

# AB INITIO CALCULATIONS ON THE GROWTH AND SUPERCONDUCTING PROPERTIES OF Nb<sub>3</sub>Sn\*

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## Abstract

In this work, we employ theoretical *ab initio* techniques to solve mysteries and gain new insights in Nb<sub>3</sub>Sn SRF physics. We determine the temperature dependence of Nb<sub>3</sub>Sn antisite defect formation energies, and discuss the implications of these results for defect segregation. We calculate the phonon spectral function for Nb<sub>3</sub>Sn cells with different combinations of antisite defects and use these results to determine  $T_c$  as a function of stoichiometry. These results allow for the first-ever determination of  $T_c$  in the tin-rich regime, where experimental measurements are unavailable and which is critical to understanding the impact of tin-rich grain boundaries on superconducting cavity performance. Finally, we propose a theory for the growth mechanism of Nb<sub>3</sub>Sn growth on a thick oxide, explaining the puzzling disappearing droplet behavior of Sn on Nb oxide and suggesting how in general an oxide layer reacts with Sn to produce a uniform Nb<sub>3</sub>Sn layer.

## INTRODUCTION

Density Functional Theory (DFT) is a versatile tool that can be used to calculate, without any needed experimental input and to within a few percent accuracy, an incredibly wide of array of fundamental material properties from electronic structure to activation energies to superconducting properties. This makes DFT a very useful complement to experimental research of advanced materials such as those used in SRF cavities. This paper presents pertinent results the on point defects, the growth mechanisms and the superconductivity of Nb<sub>3</sub>Sn and the implications of these results to the future of Nb<sub>3</sub>Sn SRF cavities [1–4].

## CALCULATIONS ON POINT DEFECTS

As part of an in-depth effort to calculate the A15 region of the Nb-Sn phase diagram (which includes Nb<sub>3</sub>Sn at 25% Sn), we have gained a rich understanding of the behavior of antisite defects in the material, which are the primary cause of off-stoichiometry. In particular, both kinds of antisite defects dramatically change the Fermi level density of states, which has major implications for their behavior at high temperature during the Nb<sub>3</sub>Sn growth process, and at low temperature during cavity operation (discussed in a later section). At the root of this phenomenon is the fact

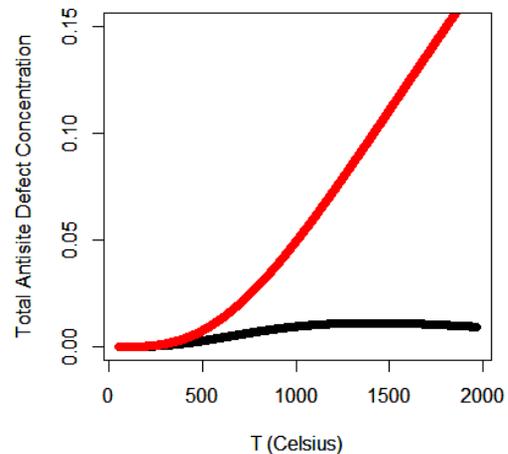


Figure 1: Calculated antisite defect concentrations in 25% Sn Nb<sub>3</sub>Sn as a function of temperature, assuming defect formation energies that have been corrected (Black) and have not been corrected (Red) for electronic entropy effects.

that perfect, stoichiometric Nb<sub>3</sub>Sn has a very high Fermi level density of states, a property directly related to its excellent superconducting properties. This means that at high temperatures Nb<sub>3</sub>Sn has a very high electronic entropy, and thus very low electronic free energy. Therefore, defects that reduce the Fermi level density of states are significantly less energetically favorable than they would otherwise be.

In general, electronic entropy increases linearly with temperature, and the corresponding contribution  $T \cdot S$  to the free energy increases quadratically with temperature. Because the configurational entropy contribution to the free energy increases only linearly with temperature, the quadratic electronic free energy term dominates at high temperature, preventing defect concentration from growing indefinitely. Using DFT, we calculate antisite defect formation energies at zero temperature, and at the coating temperature of approximately 1200 Celsius. We find that, over this range of temperatures, the free energy for a niobium atom to occupy a tin site increases by 0.24 eV, while the free energy for a tin atom to occupy a niobium site increases by 0.22 eV. Figure 1 shows the impact of these effects on defect concentrations: although zero-temperature defect formation energies would suggest that at the coating temperature nearly 10% of all sites in stoichiometric Nb<sub>3</sub>Sn should be occupied by antisite defects, proper accounting of electronic entropy indicates that, actually, only about 1% of all sites are occupied by antisite defects at this temperature.

\* This work was supported by the US National Science Foundation under award PHY-1549132, the Center for Bright Beams.

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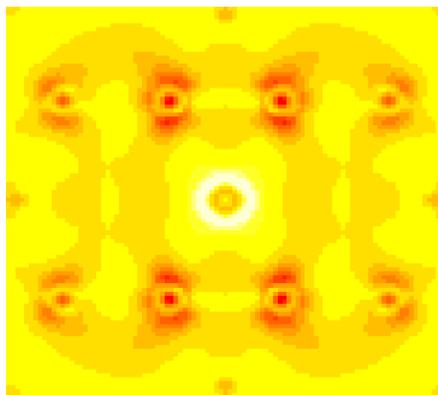


Figure 2: Real-space change in Fermi level density of states due to a niobium-on-tin antisite defect (center). The effect is strongly negative around nearest-neighbor atoms, and still significant for next-nearest-neighbor atoms.

The effect of antisite defects on the Fermi level density of states not only determines their equilibrium concentrations, but also determines how they interact with other defects and with each other. In particular, defects that suppress the Fermi level density of states in some volume of radius  $r$  (visualized in Fig. 2) can reduce their effect on electronic entropy and lower their free energy by moving within  $r$  of other defects or disorder that also reduce the Fermi level density of states. This means that interactions between antisite defects which are strongly repulsive at low temperatures are only mildly repulsive at high temperatures. This effect plays a key role in determining the tin-poor limit of the A15 phase at high temperatures.

Another interesting implication is that antisite defects will tend to be attracted to grain boundaries and dislocations, where Fermi level density of states is lower due to disorder and strain. This is one possible explanation for the observed 3 nm sheath of tin-rich stoichiometry observed around grain boundaries in some Nb<sub>3</sub>Sn layers [5]. Such Fermi level effects also apply for common impurities such as oxygen, nitrogen, and carbon. Preliminary calculations suggest that these impurities prefer Nb interstitial sites to Nb<sub>3</sub>Sn interstitial sites at zero temperature and that this preference gets stronger at high temperature because they tend to suppress the Fermi level density of states in Nb<sub>3</sub>Sn. We would therefore expect such impurities to gather near grain boundaries in in Nb<sub>3</sub>Sn to the extent that they are present in Nb<sub>3</sub>Sn at all.

## GRAIN BOUNDARIES AND $T_c$ CALCULATIONS

The most important effect of antisite disorder in Nb<sub>3</sub>Sn is to reduce  $T_c$ , and thus reduce cavity quality factor and quench field. Measurements have shown that  $T_c$  drops from its maximum value of about 18K to 6K at the tin-poor stoichiometry limit of 17-18% Sn [6]. This makes it possible to understand the effect of tin-poor regions on the superconducting properties of the Nb<sub>3</sub>Sn layer. Tin-rich stoichiometry has also

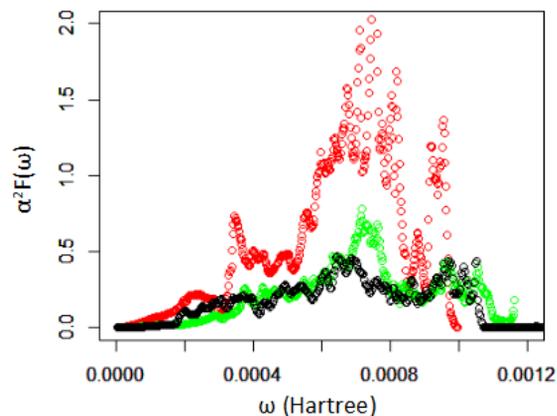


Figure 3: Example Phonon spectral function calculation for stoichiometric (red) tin-poor (green) and tin-rich (black) A15 cells.

been observed, but only in nanometer-scale regions around grain boundaries [5]. The properties of tin-rich Nb<sub>3</sub>Sn are difficult to measure directly as it only exists in such small regions, but they can be calculated using DFT.

To do this, we calculate electron-phonon scattering amplitudes in A15 cells across the observed stoichiometry range. In order to make these calculations as precise as possible, we employ Wannier function methods to integrate smoothly over all scattering processes in momentum space [7]. These scattering amplitudes are used to calculate the phonon spectral function (Fig. 3), which can be used to estimate  $T_c$  using the McMillan formula [8]. We accurately reproduced  $T_c$  for bcc niobium as well as the experimentally measured stoichiometry range of the A15 phase, as described in Table 1. For experimentally inaccessible tin-rich stoichiometry, we find that  $T_c$  falls to a minimum of about 5K at 31.25% Sn stoichiometry (Fig. 4). This information, taken together with experimental measurements of the stoichiometry profile around grain boundaries from the Seidman group at Northwestern, has enabled simulations of magnetic flux entry at grain boundaries performed by the Transtrum group at Brigham Young (Fig. 5). These simulations show that tin-rich grain boundaries can admit flux vortices even at modest fields, resulting in undesirable Q-slope behavior.

Table 1: Calculated vs. Measured  $T_c$

Composition	Experimental $T_c$ (K)	Calculated $T_c$ (K)
Pure Nb	9.3	10.5
18.75% Sn	6	5.7
20.83% Sn	7.1	5.7
Nb <sub>3</sub> Sn	18	17.8

It is important to consider other contributing factors to  $T_c$  suppression in grain boundaries, both to accurately predict the overall effect of a tin-rich grain boundary on  $T_c$ , and to accurately compare tin-rich grain boundaries to "clean"

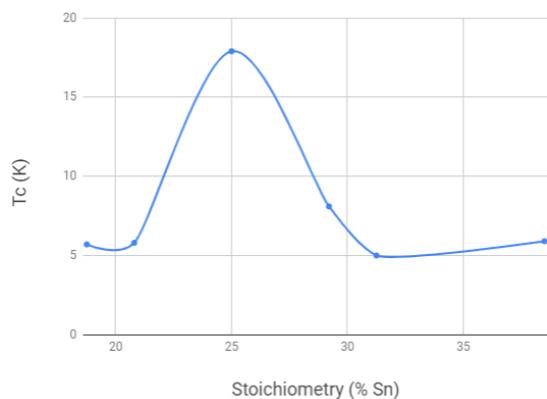


Figure 4: Calculated  $T_c$  as a function of stoichiometry, showing a minimum of about 5 Kelvin in the tin-rich regime.

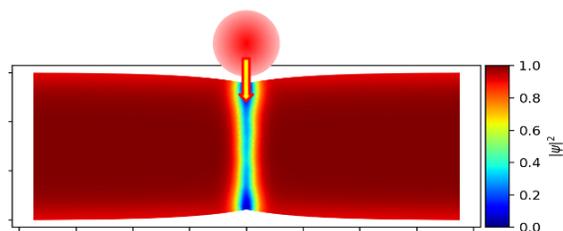


Figure 5: Example simulation of the superconducting order parameter in a tin-rich grain boundary.

25% Sn stoichiometry grain boundaries. While it is difficult to directly calculate  $T_c$  for grain boundaries due to their extremely complicated phonon spectra, it is possible to estimate their effect on  $T_c$  by calculating their effect on Fermi level density of states (Fig. 6), which we find to be correlated with  $T_c$  (Fig. 7). Given this relationship between the Fermi level density of states and  $T_c$ , we estimate that a clean grain boundary would suppress  $T_c$  by 25% over a core region of 2.2 nm, a mild effect compared to the 70% reduction inflicted by severe off-stoichiometry. We can also estimate the effect of strain on  $T_c$ , and we find that it would

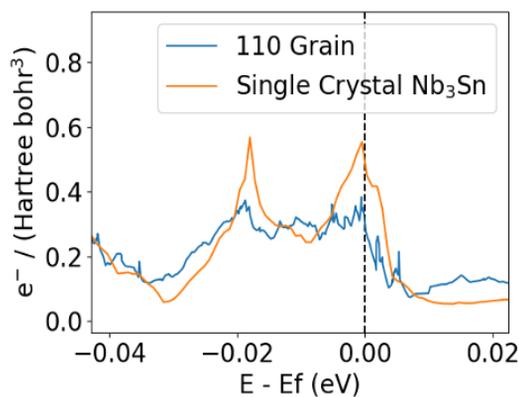


Figure 6: Calculated density of states in bulk  $Nb_3Sn$  as contrasted to that in the immediate vicinity of a grain boundary.

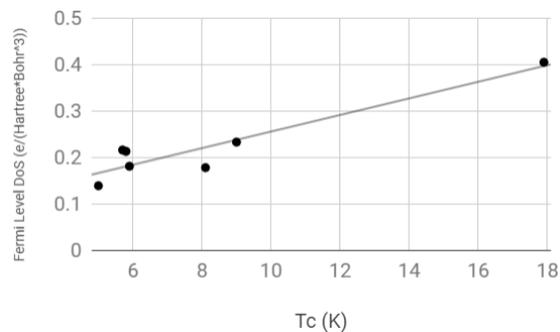


Figure 7: Correlation between  $T_c$  and Fermi level density of states in A15 cells of different stoichiometries.

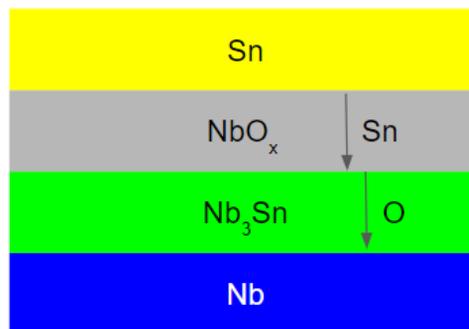


Figure 8: Schematic of the proposed  $Nb_3Sn$  growth mechanism, showing the development of a  $Nb_3Sn$  layer underneath the oxide along with the relevant diffusion processes.

take a very large 4% strain to reduce  $T_c$  by a factor of 2. Given these results, we conclude that clean grain boundaries are likely to result in significantly better cavity performance than dirty grain boundaries. Future experiments and calculations will further elucidate the relationship between grain boundary composition and cavity performance.

## NUCLEATION

It has long been known that effective nucleation of  $Nb_3Sn$  is an essential step in the growth of a high-quality  $Nb_3Sn$  layer. A recent layer growth study by the Liepe group at Cornell showed that some features of finished  $Nb_3Sn$  layers, in particular undesirable thin regions, can be traced back to features at the nucleation stage [9, 10]. Specifically, they found that the presence of a thick niobium oxide changed the nucleation process in a beneficial way, leading for more uniform  $Nb_3Sn$ . Based on these results, we have developed a new theory for the mechanism of  $Nb_3Sn$  growth on an oxidized surface. In this mechanism, illustrated in Fig. 8, tin diffuses through the oxide layer to the bulk niobium interface. There, it interacts with the oxide to produce  $Nb_3Sn$  and oxygen interstitials, which diffuse down into the bulk niobium. At the standard nucleation temperature of 500 C, this process is energetically favorable.

In contrast with the growth process of  $Nb_3Sn$  on pure Nb, the above process does not involve the diffusion of Sn

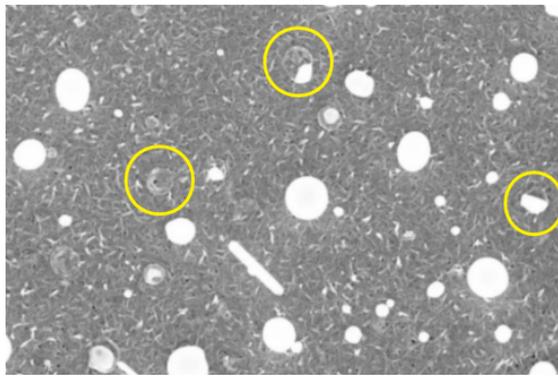


Figure 9: Image of tin droplets on an oxide surface, showing some droplets at various stages of adsorption.

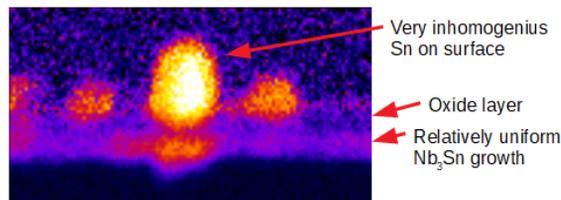


Figure 10: Cross section of nucleated Nb<sub>3</sub>Sn layer, showing a relatively uniform layer forming underneath the oxide.

through the growing Nb<sub>3</sub>Sn layer. This is important because Sn diffusion through Nb<sub>3</sub>Sn is known to be extremely inhomogeneous, occurring almost exclusively along grain boundaries and therefore inevitably leading to inhomogeneities within grains. Depending on the nature of Sn diffusion through niobium oxide and oxygen diffusion through Nb<sub>3</sub>Sn, growth on an oxide may in general be more uniform than growth on pure Nb. This appears to be the case for nucleation on niobium oxide. Figure 9 shows that tin droplets on a niobium oxide surface tend to "disappear" from the surface, and Fig. 10 shows that this tin is in fact reacting underneath the oxide. The Liepe group plans to explore the possibility of growing complete Nb<sub>3</sub>Sn layers from oxide in the future.

The Liepe group is also exploring the possibility of nucleating Nb<sub>3</sub>Sn layers electrochemically [11]. Early results have been promising, showing that under the right conditions of voltage, pH, and surface oxidation, a chemical reaction occurs that strongly binds electroplated tin to the niobium substrate. In theory, it is possible for Sn ions to react with niobium oxide to grow Nb<sub>3</sub>Sn from solution, a process that may be sufficiently catalyzed by the solution to grow a substantial Nb<sub>3</sub>Sn layer in spite of the low temperature. We have begun calculations using Joint Density Functional Theory to investigate this system in detail (Fig. 11) [12].

## CONCLUSIONS

We have demonstrated the fruitful application of DFT to open problems in Nb<sub>3</sub>Sn SRF related to point defect behavior, the superconducting properties of grain boundaries, and Nb<sub>3</sub>Sn layer nucleation. By improving our understanding

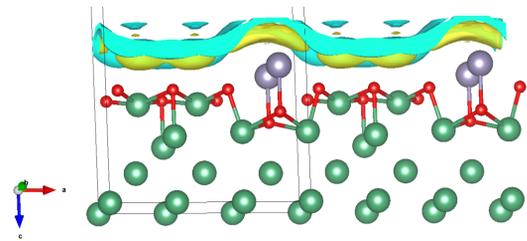


Figure 11: Calculated bound charge density surrounding tin ad-atoms (purple) on a realistic niobium oxide (green and red) surface in aqueous solution.

of the fundamental physics at work, these results will help develop new recipes to maximize cavity Q and quench field.

## ACKNOWLEDGMENT

We would like to thank Jae-yel Lee and David Seidman of Northwestern University for sharing their grain boundary segregation measurements with us, and thank Sam Posen of Fermilab for organizing, facilitating, and contributing valuable insights to the many meetings we had with them.

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