

# **Current-blocking grain boundaries in SRF cavities and RF dissipation due to nonlinear dynamics of Josephson vortices under strong RF fields**

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# Outline

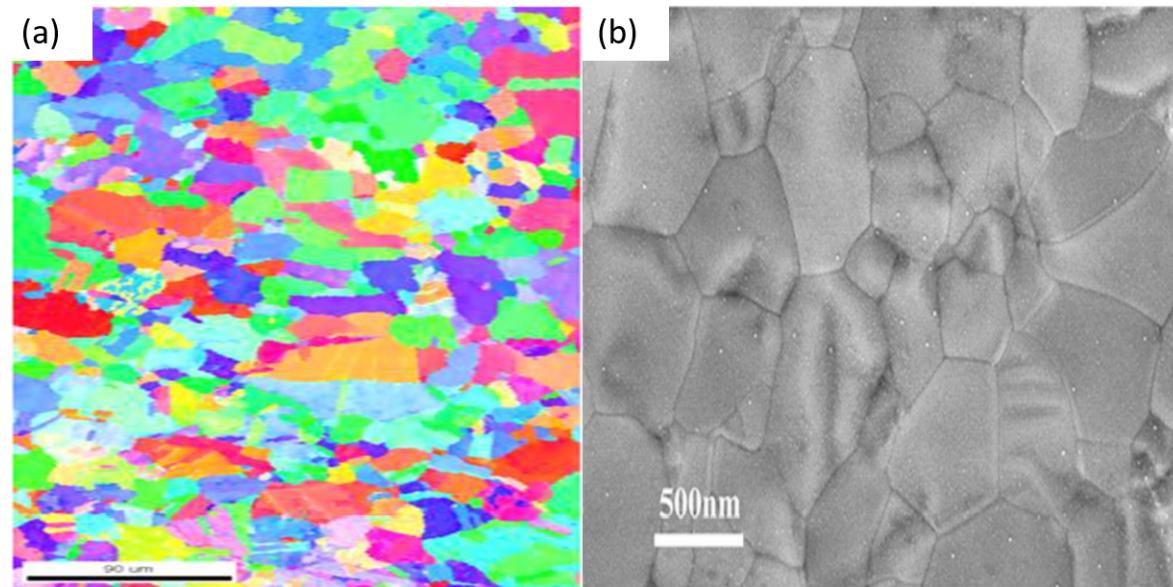
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- Is there a grain boundary problem in SRF materials?
  - Nb – usually not but it may appear due to segregation of impurities on GBs
  - $\text{Nb}_3\text{Sn}$  – likely yes, and it can be a serious problem at high fields
- How can current-blocking grain boundaries reduce the SRF breakdown field?
- Effect of grain boundaries on  $Q(H)$  slope.
- Strongly and weakly-coupled grain boundaries.
- Dynamics of mixed Abrikosov-Josephson vortices in strongly coupled GBs.
- Cherenkov instability and dynamic transition of AJ vortices into phase slips.
- RF dissipation and deterioration of SRF performance by GBs

A. Sheikhzada and A. Gurevich:

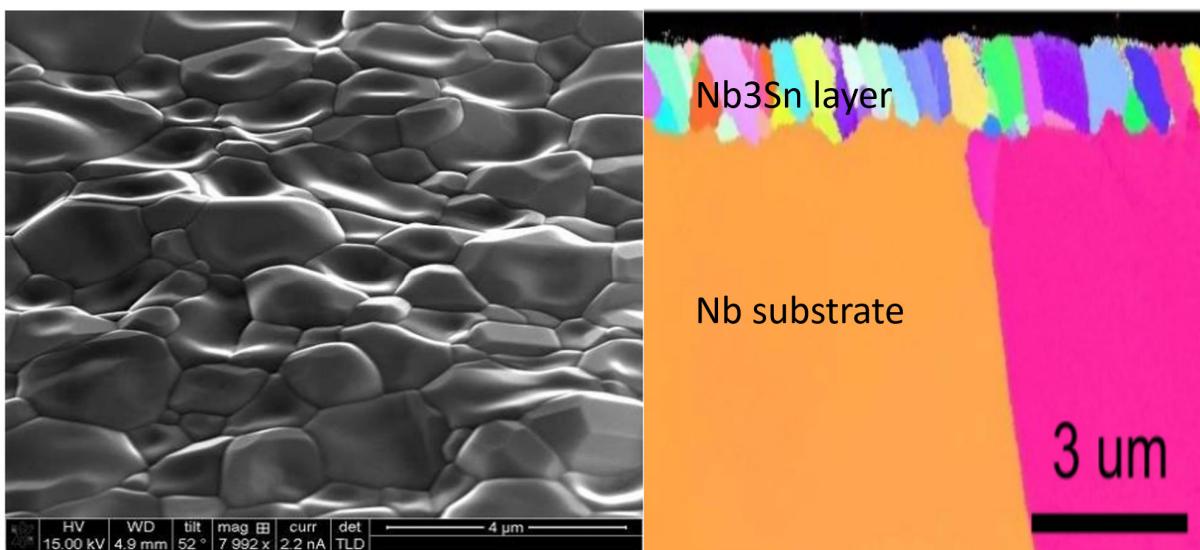
1. [Physica C 506, 59-68 \(2014\)](#)
2. [Nature Scientific Reports 5, 17821 \(2015\)](#)
3. [Physical Review B 95, 214507 \(2017\)](#)

# Do these grain boundaries impede RF currents?



EBSD map and optical microscopy of polycrystalline Nb films on Cu  
A-M Valente-Feliciano,  
SUST 29, 113002 (2016)

GBs in Nb do not block RF currents up to  $H = H_c$

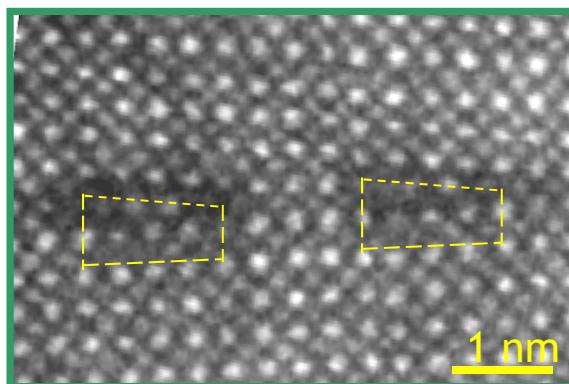
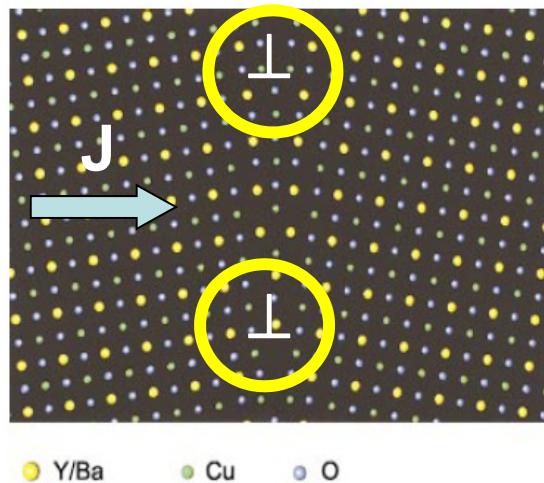


EBSD map and optical microscopy of polycrystalline Nb<sub>3</sub>Sn films on Nb  
S. Posen and D. Hall,  
SUST 30, 033004 (2017)

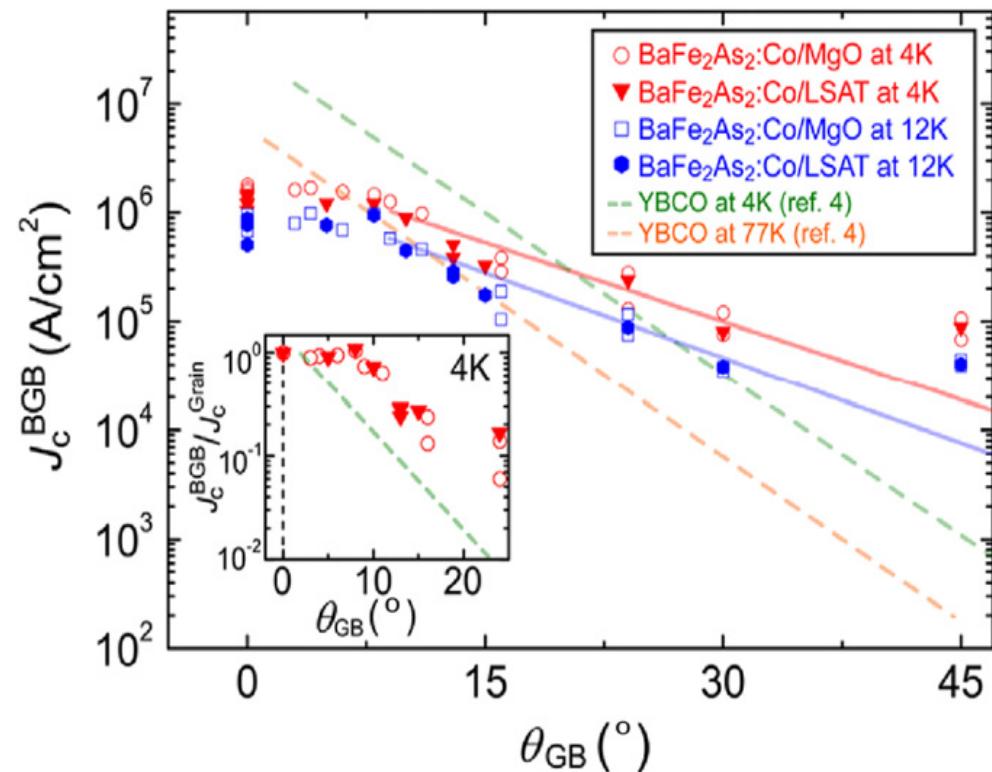
GBs in Nb<sub>3</sub>Sn partly obstruct strong RF currents and pin vortices

# GB problem in cuprates and pnictides

16° [001] tilt grain boundary in YBCO



X. Song et al. Nature Mat. 4, 470 (2005)

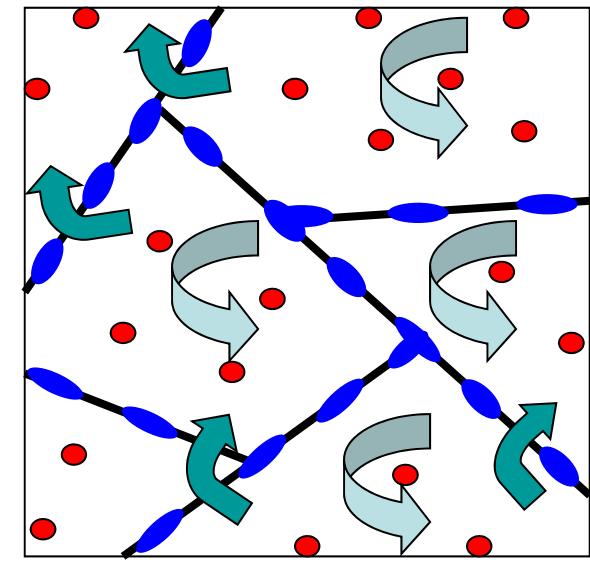
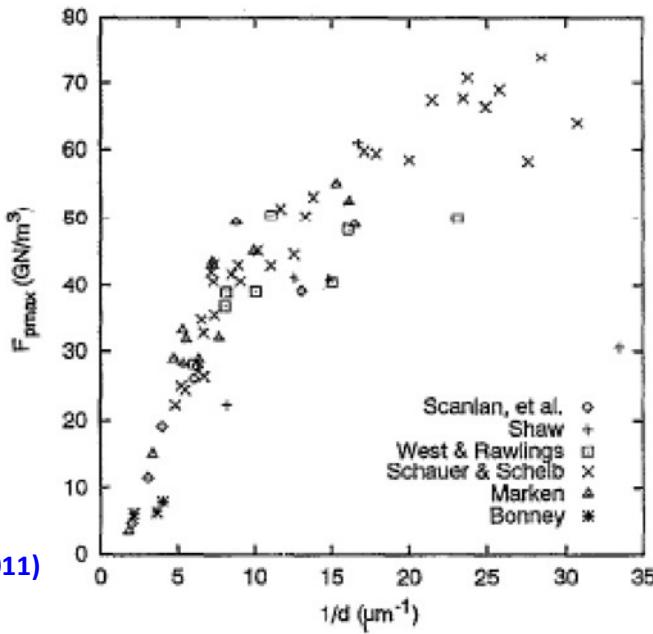
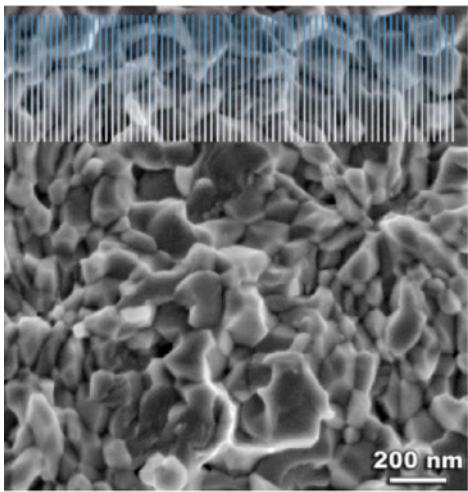


Exponential decrease of  $J_c$  with the misorientation angle  
GBs behave as weakly-coupled Josephson junctions

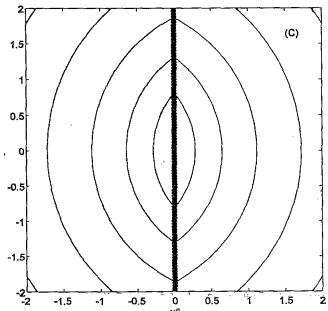
$J_c = 10^4 - 10^5 \text{ A/cm}^2$  at 15-30° are more than 3 orders of magnitude smaller than the SRF screening current densities at  $B_a = 100 \text{ mT}$ :

$$J_{SRF} \sim \frac{B_a}{\mu_0 \lambda} \sim 10^8 \frac{A}{cm^2}$$

# Grain boundaries pin vortices in $\text{Nb}_3\text{Sn}$



$\uparrow$  **J**



GB cuts currents circulating around a vortex causing its attraction to the GB at  $r < L$ :

$$J(r) \simeq J_d \frac{\xi}{r}, \quad L \simeq \xi \frac{J_d}{J_c}$$

Gurevich and Cooley, Phys. Rev. B 50, 13563 (1994)

- Vortices caged in the grains and pinned by GBs
- GBs critical current density  $J_c$  is much smaller than the bulk depairing current density  $J_d$ :

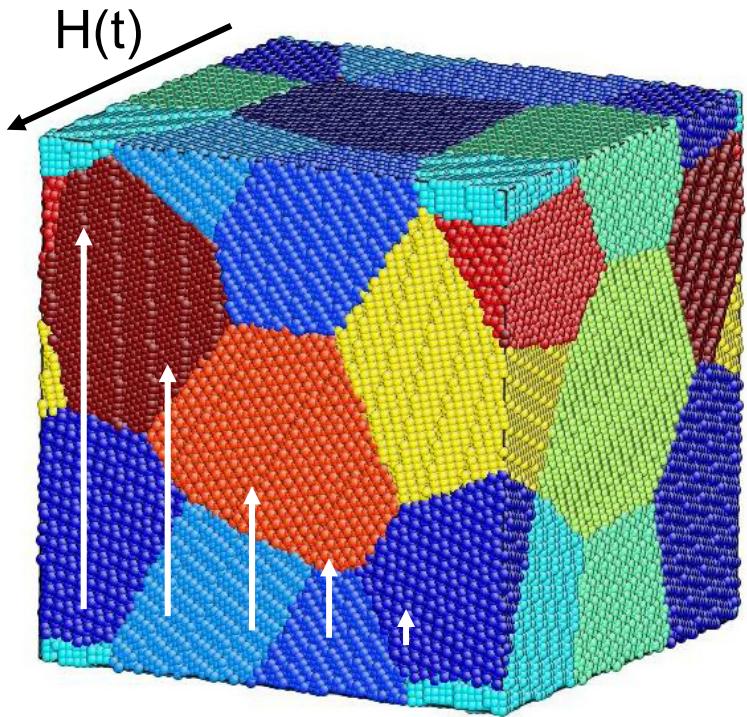
$$J_c \ll J_d = H_c/\lambda$$

- Bulk pinning force depends on the grain size  $d$ :

$$F_p = BJ_p \propto 1/d^2, \quad d > \lambda,$$

$$F_p = BJ_p \propto 1/d, \quad d < \lambda$$

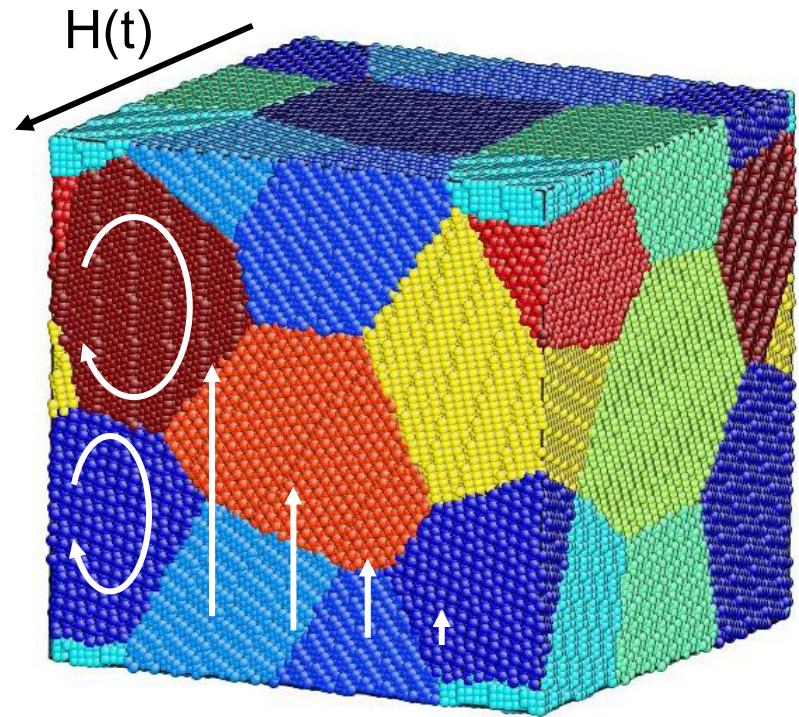
# Grain boundaries impeding SRF currents



GBs are transparent to the RF current density

$$J(x) = (H/\lambda)e^{-x/\lambda}$$

Critical current density  $J_c$  of GB is larger than  $J(x)$  up to  $H \approx H_c$



GBs are partly transparent to the RF current density

$J_c$  of GB is not large enough to transmit  $J(x)$  above a penetration field:

$$H > H_p \simeq \lambda J_c \simeq H_c J_c / J_d$$

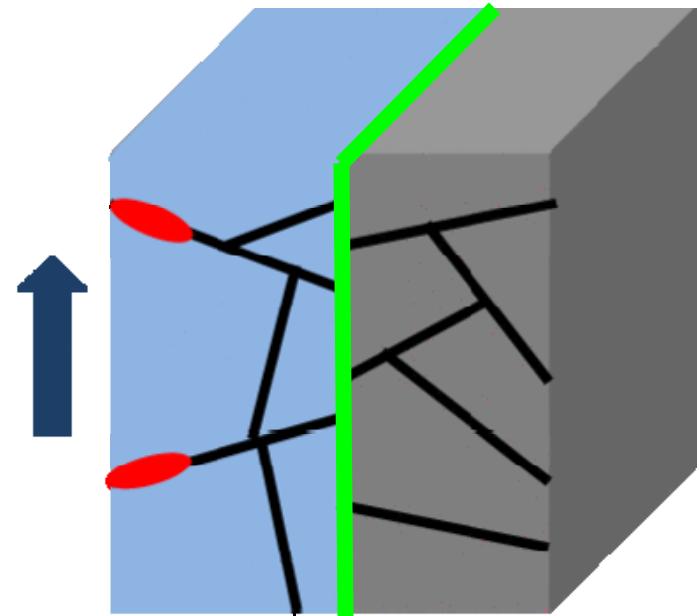
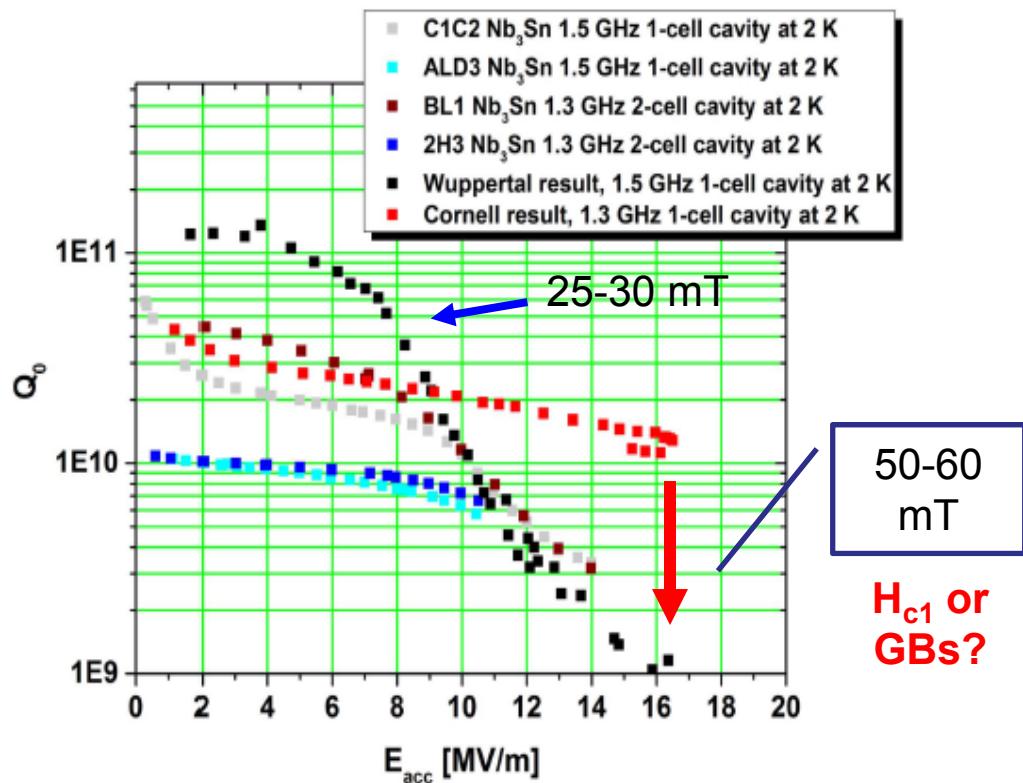
For  $\mathbf{J}_c = 0.1 \mathbf{J}_d$ , Abrikosov-Josephson vortices penetrate in and out of GBs in  $\text{Nb}_3\text{Sn}$  at  $H > 20-50$  mT

# Penetration of vortices along GBs

Field onset of penetration for Josephson vortices

$$H > H_p \simeq \lambda J_c \simeq H_c J_c / J_d$$

Higher  $H_c = 540$  mT for  $\text{Nb}_3\text{Sn}$  can result in better high-field SRF performance only if GBs are strongly coupled,  $J_c > 0.2J_d$



Mismatch of GB structures in S coating layers and the Nb cavity:  
1 layer intercepts AJ vortices

Penetration of AJ vortices along  
GBs does not go beyond the first  
S layer

# Josephson vortex in a weak JJ

$$\ddot{\theta} + \eta \dot{\theta} = \theta'' - \sin \theta + \beta(t)$$

- sine-Gordon equation for the Josephson phase difference. Moving J vortex at  $\eta = 0$ :

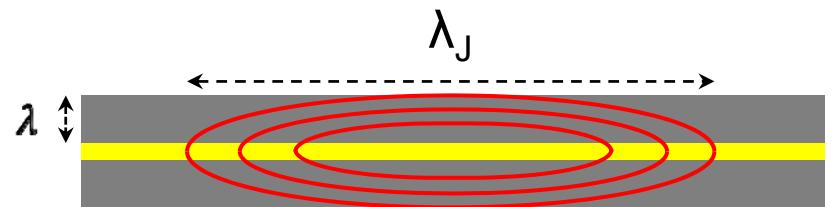
$$\theta(x, t) = 4 \tan^{-1} \exp \left[ \frac{x - vt}{\lambda_J \sqrt{1 - (v/c_s)^2}} \right]$$

$$\lambda_J = \left( \frac{c\phi_0}{16\pi^2\lambda J_c} \right)^{1/2} \sim \sqrt{\xi\lambda} \left( \frac{J_d}{J_c} \right)^{1/2}$$

- $c_s = \omega_J \lambda_J$  (Swihart velocity)
- Weak low- $J_c$  junction:

$$\lambda_J \gg \lambda, \quad J_c \ll \frac{J_d}{\kappa}, \quad \kappa = \frac{\lambda}{\xi} \simeq 20 - 40$$

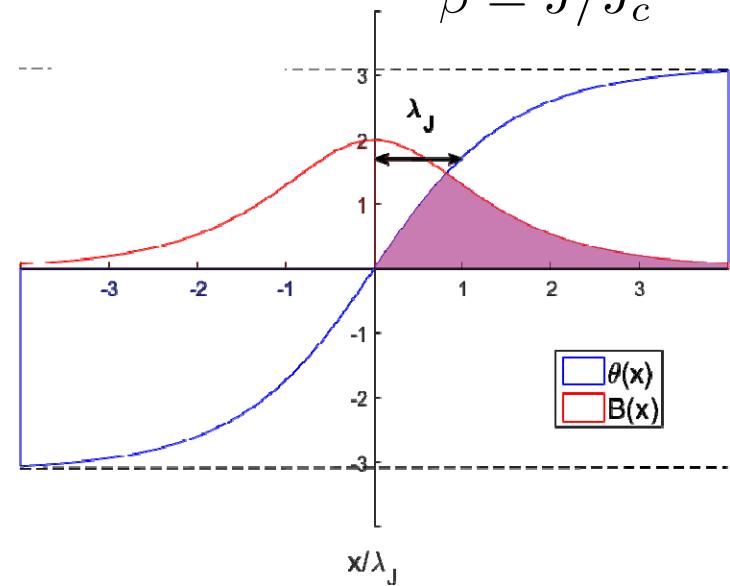
- Fields  $H \ll H_c/\kappa \simeq 20 - 30$  mT for Nb<sub>3</sub>Sn
- As  $v$  increases J vortex **shrinks** at  $\eta \ll 1$  and **expands** at  $\eta \gg 1$



$$\omega_J = (2\pi c J_c / C \phi_0)^{1/2},$$

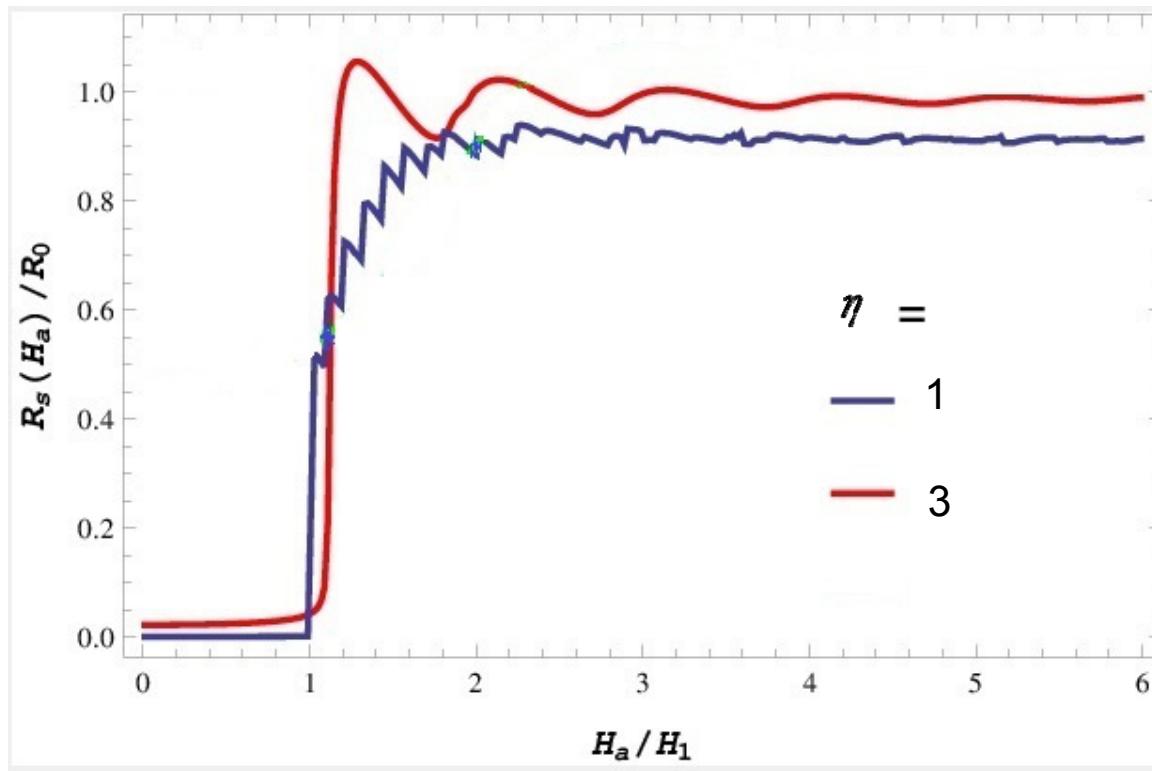
$$\eta = 1/RC\omega_J,$$

$$\beta = J/J_c$$



**J vortex described by SG eq. cannot move faster than  $c_s$  and remains stable at any current  $J < J_c$**

# Field-dependent surface resistance



- Sharp jump at the onset of penetration of J vortices
- $R_s(H)$  is not linear in the field amplitude.
- Penetration of J vortices can trigger transition to the normal state

# Strongly coupled GBs

- General eq. for the phase difference  $\theta(x, t)$

$$\ddot{\theta} + \eta \dot{\theta} = \frac{\lambda_J^2}{\pi \lambda} \int_{-\infty}^{\infty} K_0 \left[ \frac{|x - u|}{\lambda} \right] \frac{\partial^2 \theta}{\partial u^2} du - \sin \theta + \beta$$

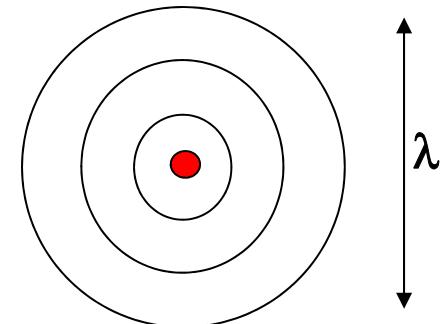
Gurevich, Phys. Rev. B 46, 3187 (1992); 65, 214531 (2002)

**Low- $J_c$  JJ:** Josephson vortices in which  $\theta(x, t)$  varies slowly over  $\lambda \ll \lambda_J$ , so that  $K_0(|x - u|/\lambda) \rightarrow \pi \lambda \delta(x - u)$  and the integral eq. reduces to the sG eq.

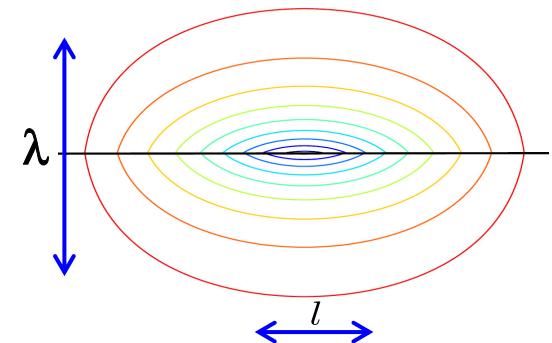
**High- $J_c$  JJ:** Mixed Abrikosov-Josephson (AJ) vortices with no normal core but a phase core of length:

$$l = \frac{\lambda_J^2}{\lambda} \simeq \xi \frac{J_d}{J_c}$$

A



AJ

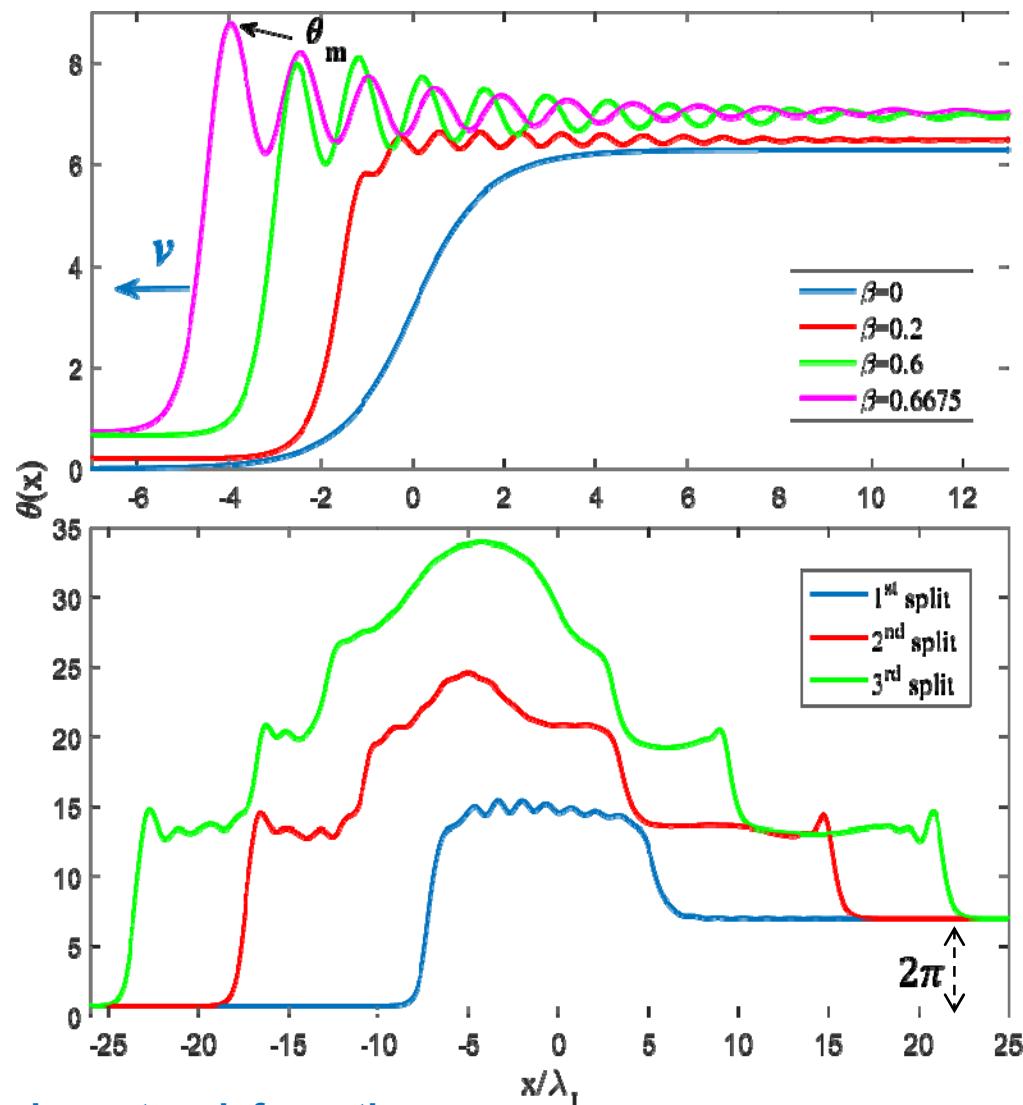


The general integral equation take into account Cherenkov radiation of vortices which is missing in the sine-Gordon equation



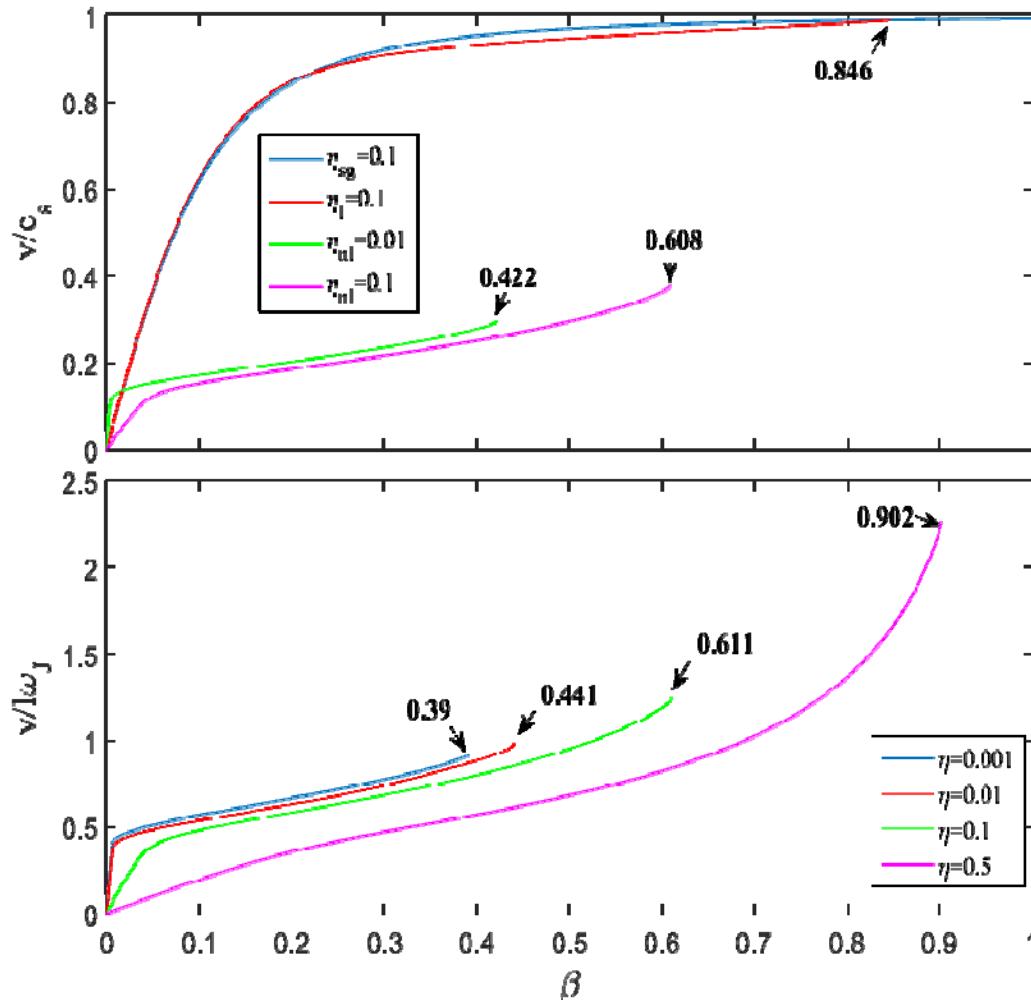
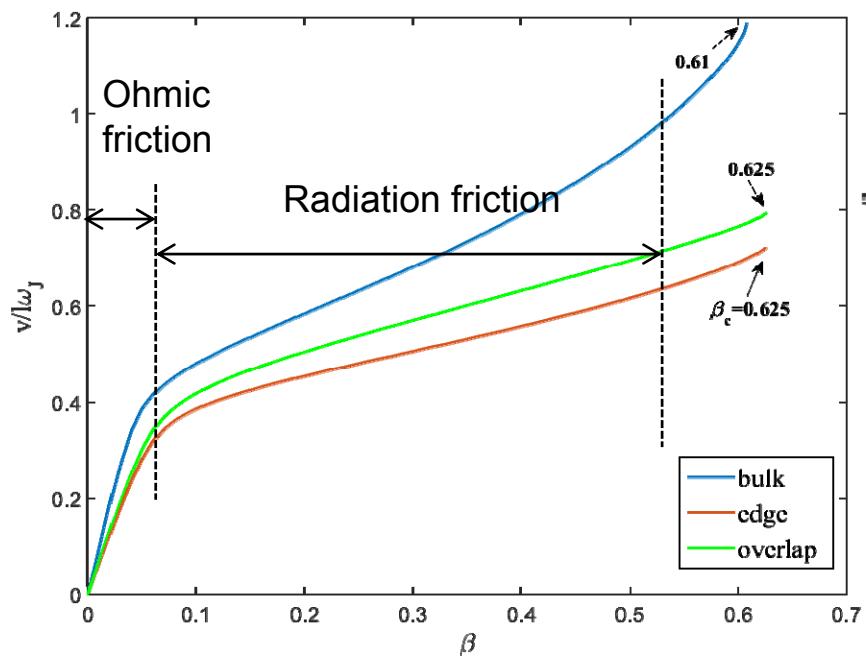
# Instability of underdamped J vortex

- Trailing tale of **Cherenkov radiation** with the amplitude increasing with  $\beta$
- Instability occurs as a  **$\pi$  junction** critical nucleus with  $5\pi/2 < \theta < 3\pi$  appears behind a moving vortex
- Cascade of **v-av pairs**
- Formation of an expanding **phase pile**, initial  $2\pi$  phase difference is preserved
- Vortices at the edges move close to **Swihart velocity** or even higher



# I-V characteristics

- Small currents  $\rightarrow$  Ohmic friction
- Large currents  $\rightarrow$  Cherenkov radiation friction
- Instability at  $J \ll J_c$
- Similar behavior in different geometries



# $H_{c1}$ and superheating field for GB vortices

**Weakly coupled GBs with**  $J_c \ll J_d/\kappa$

$$B_{c1}^J = \frac{\phi_0}{\pi^2 \lambda \lambda_J} \approx B_c \sqrt{\frac{J_c}{\kappa J_d}}, \quad B_s^J = \frac{\pi}{2} B_{c1}^J$$

Both  $B_{c1}^J$  and  $B_s^J$  are much smaller than  $B_c$ , particularly for  $\text{Nb}_3\text{Sn}$  with  $\kappa = \lambda/\xi = 20\text{--}30$

**Strongly coupled GBs with**  $J_c > J_d/\kappa$

Gurevich, Phys. Rev. B 46, 3187 (1992)

$$B_{c1}^{AJ} = \frac{\phi_0}{4\pi\lambda^2} \left( \ln \frac{\lambda}{l} + 0.423 \right), \quad B_s^{AJ} = \mu_0 \lambda J_c = B_c \frac{J_c}{J_d}$$

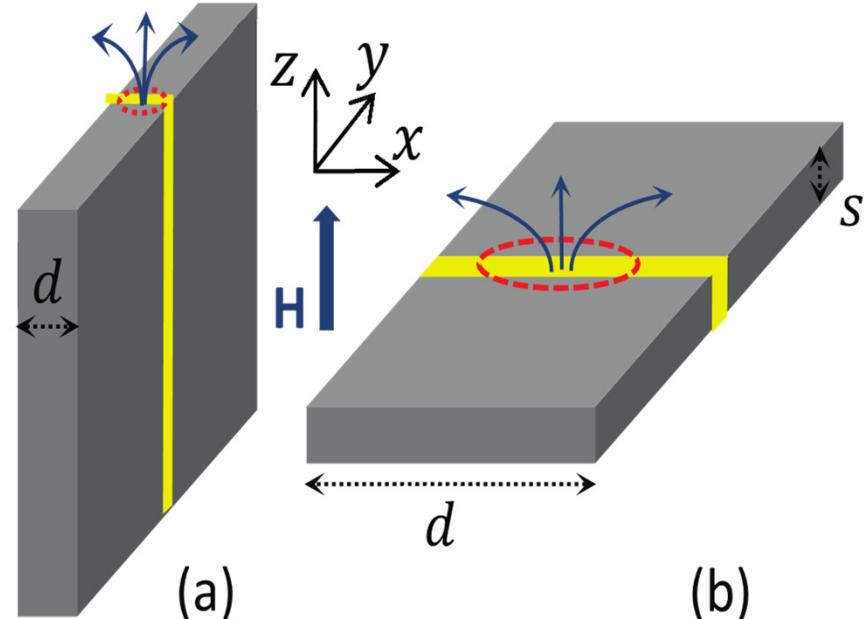
Here  $B_{c1}^{AJ} \simeq B_{c1}^A$  but the AJ superheating field  $B_s^{AJ}$  can be much larger than  $B_{c1}^{AJ}$  if:

1.  $J_c$  is of the order of  $J_d$  for a large  $-\kappa$  superconductor like  $\text{Nb}_3\text{Sn}$

2. GBs do not have edge defects which locally reduce  $J_c$  down to  $J_c \sim J_d/\kappa$

# V-AV pair production in a GB in a film

- What happens when expanding phase pile hits the edges of the junction?
- How is the Cherenkov vortex pair production affected by the size effects?
- Acceleration of vortices due to their attraction to the edges causes additional radiation (vortex bremsstrahlung)
- Change of the structure of moving vortices in a finite junction
- Dynamic transition of driven vortices into phase slips



Vortices parallel or perpendicular  
to a thin film JJ

# Numerical simulations

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- Strongly-coupled GB in a thin film: length  $d$  is smaller than  $\lambda$ :

$$\ddot{\theta} + \eta \dot{\theta} + \sin \theta - \beta(x, t) = \epsilon \int_{-1/2}^{1/2} \ln \left| \frac{2}{\sin \pi x - \sin \pi u} \right| \theta''(u) du,$$
$$\epsilon = \frac{l}{\pi d} = \frac{c\phi_0}{16\pi^3 \lambda^2 d J_c}$$

- Small gradient in  $\beta(x,t) = (1-kx)\beta(t)$  due to weak screening facilitates penetration of vortices from the edge at  $x = -1/2$ .
- Both dc and ac currents  $\beta(t) = \beta_0 \cos \omega t$
- Interaction of vortices with either edge or bulk defects
- Simulations were done at  $\epsilon = 0.002$ ,  $k = 0.02$ ,  $\omega = \pi/30$  and different  $\eta$

# Exact dynamic solution for $\eta \gg 1$

$$\theta(x, t) = \chi + 2 \tan^{-1} \left[ \frac{\sin \pi u \sinh \pi l}{\cos \pi u \cosh \pi l - \cos \pi x} \right]$$

$u(t)$  is the coordinate of the AJ vortex core

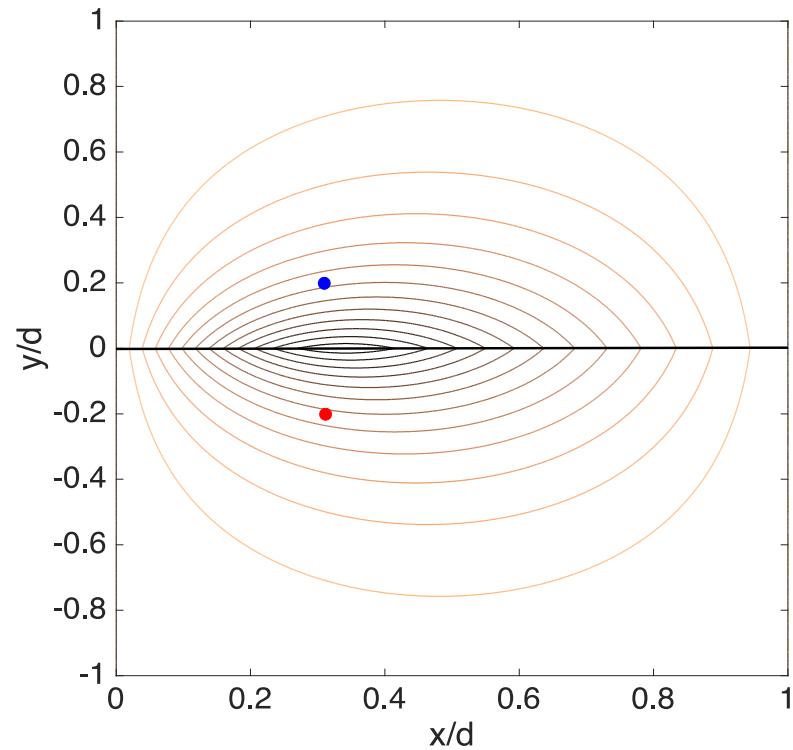
$l(t)$  is the length of the AJ vortex core

$x(t)$  is the phase difference due to external current

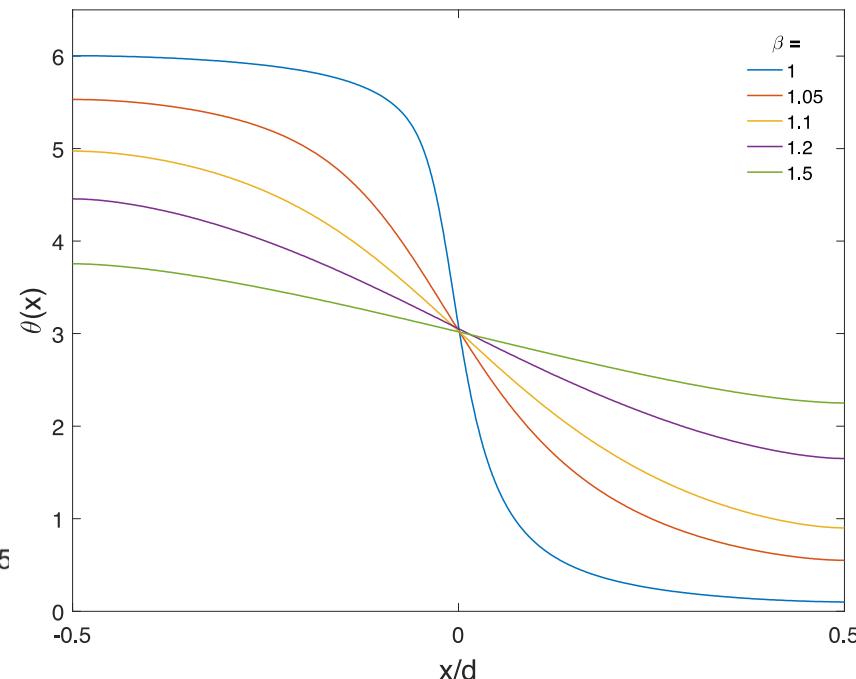
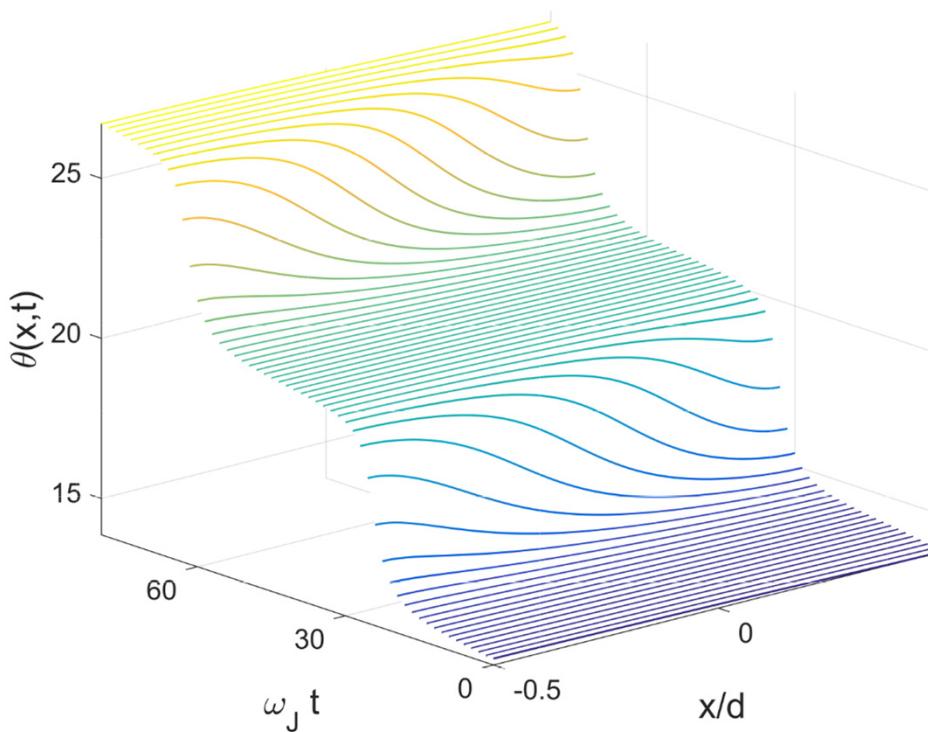
Vortex solution disappears if :

$$d < d_c = \frac{\pi l_0}{\sqrt{1 - (J/J_c)^2}}$$

- Similar to A vortices in films  
[\(K.K. Likharev, RMP, 51, 101 \(1979\)\).](#)
- Transition of the AJ vortex into a phase slip in which the whole junction switches to a uniform resistive state.

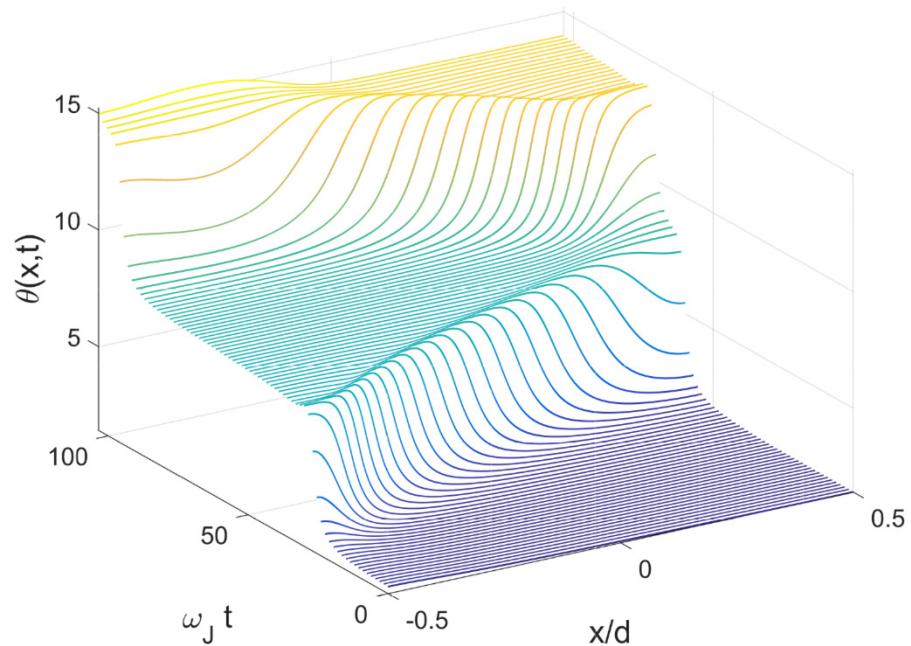


# Overdamped GB ( $\eta=2$ )

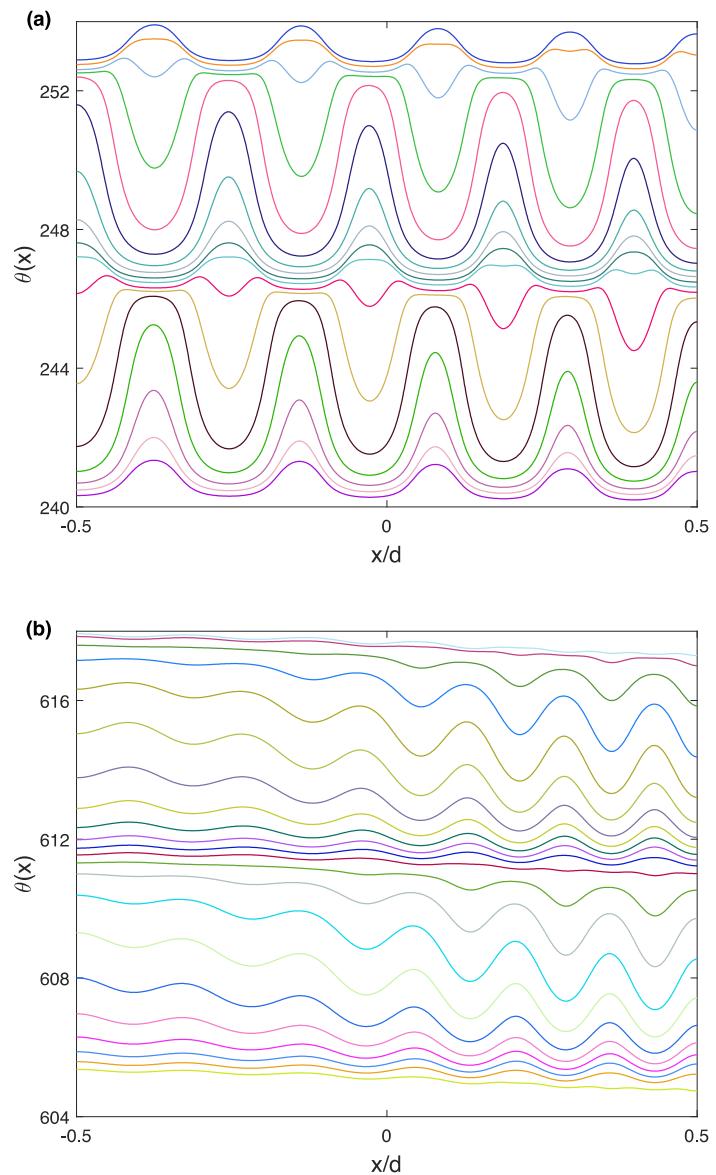


- Overdamped vortex expands as it moves faster
- Dynamic transition of a moving vortex to a phase slip as current increases
- Cherenkov radiation and vortex bremsstrahlung is suppressed
- No V-AV pair production

# Weaker damping ( $\eta=1$ )

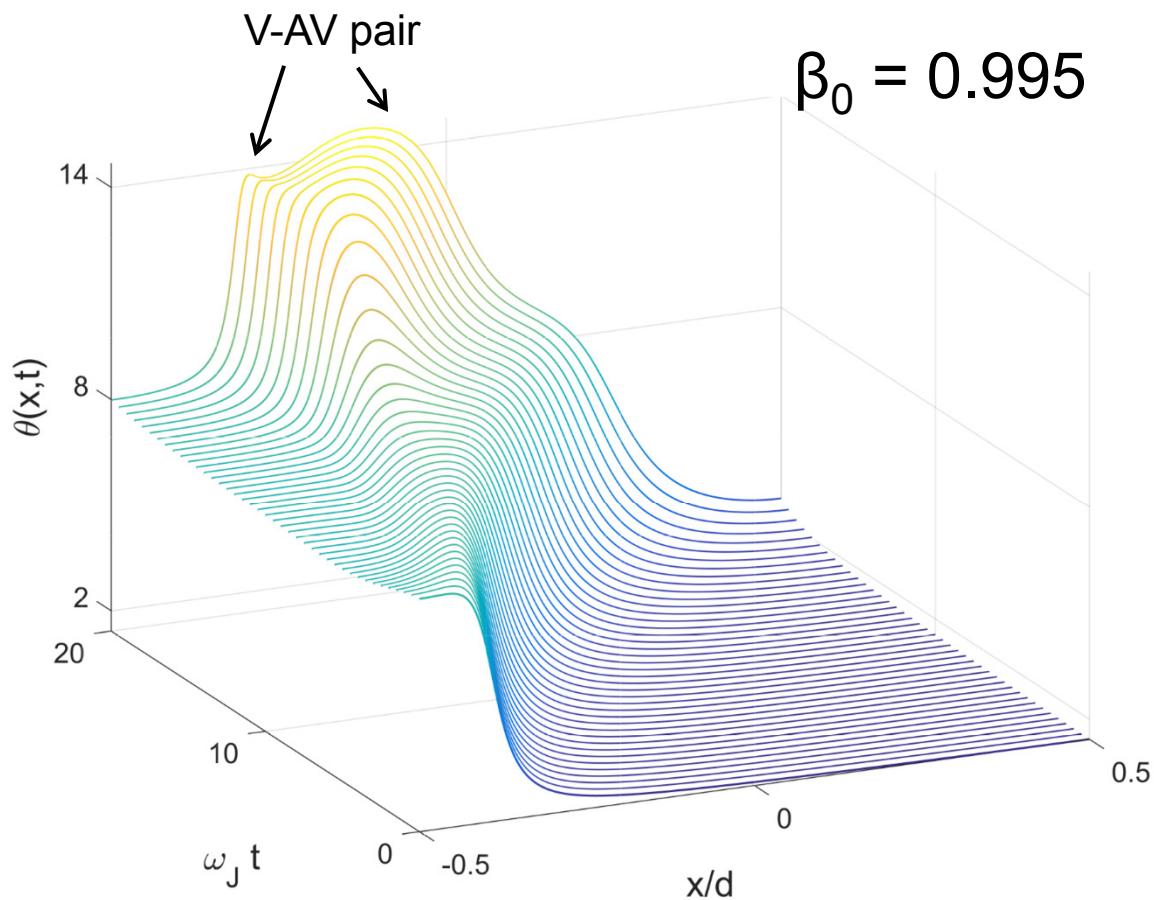


- Radiation wakes become apparent but no generation of V-AV pairs occurs
- Standing nonlinear waves as  $I$  increases
- Phase slips at higher currents



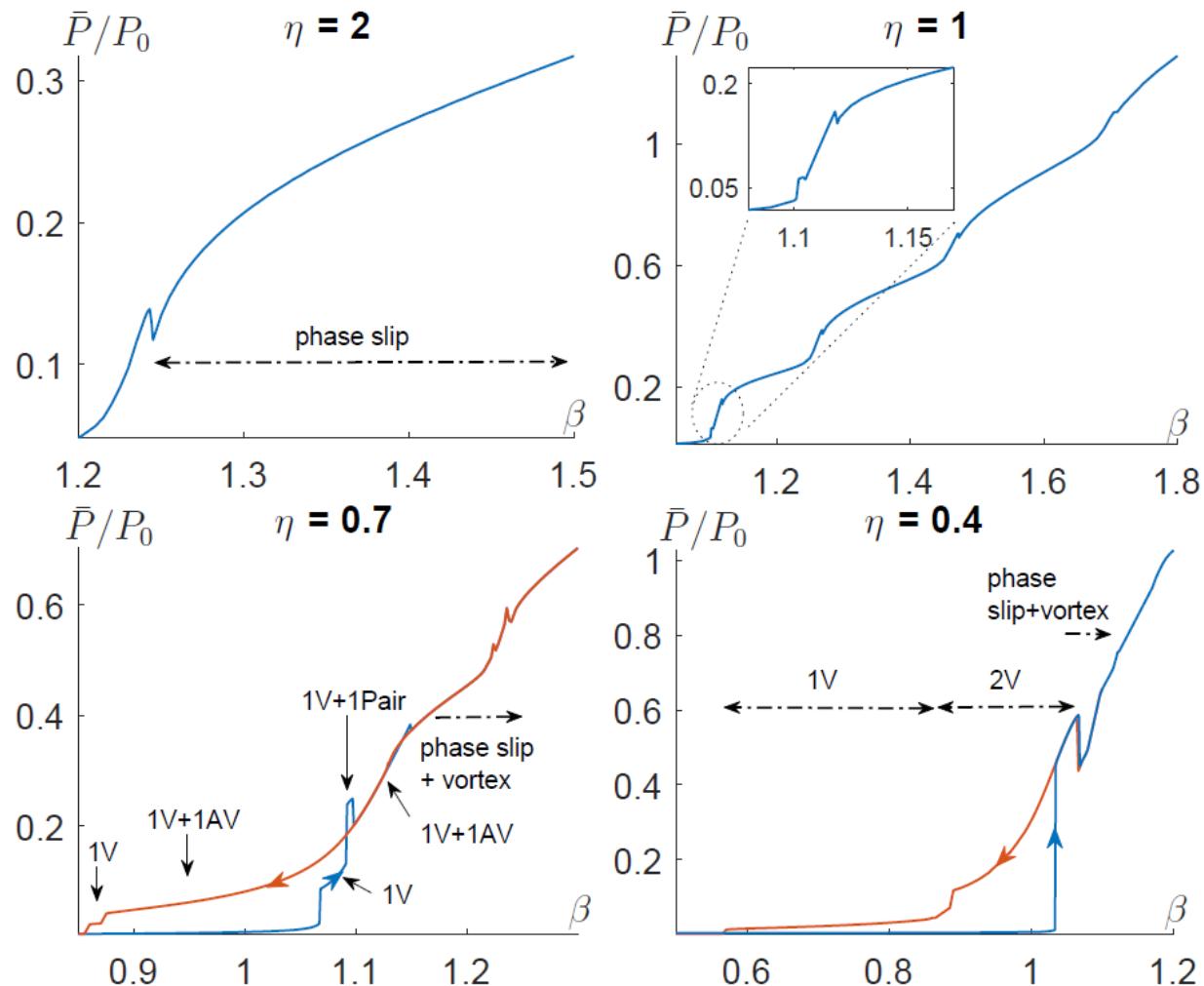
# Moderately under-damped GB ( $\eta = 0.7$ )

- Vortex generates V-AV pairs at  $I$  slightly above the penetration threshold
- Multiple pair production and reflection from edges result in standing waves
- Standing waves evolve into phase slips as current increases
- V-AV pair production in a finite JJ occurs at larger  $I$  than in infinite JJ



# RF power

- Transition from oscillating to ballistic penetration of vortices as the field increases
- Staircase dependence of  $P(H)$  due to increasing the number of vortices in the junction
- Hysteretic dependencies of  $P(H)$  in underdamped junctions
- Dynamic transition of vortices into phase slips switches the entire GB into highly resistive state.

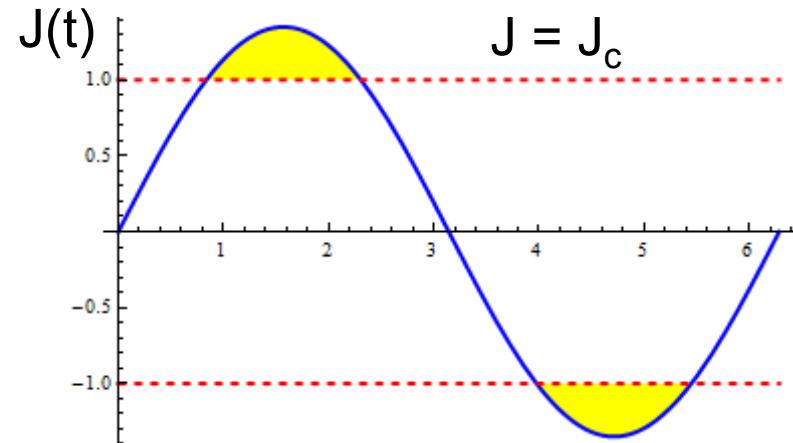


# RF losses in the phase slip state

- Dynamic transition from vortices to phase slips occurs due to current-induced expansion of vortices in overdamped GB and Cherenkov instability in underdamped GBs.
- Dissipated power for  $J(t) = J_0 \sin \omega t$

$$P \simeq \frac{R}{2} (J_0^2 - J_c^2)$$

Estimate for Nb<sub>3</sub>Sn, assuming  $R_{\square} \approx \rho t$ , with  
 $t = 1 \text{ nm}$ ,  $\rho = 1 \mu\Omega\text{m}$ ,  $J_0 = 1.1J_c$ ,  $J_c = 0.5J_d$   
 $J_d = B_c/\mu_0\lambda$ ,  $B_c = 540 \text{ mT}$ ,  $\lambda = 100 \text{ nm}$ :



$P = 400 \text{ MW/m}^2$ , so that a single  $1 \times 1 \mu\text{m}$  GB dissipates  $0.4 \text{ mW}$ .

RF losses above the AJ superheating field in a polycrystal with strongly coupled GBs and a micron grain size can be some 6 orders of magnitude higher than  $P = R_s B_a^2 / 2\mu_0^2$  for  $B_a = 200 \text{ mT}$  and  $R_s = 20 \text{ nOhm}$ .

GB losses above  $H_s^{\text{AJ}}$  are of the order of the losses produced by avalanche penetration of A vortices which quench the cavity

# Summary

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- Grain boundaries do not affect much the surface resistance at low fields but can become crucial performance-limiting defects in  $\text{Nb}_3\text{Sn}$  even at moderate fields.
- Achieving breakdown fields in  $\text{Nb}_3\text{Sn}$  clad Nb cavities above 50 mT may require strongly-coupled grain boundaries with  $J_c \sim J_d$  which do not block RF currents up to  $H \simeq H_c$
- New physics of vortices in strongly-coupled GBs: Cherenkov instability and vortex-antivortex pair production in underdamped GBs at currents below  $J_c$
- Dynamic vortex-phase slip transition in thin film GBs under RF fields
- Hysteretic power losses as a function of RF field
- Explosive increase of RF losses above the GB vortex penetration field