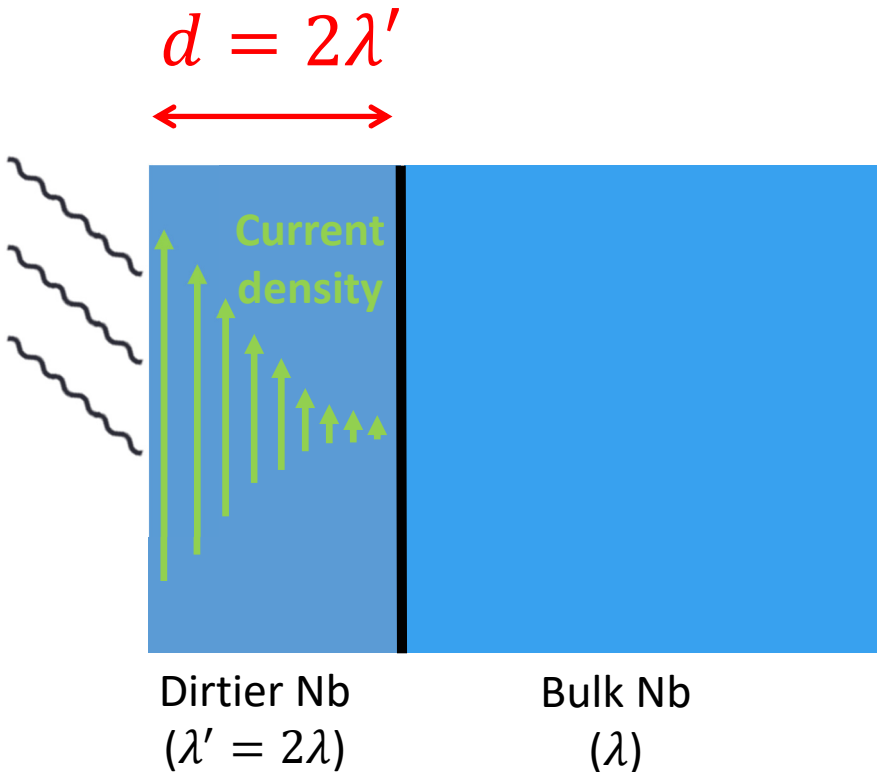
One of the most striking effects is *the reduction of the current density* at the top layer with $\lambda'(>\lambda)$

Example 1: Nb-Nb structure



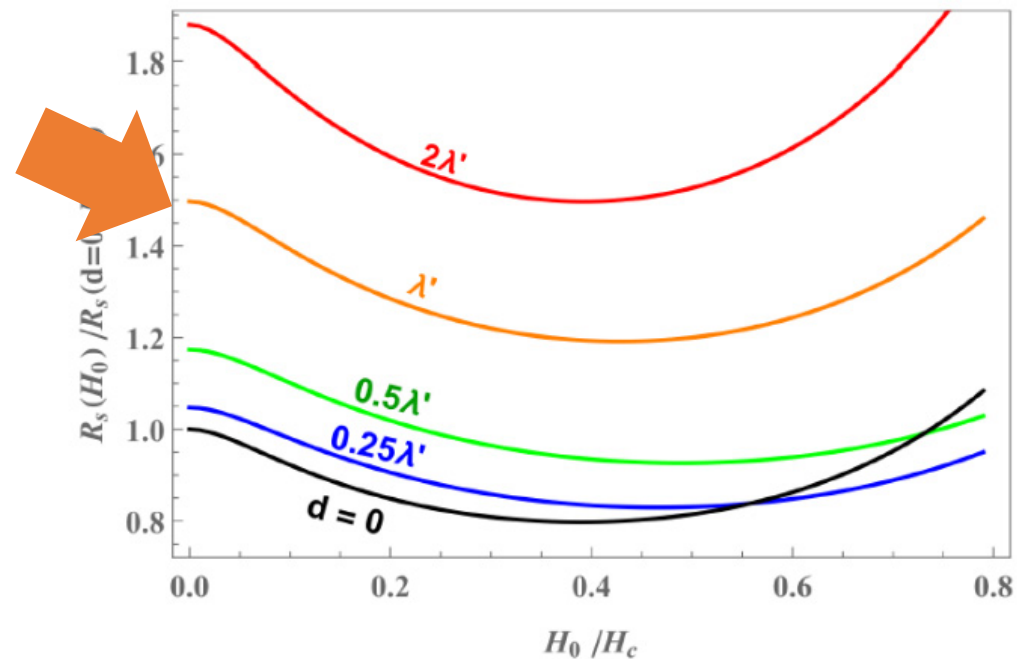
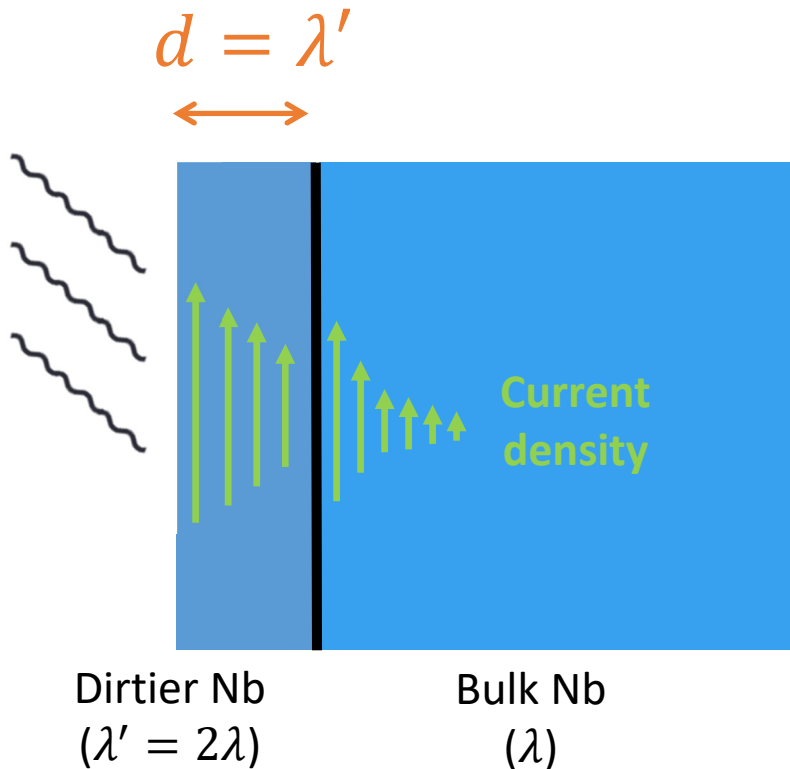
One of the most striking effects is *the reduction of the current density* at the top layer with $\lambda'(>\lambda)$

Example 1: Nb-Nb structure



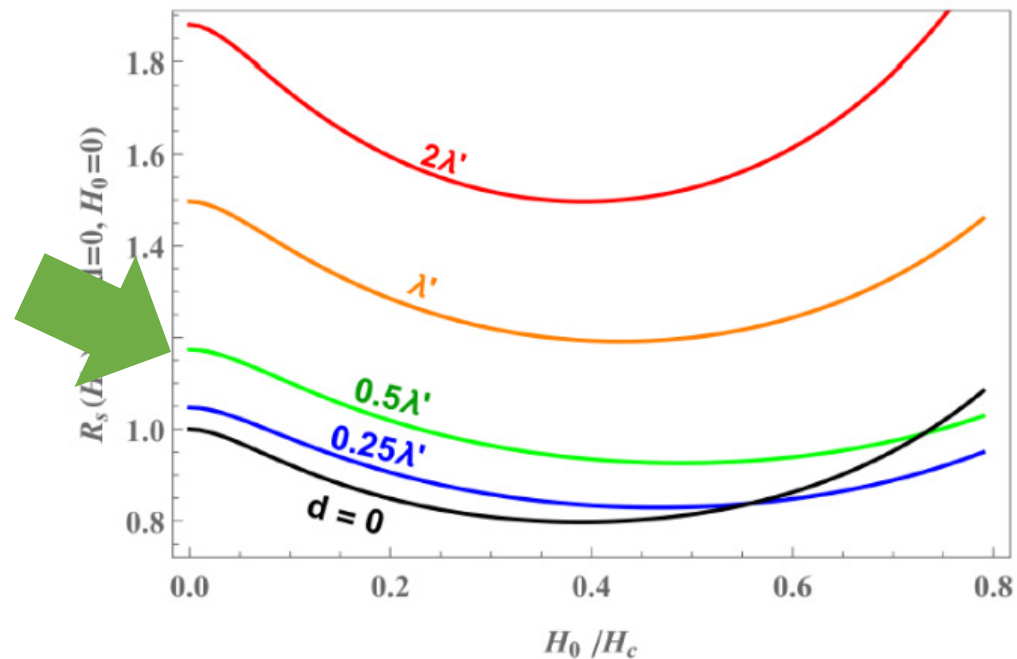
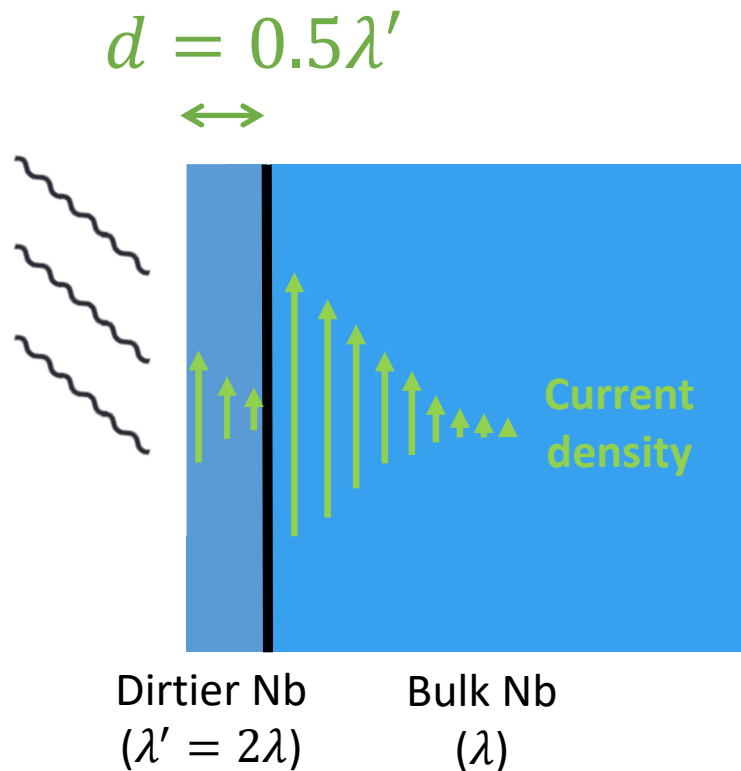
One of the most striking effects is *the reduction of the current density* at the top layer with $\lambda'(>\lambda)$

Example 1: Nb-Nb structure



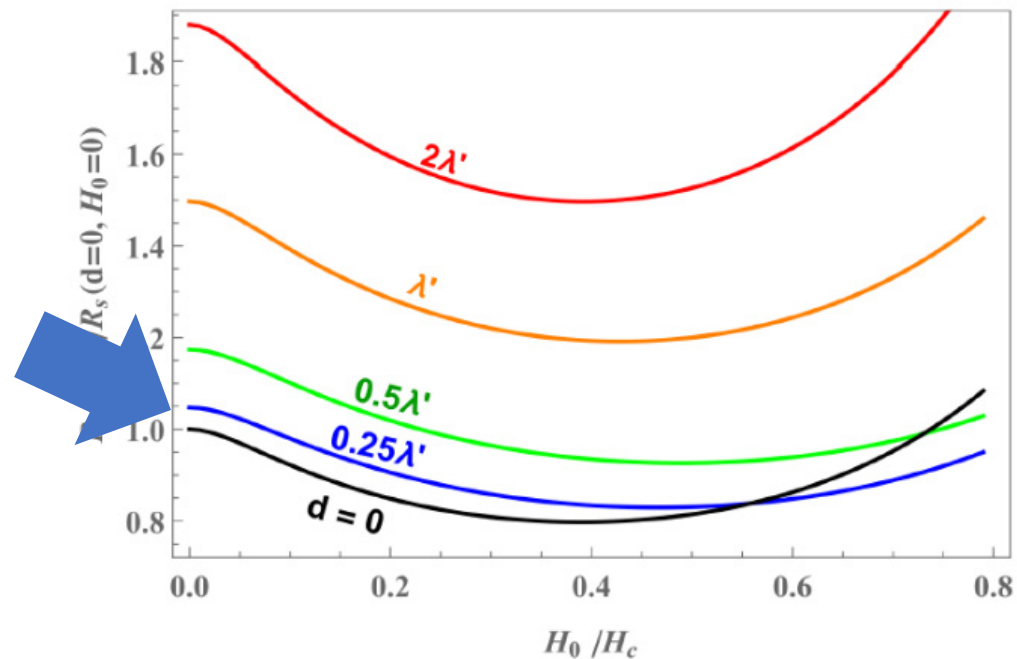
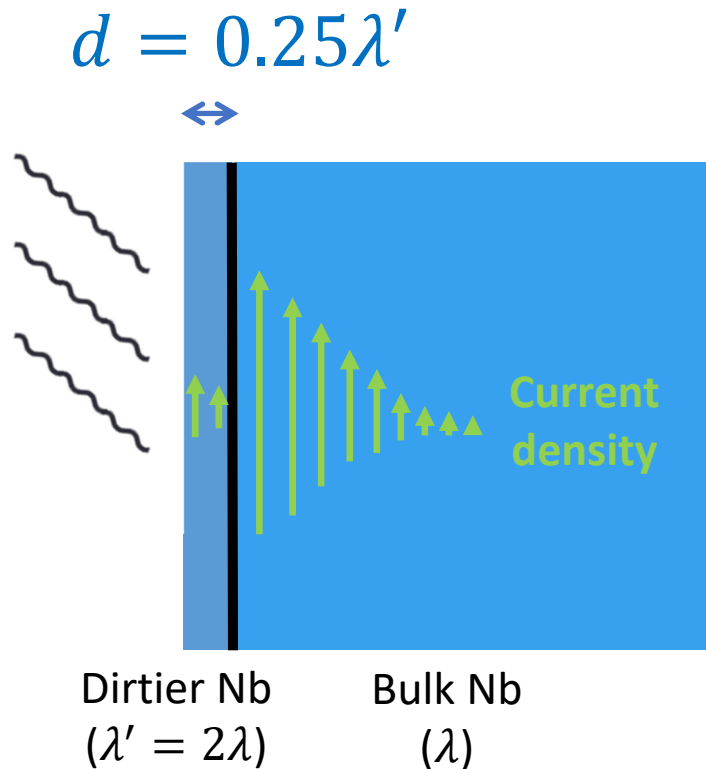
One of the most striking effects is *the reduction of the current density* at the top layer with $\lambda'(>\lambda)$

Example 1: Nb-Nb structure



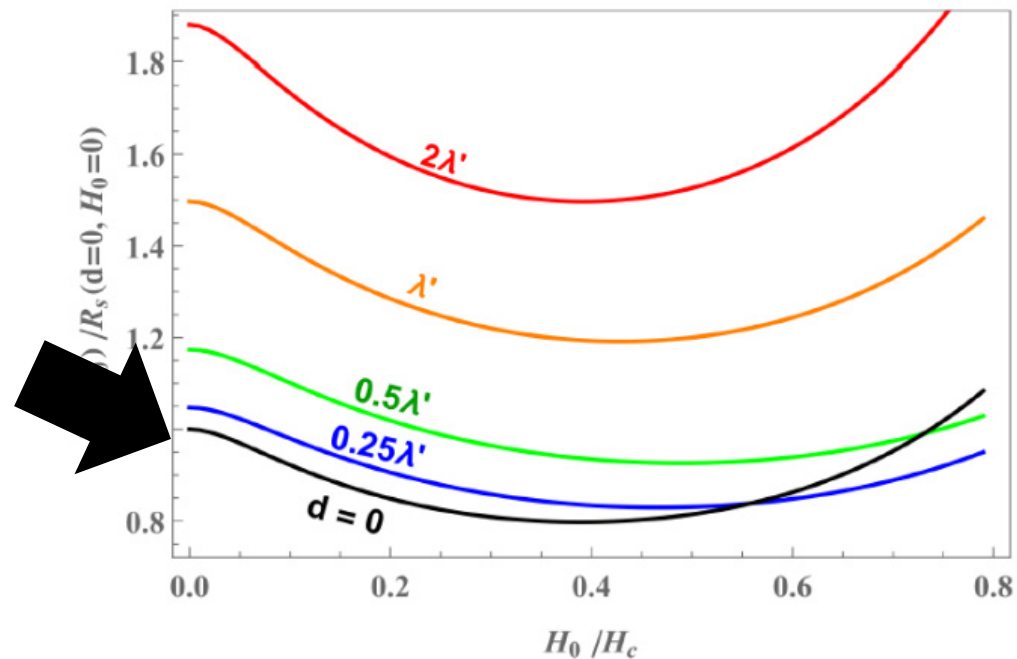
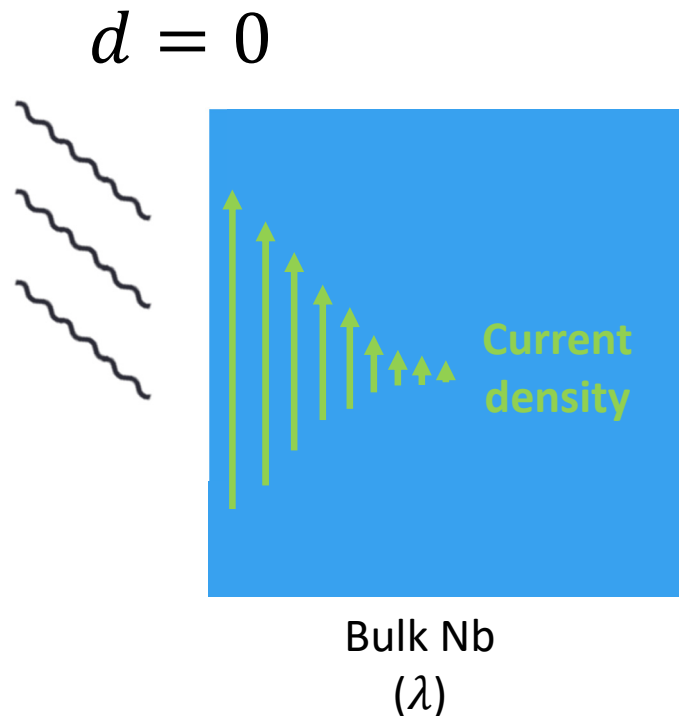
One of the most striking effects is *the reduction of the current density* at the top layer with $\lambda'(>\lambda)$

Example 1: Nb-Nb structure



One of the most striking effects is *the reduction of the current density* at the top layer with $\lambda'(>\lambda)$

Example 1: Nb-Nb structure

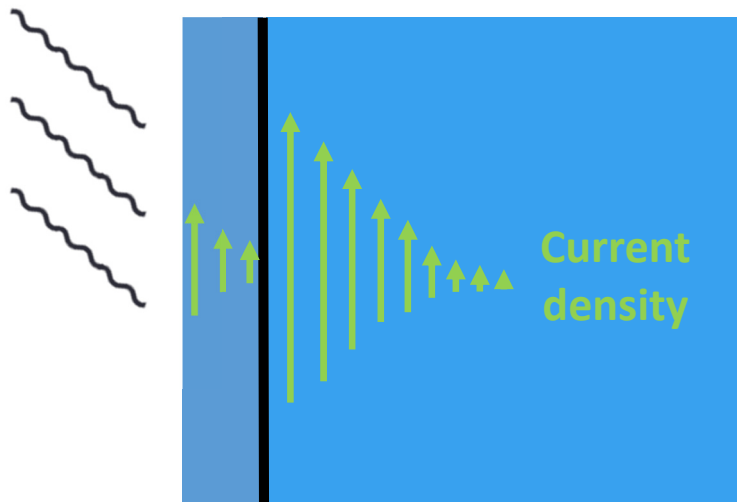


One of the most striking effects is *the reduction of the current density* at the top layer with $\lambda'(>\lambda)$

Example 1: Nb-Nb structure

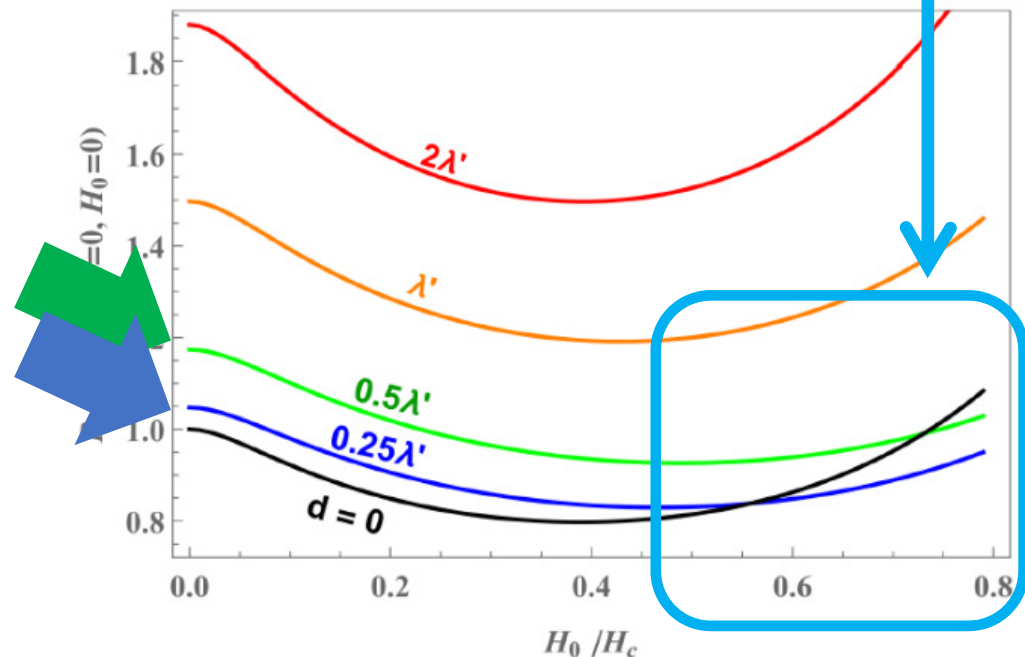
A dirtier Nb layer with $d \lesssim \lambda'$ cures the Q slope:

→ “baking-like” effect.

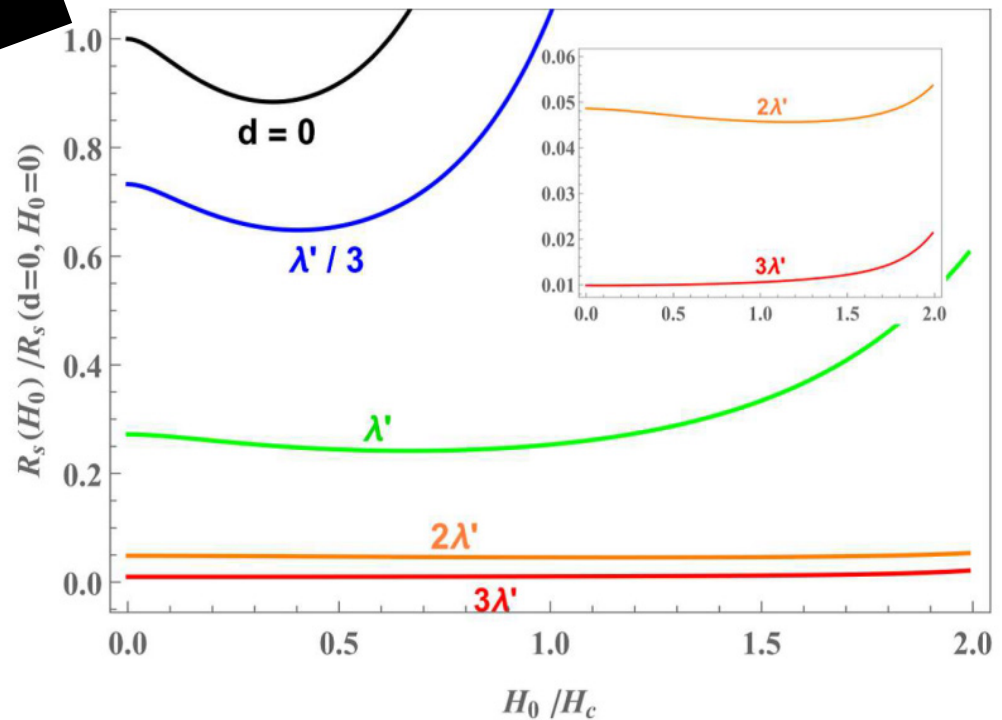
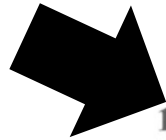
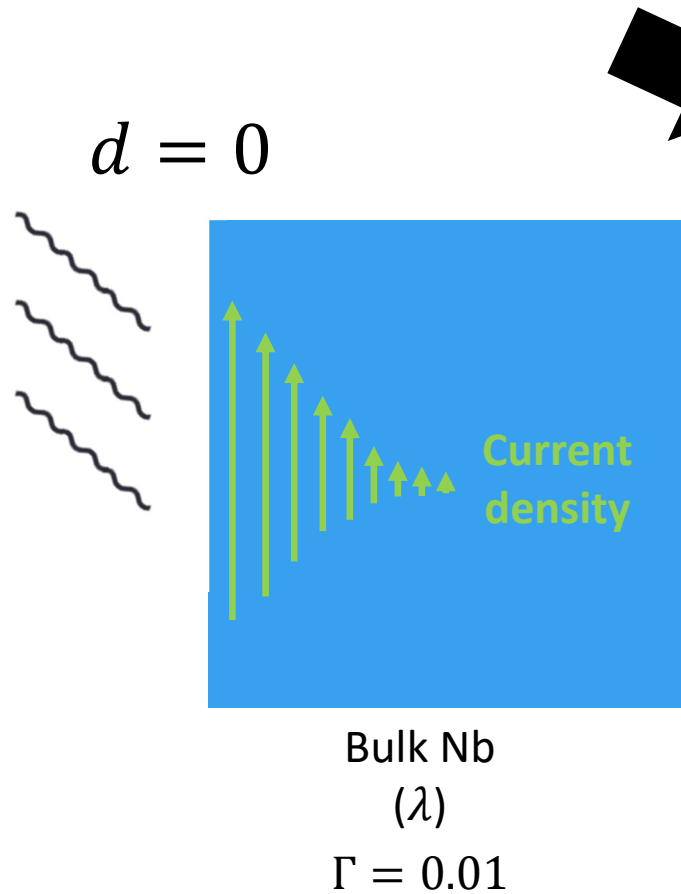


Dirtier Nb
($\lambda' = 2\lambda$)

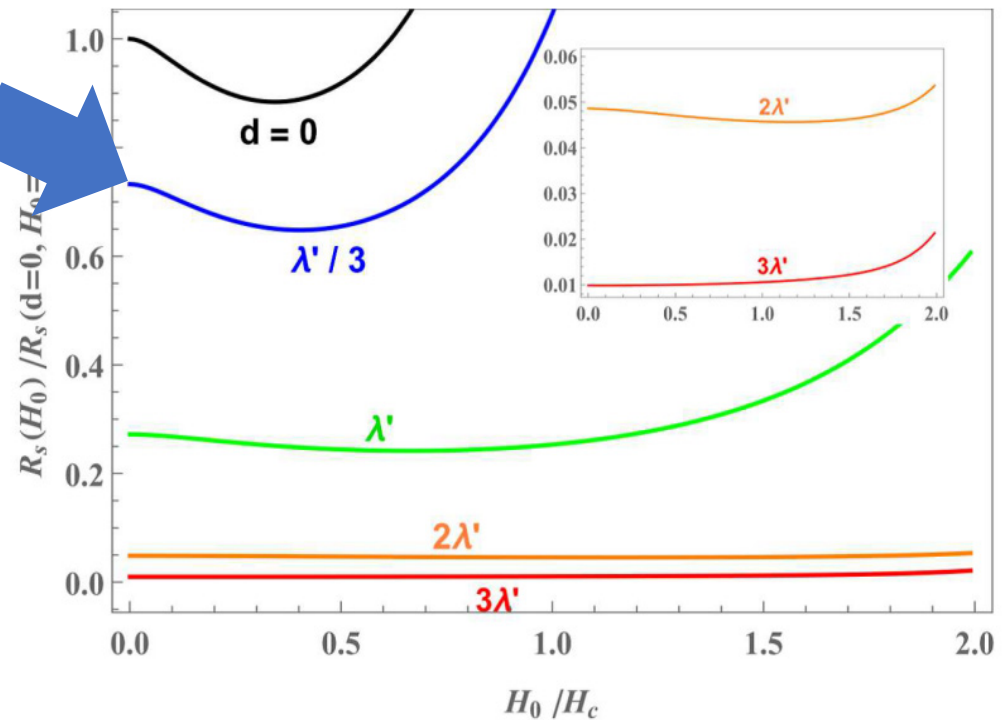
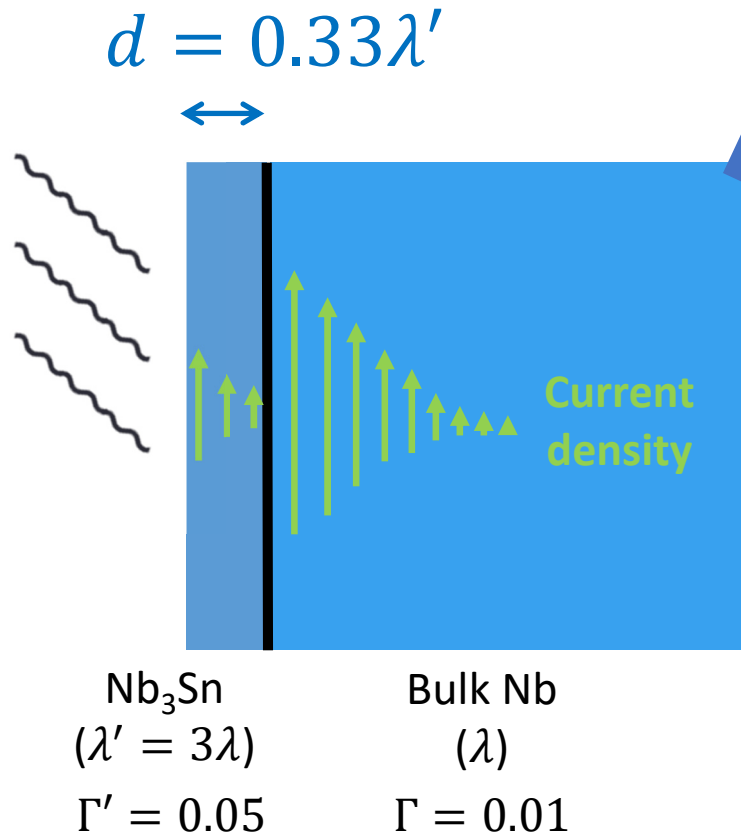
Bulk Nb
(λ)



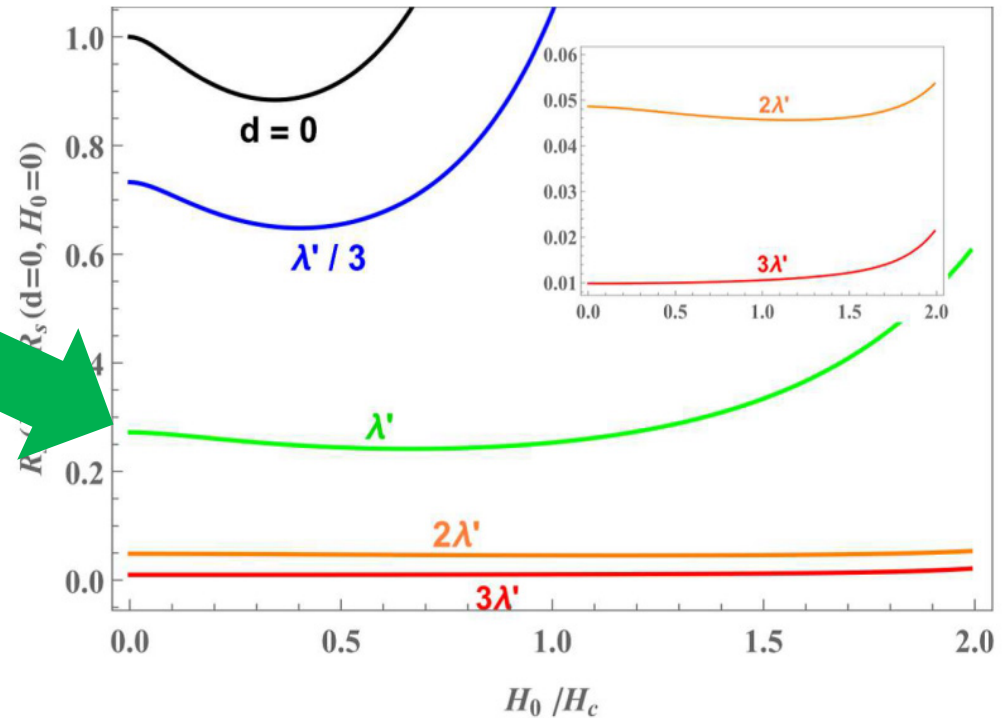
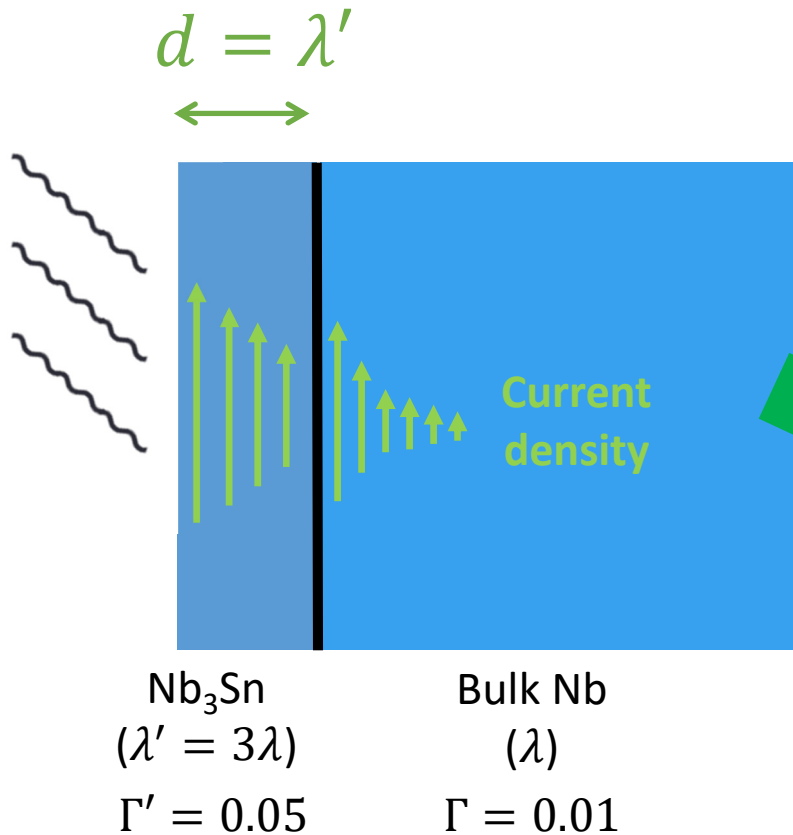
Example 2: Nb_3Sn -Nb structure



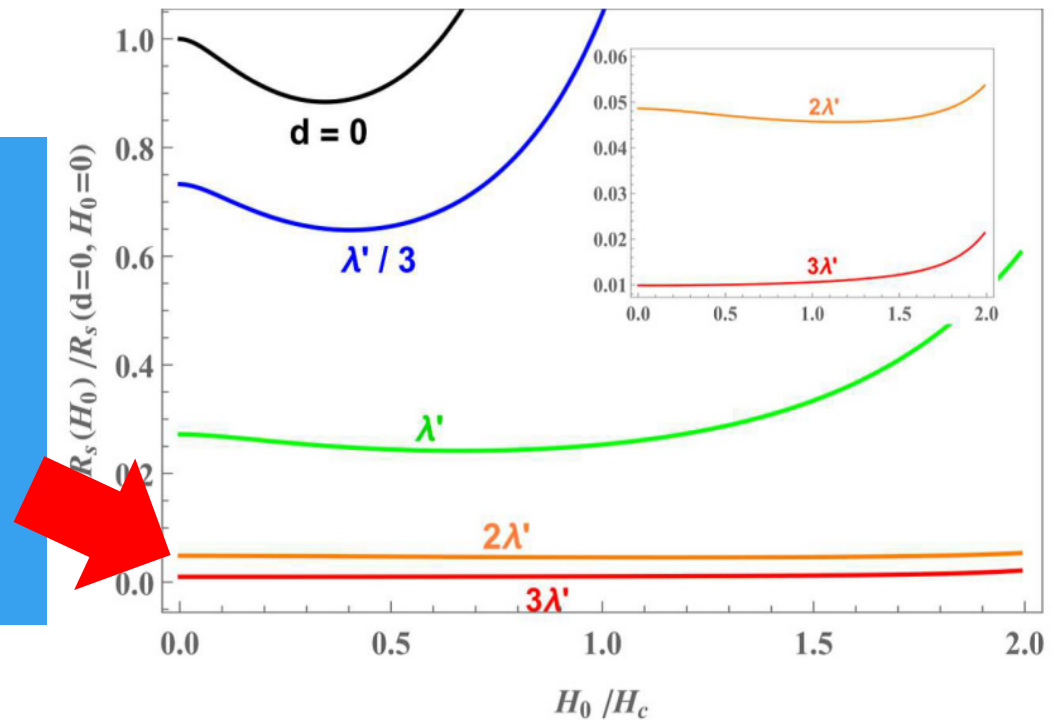
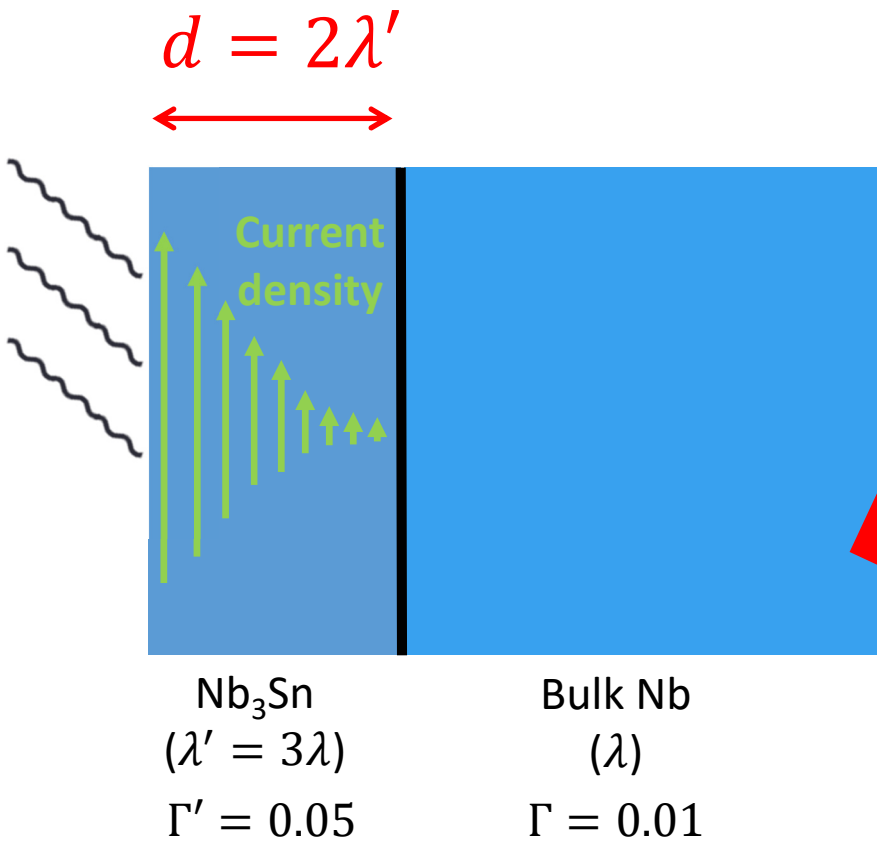
Example 2: Nb_3Sn -Nb structure



Example 2: Nb_3Sn -Nb structure



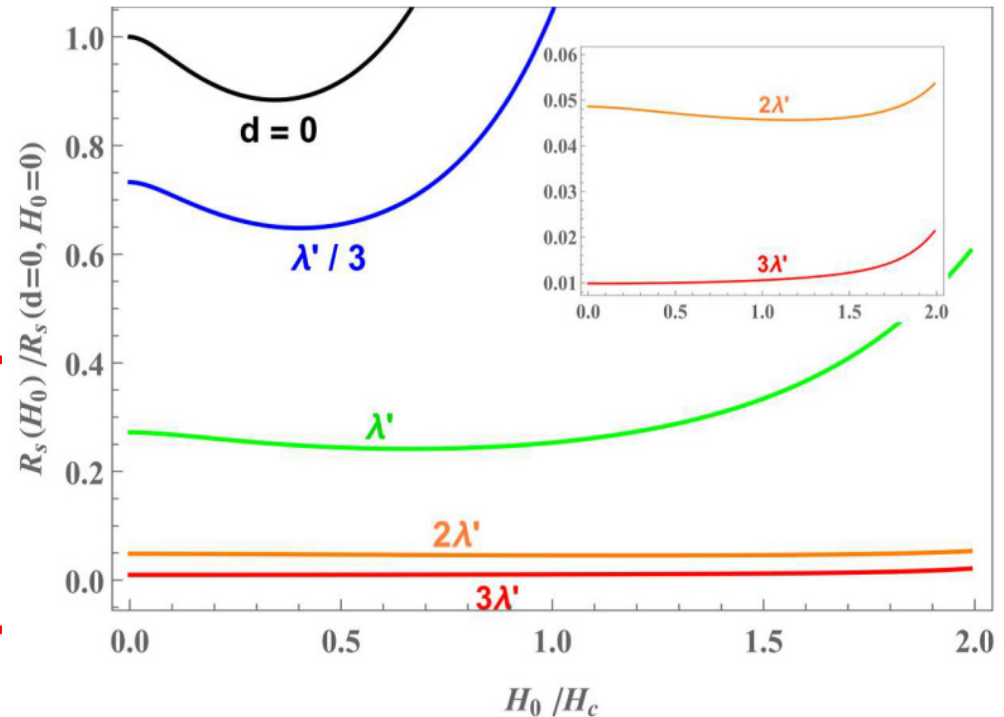
Example 2: Nb_3Sn -Nb structure



Example 2: Nb_3Sn -Nb structure

A. Gurevich, Appl. Phys. Lett. 88, 012511 (2006).
T. Kubo, Y. Iwashita, and T. Saeki, Appl. Phys. Lett. 104, 032603 (2014).
A. Gurevich, AIP Adv. 5, 017112 (2015).
T. Kubo, Supercond. Sci. Technol. 30, 023001 (2017).

The optimum thicknesses
for higher fields are given
by $d \sim \lambda'$



Here, Q drops are not
significant even at $H > H_c^{(Nb)}$

- We have developed a theory of **field dependent surface resistance** of a dirty superconductor in a strong RF field, **taking into account realistic materials features** based on the BCS theory.
- **Many different field dependencies $R_s(H_0)$ naturally result from realistic material features** such as a finite Γ , Γ_p or a thin N or S layer on the surface.
- The surface resistance **can be minimized** by engineering optimum impurity concentration or properties of the surface normal layer.
- To compare the theory with experiments and to utilize the theoretical consequences to improve cavity performances, **we need measurements of multiple parameters characterizing a particular material** (e.g., d and σ_n of the N layer, R_B , and Γ parameters) **as well as the way these parameters change after different materials treatments.**

Wait!

*How about nonequilibrium
effects on $R_s(H)$???*

Part 3

Non-equilibrium effects under strong RF current

T. Kubo and A. Gurevich, to be published

The equation of motion for the Green's function in Keldysh-Nambu space for a dirty superconductor:

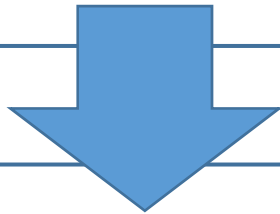
$$i\hbar D\check{\partial} \otimes [\check{G} \otimes (\check{\partial} \otimes \check{G})] = \left(\check{\tau}_z i\hbar \frac{\partial}{\partial t} + \check{\Delta} - \check{\Sigma} \right) \otimes \check{G} - \check{G} \otimes \left(\check{\tau}_z i\hbar \frac{\partial}{\partial t} + \check{\Delta} - \check{\Sigma} \right) \quad \check{G}(\mathbf{R}, t_1, t_2) = \begin{pmatrix} \hat{G}^R & \hat{G}^K \\ 0 & \hat{G}^A \end{pmatrix}$$

Consider the first order of slow variation and **equilibrium phonons**.

We can evaluate **non-equilibrium effects under the strong RF current** on the distribution function:

$$f(\epsilon) = \tanh \frac{\epsilon}{2k_B T}$$

Equilibrium (Fermi-Dirac)



$$f(\epsilon, t) = \tanh \frac{\epsilon}{2k_B T} + \delta f(\epsilon, t)$$

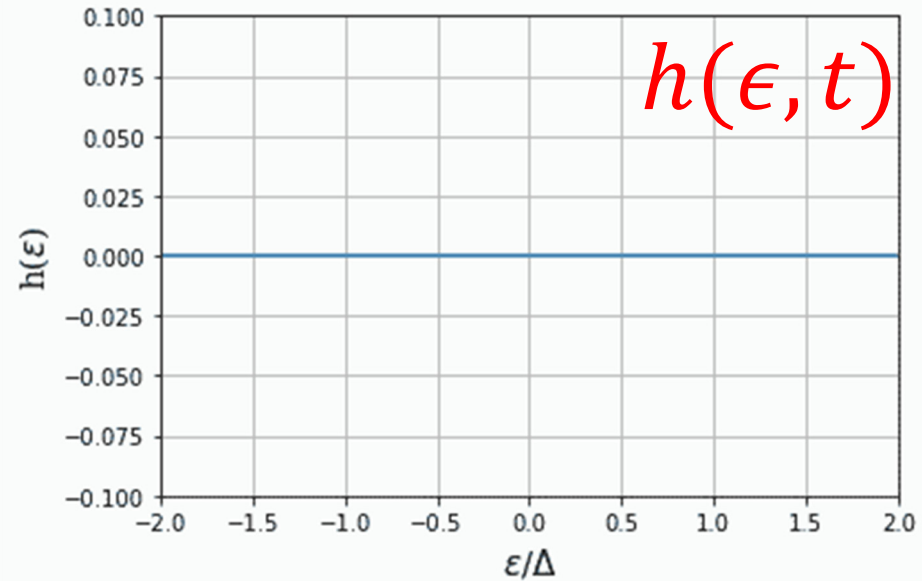
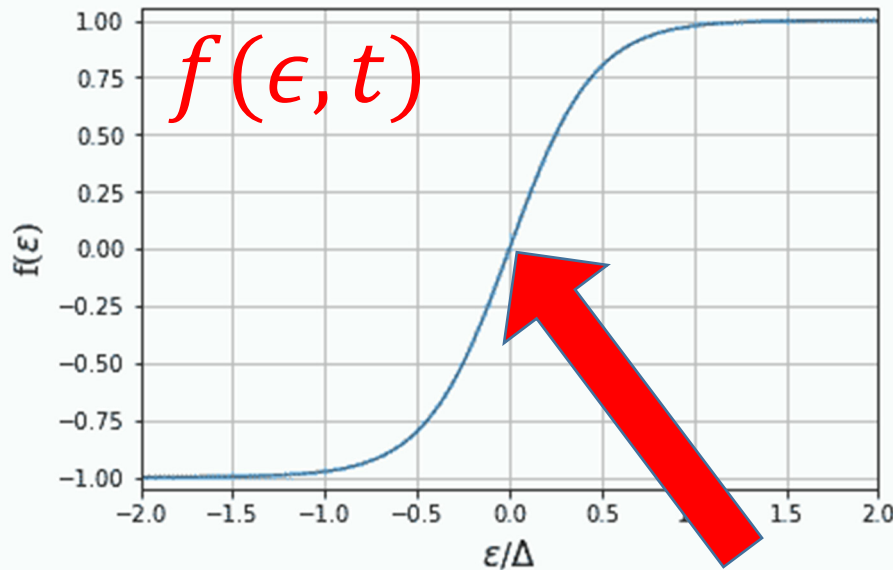
non-equilibrium effect

$$\delta f(\epsilon, t) = \frac{h(\epsilon, t)}{\cosh^2 \frac{\epsilon}{2k_B T}}$$

Example (1)

Contain animations.
See in Slide Show Mode.

- RF frequency $\sim 1\text{GHz}$
- $T=4\text{K}$
- Broadening parameter $\Gamma = 0.05$



Can you see the oscillation?
Its effect is very small.

non-equilibrium effect

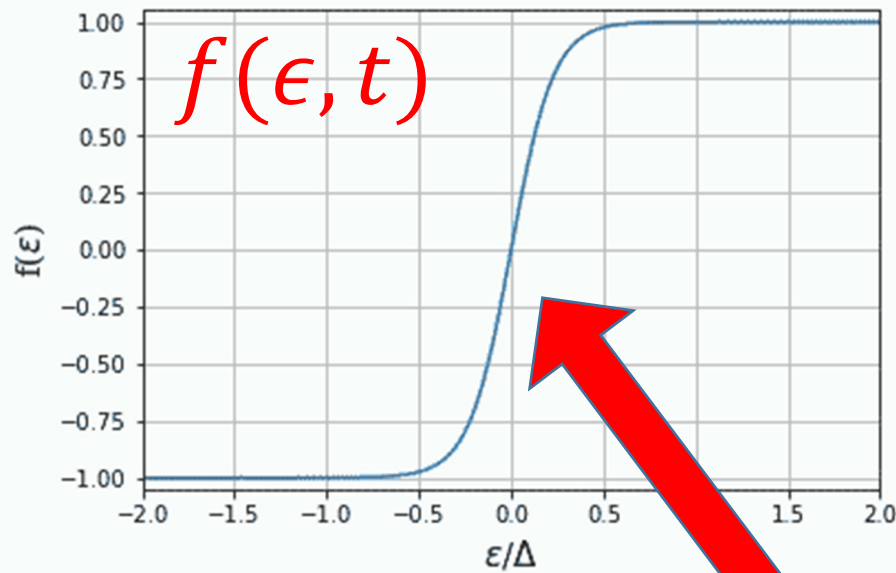
$$f(\epsilon, t) = \tanh \frac{\epsilon}{2k_B T} + \delta f(\epsilon, t)$$

$$\delta f(\epsilon, t) = \frac{h(\epsilon, t)}{\cosh^2 \frac{\epsilon}{2k_B T}}$$

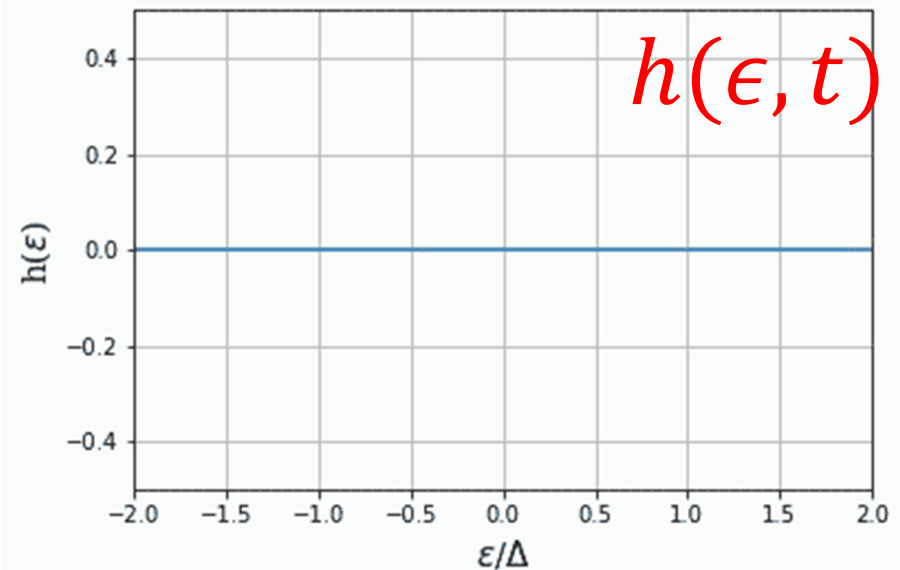
Example (2)

Contain animations.
See in [Slide Show Mode](#).

- RF frequency $\sim 1\text{GHz}$
- $T=2\text{K}$
- Broadening parameter $\Gamma = 0.1$



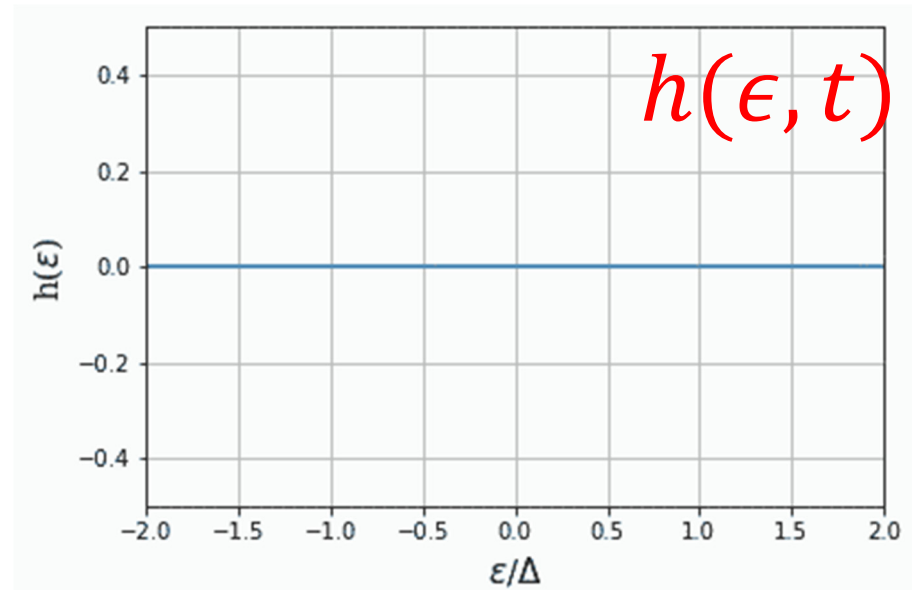
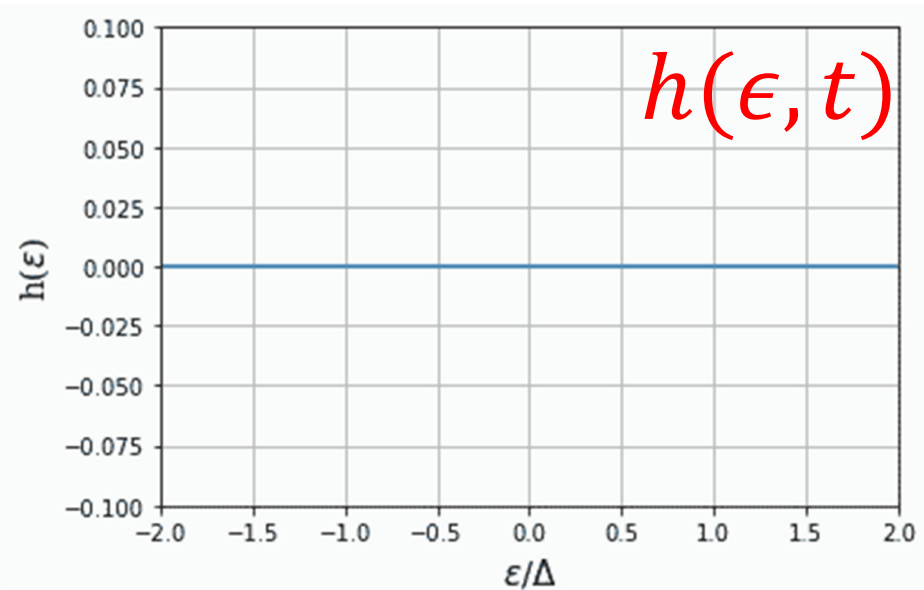
Oscillating $f(\epsilon, t)$



non-equilibrium effect

$$f(\epsilon, t) = \tanh \frac{\epsilon}{2k_B T} + \delta f(\epsilon, t)$$

$$\delta f(\epsilon, t) = \frac{h(\epsilon, t)}{\cosh^2 \frac{\epsilon}{2k_B T}}$$



- *Nonequilibrium effects in $R_s(H)$ becomes significant at lower temperature, higher frequencies, and higher fields.*
- *We started to address this problem.*
Look forward to next conferences

Vielen Dank für Ihre Aufmerksamkeit!

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- **Non-equilibrium effects** becomes significant at lower temperatures, higher frequencies and higher fields. **We have already started to address this problem.** Look forward to SRF2021!