Nonlinear Dynamics and Dissipation of Vortex Lines Driven by Strong RF Fields. A new mechanism of the extended Q(H) rise*

Manula Pathirana# and Alex Gurevich

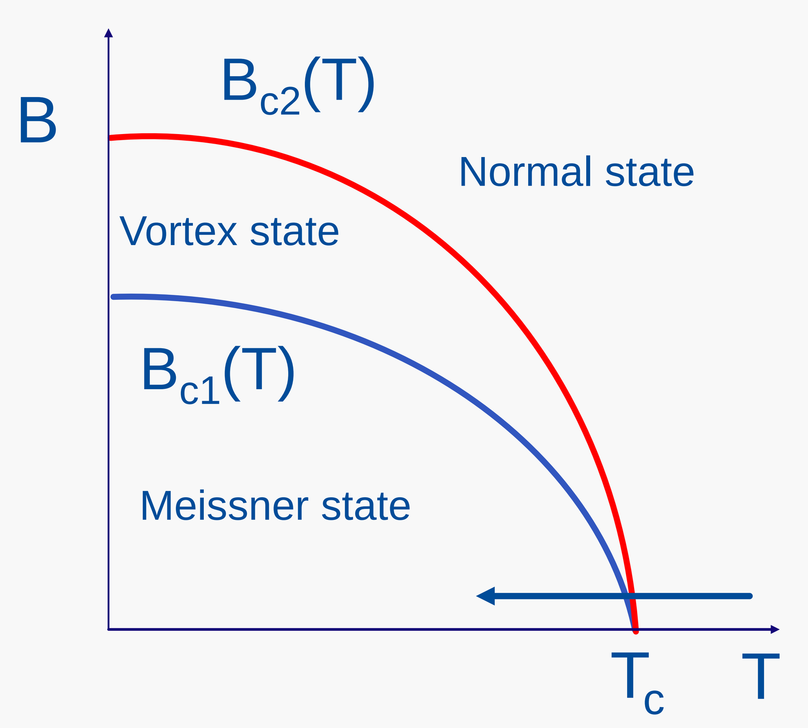
Center for Accelerator Science, Old Dominion University, Norfolk, VA 23529, USA

ABSTRACT

Trapped vortices can contribute to a residual surface resistance of superconducting radio frequency (SRF) cavities but the nonlinear dynamics of flexible vortex lines driven by strong rf currents has not been yet understood. Here we report extensive numerical simulations of large-amplitude oscillations of a trapped vortex line under strong rf magnetic fields. The rf power dissipated by an oscillating vortex segment driven by the rf Meissner currents was calculated by taking into account the nonlinear vortex line tension, vortex mass and a nonlinear Larkin-Ovchinnikov and overheating viscous drag force. We calculated the field dependence of the surface resistance R_s and showed that at low frequencies R_s (H) increases with H but as the frequency increases, R_s (H) becomes a nonmonotonic function of H which decreases with H at higher fields. These results suggest that trapped vortices can contribute to the extended Q(H) rise observed on the SRF cavities.

GENERATION OF TRAPPED VORTICES [1]

- Vortex state is favorable at B > B_{c1}(T) but because $B_{c1}(T_c) \rightarrow 0$, vortices can be generated in the cavity during the cool-down through T_c
- Most of the vortices exit as T it reduced below T_c but some vortices get pinned by materials defects.
- Trapped vortices contribute to the residual surface resistance
- How does the vortex residual resistance depend on B at strong RF field?

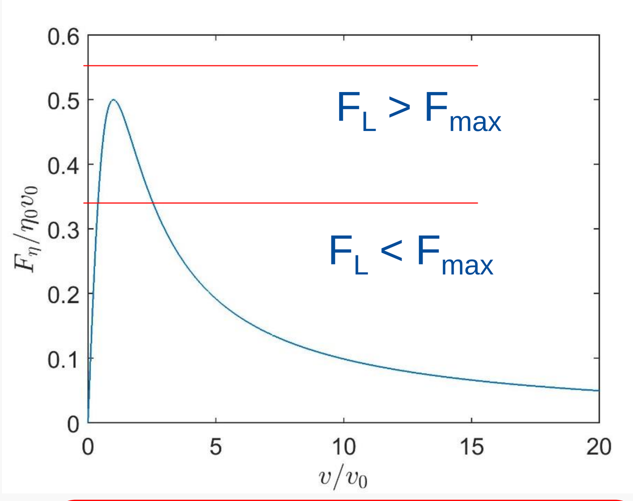


LARKIN - OVCHINNIKOV VORTEX DRAG FORCE [2]

- Quasiparticles in the core are accelerated by the applied electric field.
- As the energy of quasiparticles reach the SC gap they diffuse away from the vortex core. Because of quasiparticle depletion in the core the viscosity decreases as the velocity v increases.
- The LO instability has been observed on many superconductors [3,4]
- The viscous drag force can balance the driving Lorentz force F_L only if

$$F_L < F_{max} = \eta_0 v_0/2$$

- Jumps of straight vortices at $F_L > F_{max}$ but curved vortices are balanced by the line tension
- For thin film Nb v_0 ~ *0.1 km/s – 1km/s* [5].

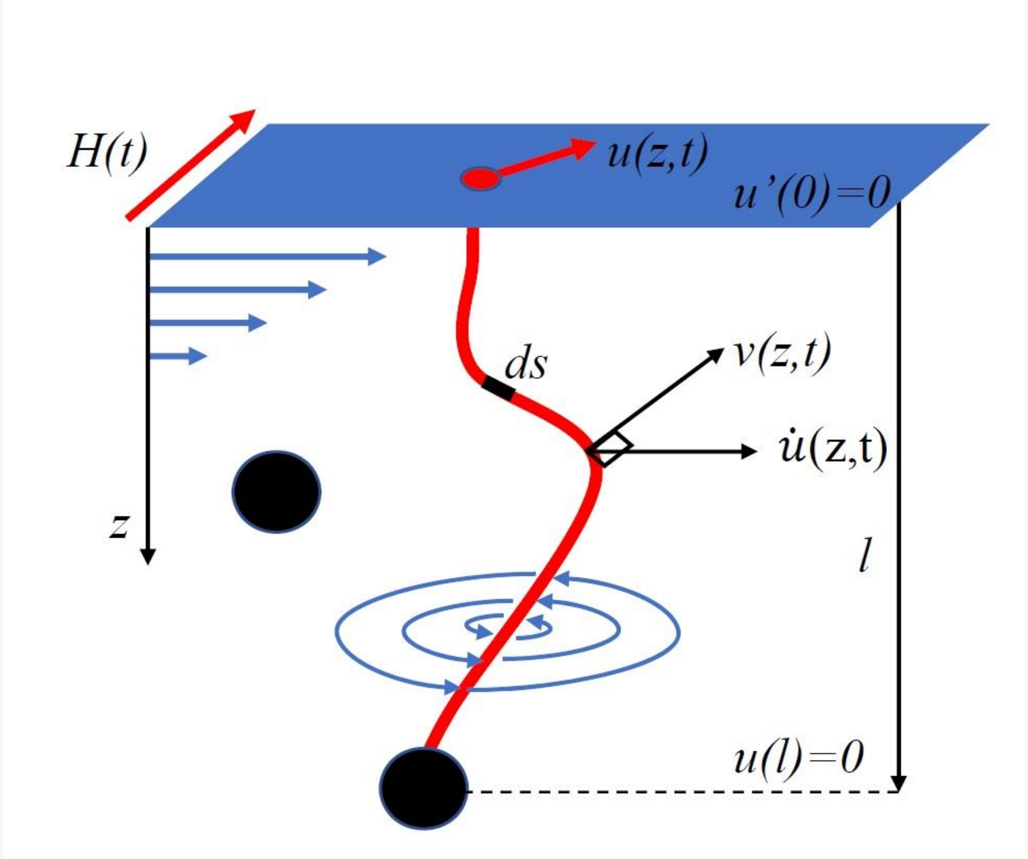


$$F_\eta = \frac{\eta_0 v}{(1+v^2/v_0^2)}$$

$$\eta_0 = \phi_0 B_{c2}/\rho_n$$

Bardeen-Stephen viscosity

DYNAMICS OF A TRAPPED VORTEX UNDER RF CURRENT



Dynamic equation for the local velocity normal to a flexible vortex line:

$$m\dot{v} + \eta(v)v = \frac{\epsilon}{R} - Fexp(-z/\lambda)sin(\omega t)$$

- m-vortex mass, η(v)-nonlinear viscosity, ε/R-elastic force, R is the local curvature radius of a vortex with the line tension ε, F- amplitude of the rf driving force
- The effective vortex mass mostly results from the kinetic energy of quasiparticles in the normal core [6]. For Nb, *m ~ 7 x 10^{-22} kg/m*

$$m_{suhl} \approx \frac{2}{\pi^3} K_F m_e$$

Full dynamic equation for the dimensionless vortex displacement u(z,t) =x(z,t)/λ:

$$\mu\ddot{u}(1+u'^2)^2 + \frac{\gamma\dot{u}\left(1+u'^2\right)^2}{1+\alpha\dot{u}^2(1+u'^2)} = u'' - \beta_1 e^{-z}(1+u'^2)^{3/2}$$

$$\gamma = f/f_0 \text{ with } f_0 = B_{c1}\rho_n/B_{c2}\lambda^2\mu_0$$

$$\alpha = \alpha_0\gamma^2 \text{ with } \alpha_0 = (\lambda f_0/v_0)^2$$

$$\beta_1 = \beta sin(2\pi t) \text{ with } \beta = B/B_{c1}$$

$$\mu = \mu_1\gamma^2 \text{ with } \mu_1 = \lambda^2 f_0^2 m\mu_0/\phi_0 B_{c1}$$

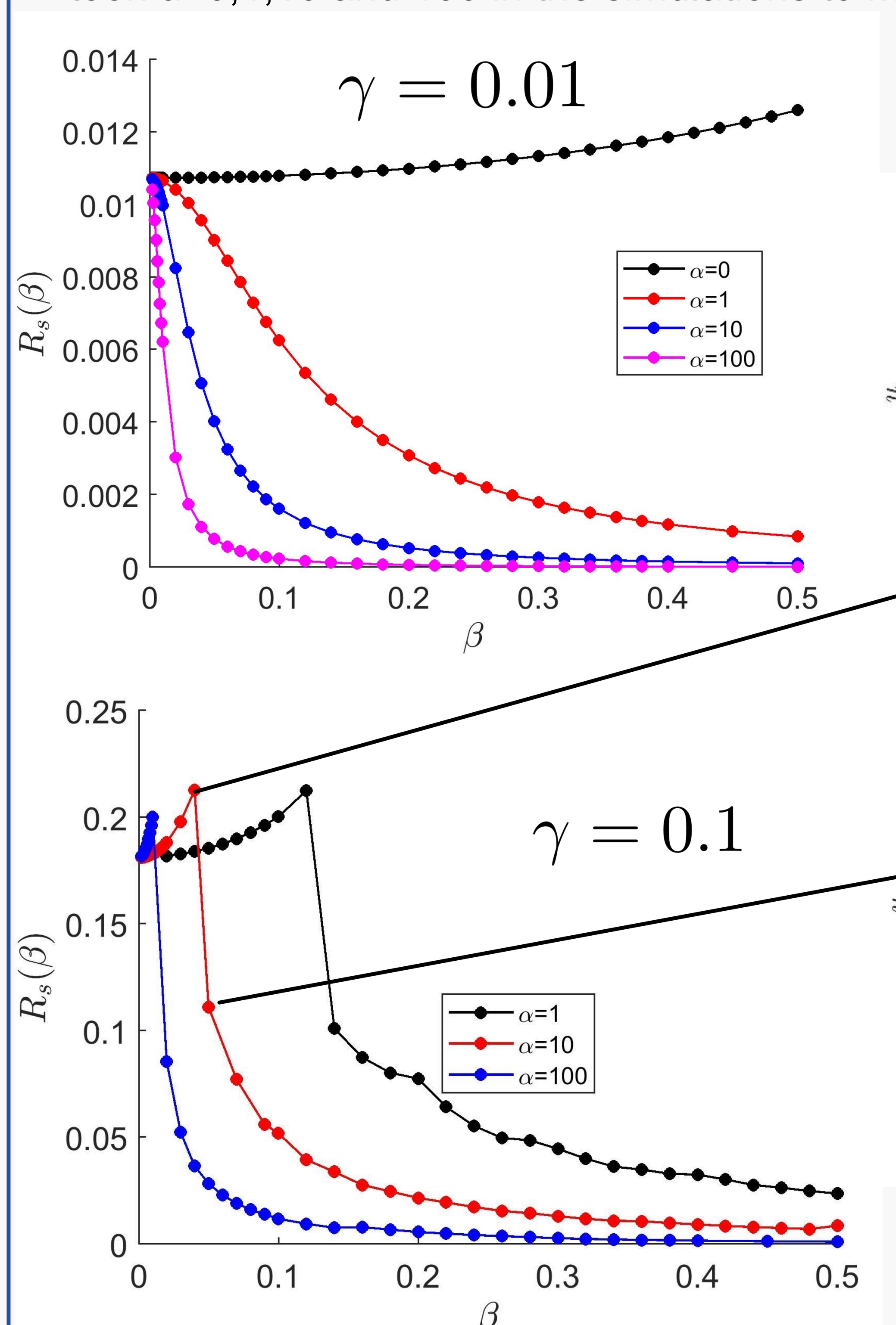
λ - London penetration depth
B - applied field
f – RF frequency
time is in units of rf period
z and L are in units of λ

Dimensionless surface resistance:

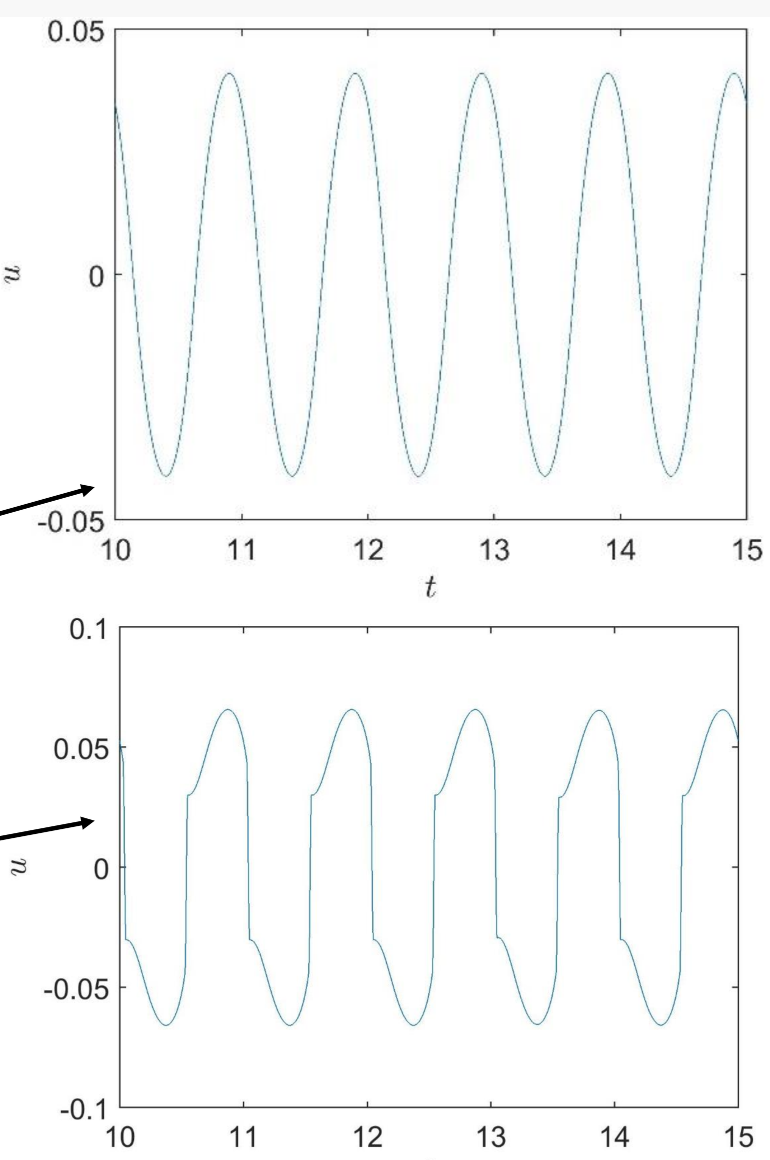
$$R_s(\beta) = \frac{2\gamma^2}{\beta^2}\int_0^1\int_0^l \frac{\dot{u}^2(1+u'^2)^{3/2}}{(1+\alpha(1+u'^2)\dot{u}^2)} dz\, dt$$

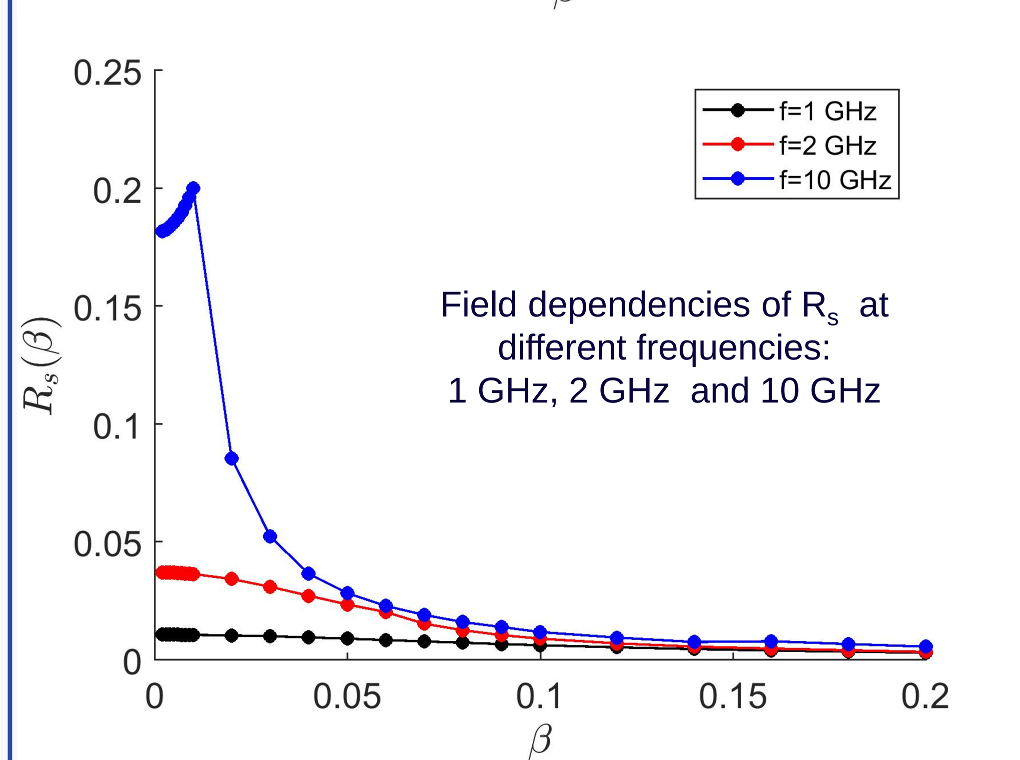
NUMERICAL RESULTS

- Taking λ/ξ=1, λ=40 nm, $\rho_n = 10^{-9}$ Ωm, $n_0 = 6\times10^{28}$ m^{-3} [1] for a clean Nb at f=1 GHz and 10 GHz, we obtain γ ≈ 0.01 and 0.1 for 1 GHz and 10 GHz respectively. We took α=0,1,10 and 100 in the simulations to model uncertainties in v_0.



Shapes of vortex tip oscillations





$$R_s(n\Omega mm^2) = R_s(\beta) \times (2\mu_0^2\lambda^3 f_0^2\eta_0/B_{c1}^2) \approx 0.5R_s(\beta)\, n\Omega mm^2 \text{ For Nb}$$

SUMMARY

- At low frequencies R_s(H) increases with field. But as the frequency rises R_s(H) becomes a non-monotonic function of H which decreases with H at higher fields.
- Trapped vortices can cause a field-induced reduction of R_s(H). This effect can contribute to the extended Q(H) rise in nitrogen or titanium doped Nb cavities.

* Work supported by NSF under Grant 100614-010

mwali003@odu.edu

[1] A. Gurevich and G. Ciovati, “Effect of vortex hotspots on the radio-frequency surface resistance of superconductors,” Phys. Rev. B, 87. 054502 (2013)
[2] A. Larkin and Y. N. Ovchinnikov, “Pinning in type ii superconductors,”Journal of Low Temperature Physics, 34, 409–428, (1979).
[3] W. Klein, R.P. Huebener, S. Gauss, and J. Parisi, “Nonlinearity in the flux-flow behavior of thin-film superconductors”. J. Low Temp. Phys. 61, 413–432 (1985).
[4] S. G. Doettinger, S. G. et al. “Electronic instability at high flux-flow velocities in high-Tc superconducting films”. Phys. Rev. Lett. 73, 1691–1694 (1994).
[5] C. Villard, C. Peroz, and A. Sulpice, “Viscous motion of vortices in superconducting niobium films,” J. Low Temp. Phys, 131. 957–961, (2003).
[6] H. Suhl, “Inertial mass of a moving fluxoid,” Physical Review Letters, vol. 14, no. 7, p. 226, 1965.