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Electron Mean-Free-Path Dependence of the Vortex Surface Impedance

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Trapped flux surface resistance

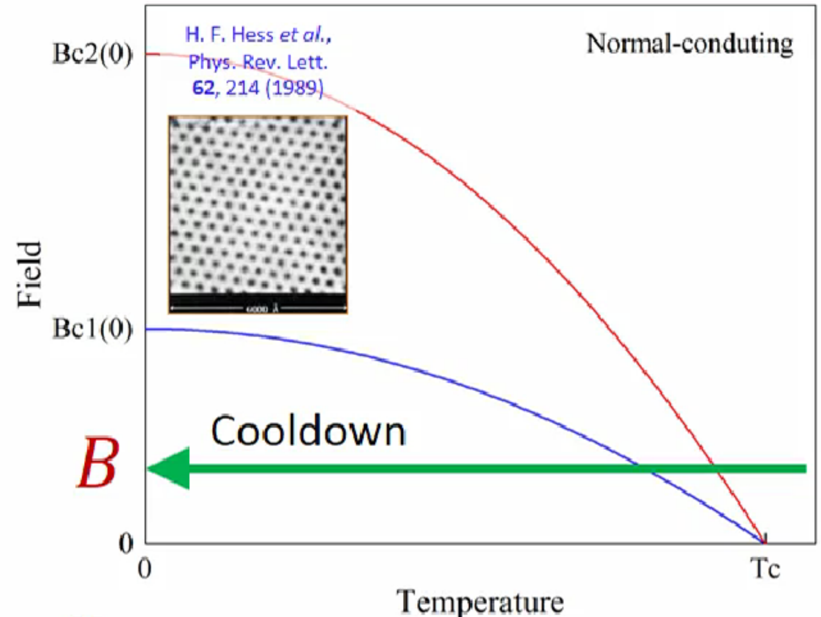
$$R_s(T,B) = R_{BCS}(T) + \boxed{R_{fl}(B)} + R_0$$

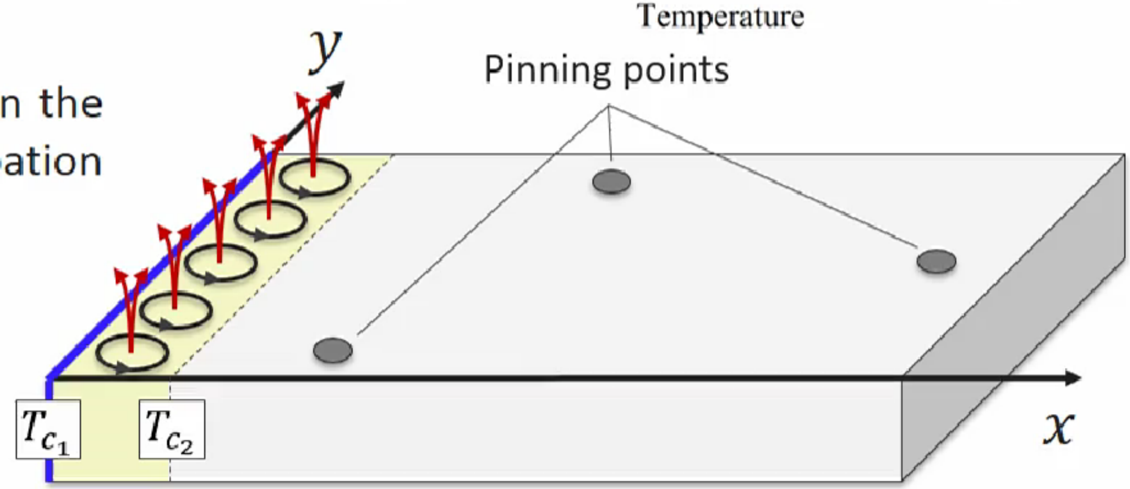
R_{BCS} ⇒ temperature-dependent part of the surface resistance

R_0 ⇒ intrinsic residual resistance

$R_{fl} = \eta_t S B$ ⇒ trapped flux surface resistance:

- If pinned, vortices may survive in the Meissner state introducing dissipation
- η_t—flux trapping efficiency
- S—trapped flux sensitivity
- B—external magnetic field





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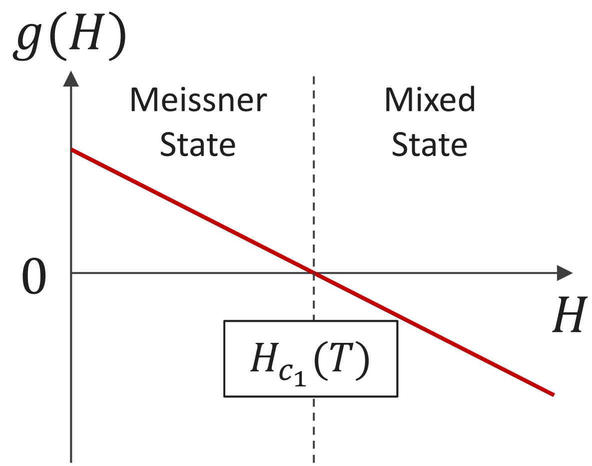
Flux Trapping Efficiency

—η_t—

Thermodynamic force during cooldown

The Gibbs free energy density defines the stability of vortices in the SC:

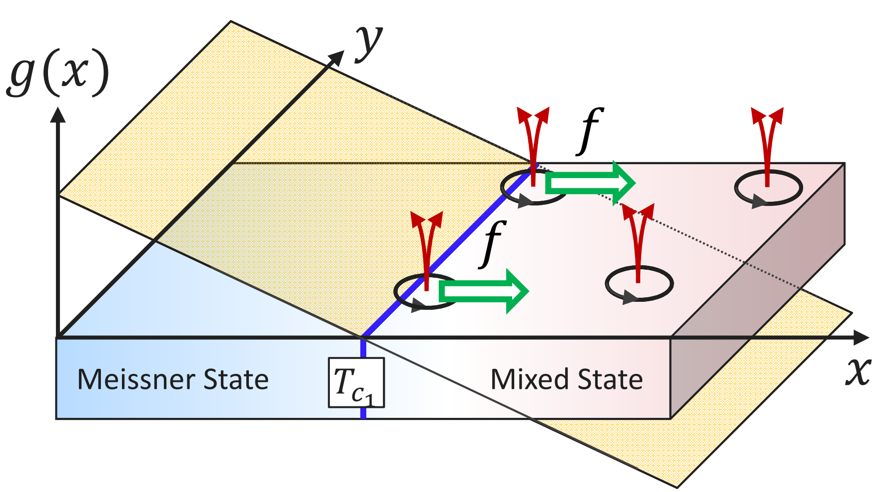
$$g = B(H_{c_1}(T) - H)$$



We can define the *thermodynamic force* acting on the vortex as:

$$f = -\frac{\partial g}{\partial x} = -\frac{\partial g}{\partial T}\frac{\partial T}{\partial x}$$

$$f = \frac{2BH_{c_1}(0)T}{T_c^2}\nabla T$$



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Critical thermal gradient

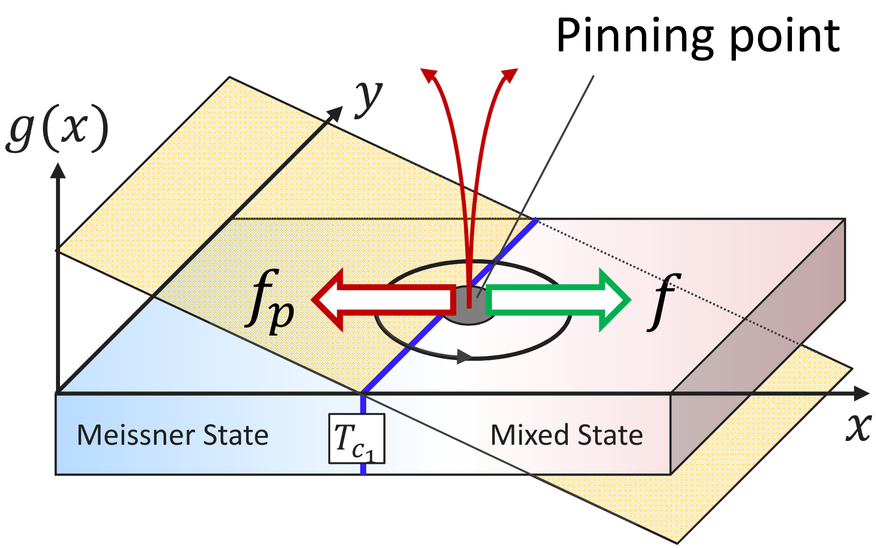
The *pinning force acting against the expulsion* is defined in terms of critical current density J_c:

$$f_p = |\bar{J}_c \times n\bar{\Phi}_0| = J_c B$$

The *minimum thermal gradient needed to expel vortices* is the critical thermal gradient ∇T_c :

$$\nabla T_c = \frac{J_c T_c^2}{2H_{c_1}(0)T}$$

$$\nabla T_c \propto J_c \propto f_p$$



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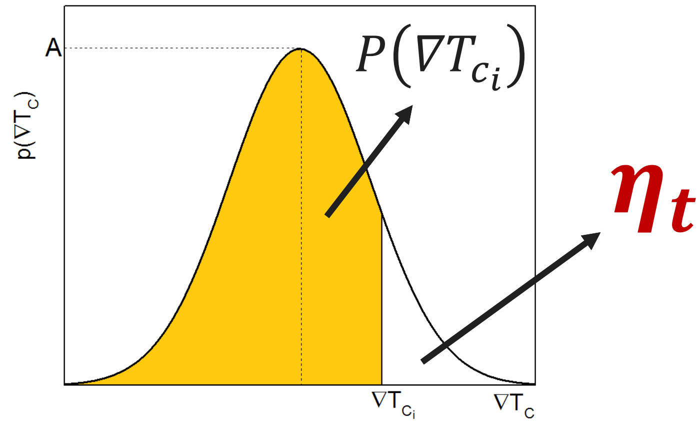
Statistical model for the expulsion ratio

SEE ALSO:
S. Posen, TUXBA02
A. Palczewski, TUXBA06

Let's define a probability density function for flux expulsion:

→ the *probability of expelling vortices with the thermal gradient ∇T_{c_i} is $P(\nabla T_{c_i})$*, hence the expulsion ratio is:

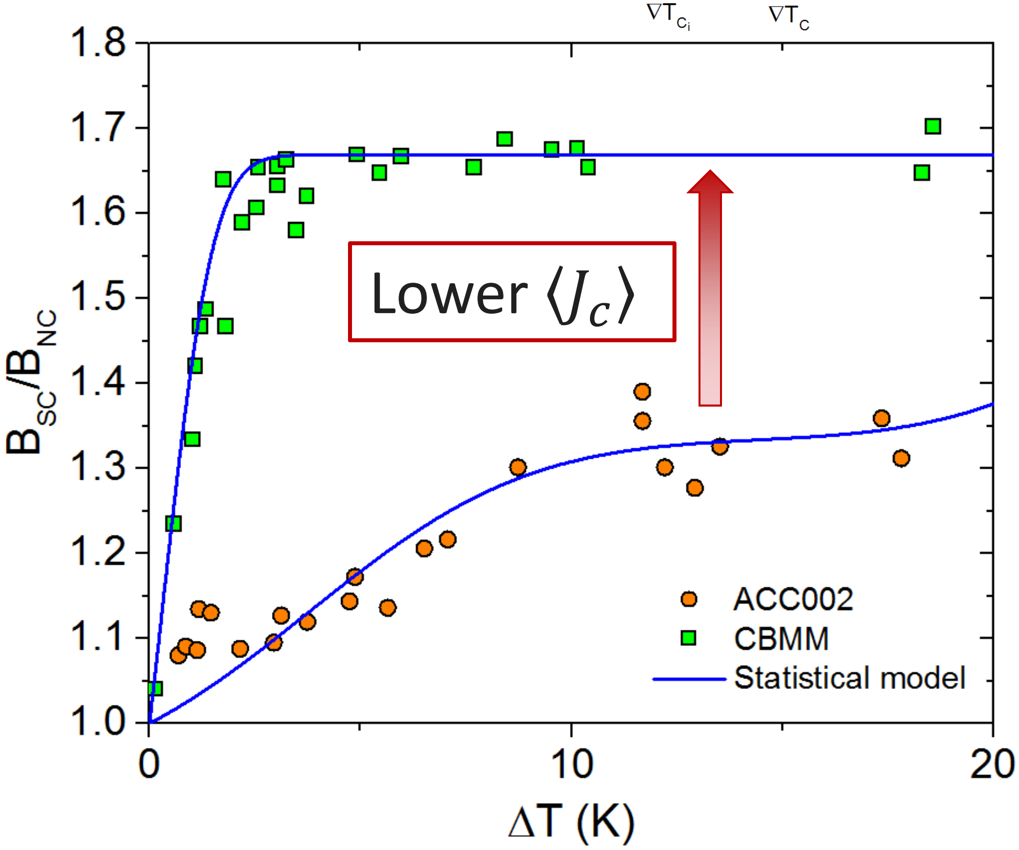


For TESLA shape

$$B_{SC}/B_{NC} = 1 + 0.74 \cdot P(\nabla T_{c_i})$$

The model predicts *$\langle J_c \rangle$ in agreement with literature*[1,2]:

Cavity name	$\langle J_c \rangle$ (A/mm^2)
CBMM	0.3
ACC002	1.6



M. Martinello, M. Checchin *et al.*, to be published

Data: S. Posen *et al.*, J. Appl. Phys. **119**, 213903 (2016)

[1] G. Park *et al.*, Phys. Rev. Lett. **68**, 12 (1992)

[2] L. H. Allen and J. H. Claassen, Phys. Rev. B **39**, 4 (1989)

Vortex Viscosity, Pinning Force and Vortex Motion Equation

Viscosity against vortex motion

The resistivity in the flux-flow regime was defined by Kim *et al.* [1] as:

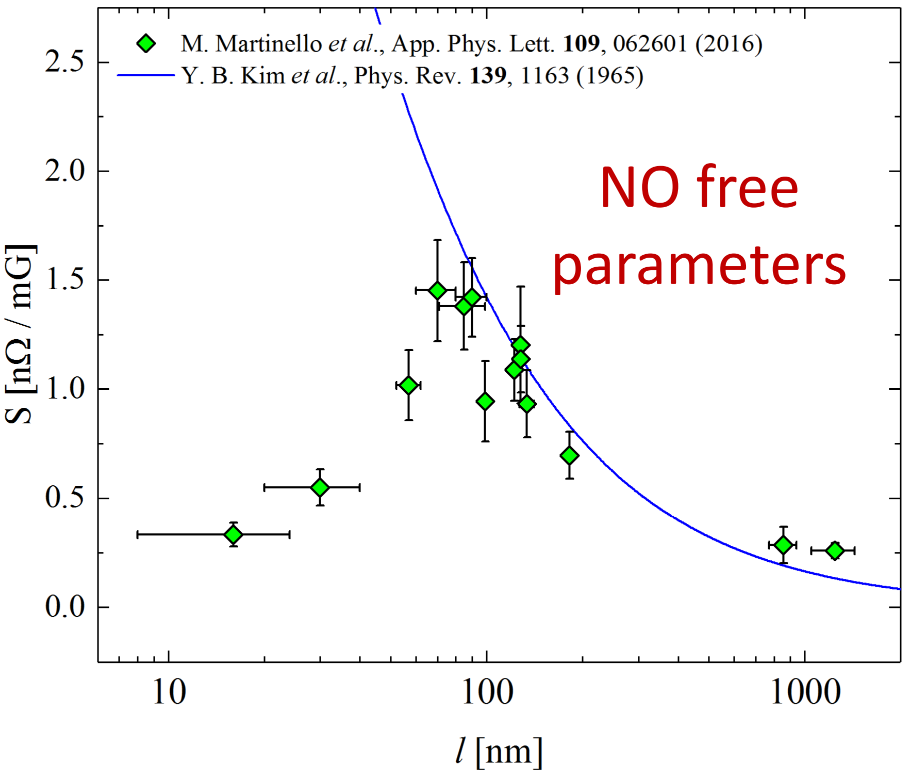
$$\rho_{flux-flow} = \frac{\Phi_0 B}{\eta}$$

Where η is the material viscosity against the vortex motion[2]:

$$\eta(l) = \frac{\Phi_0 B_{c2}(T)}{\rho_n(l)}$$

- $\rho_{flux-flow}$ describes the RF surface resistance for large mean-free-path.
- The low man-free-path region is not described:

  ⇒ *pinning plays a central role!*



[1] Y. B. Kim *et al.*, Phys. Rev. **139**, 1163 (1965)

[2] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, 1197 (1965)

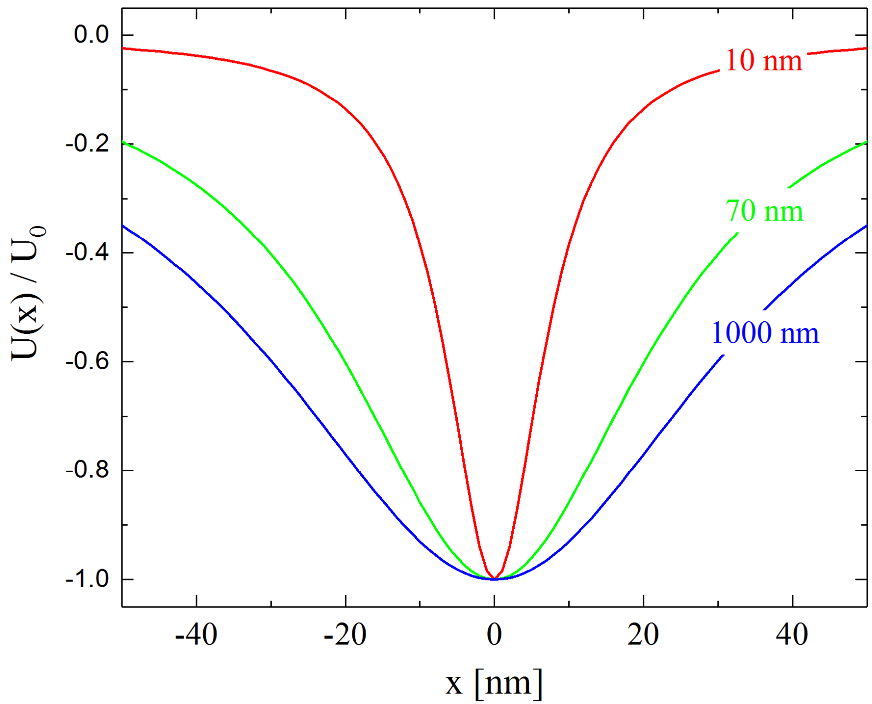
Qualitative dependence of pinning on the mean-free-path

- Pinning force: "interference" between the spatial variations of the order parameter Ψ due to vortex and pinning center (*free energy minimization*)

- *The characteristic variation length of Ψ is* the coherence length ξ

- ξ depends on the mean-free-path
  → The *pinning potential depends on the mean-free-path*

$$U(x,l) = -\frac{U_0 \xi(l)^2}{\xi(l)^2 + x^2}$$



M. Checchin *et al.*, Supercond. Sci. Technol. **30**, 034003 (2017)

More realistic bi-dimensional pinning potential

- A *vortex* is a *mono-dimensional object* oscillating in the plane xz

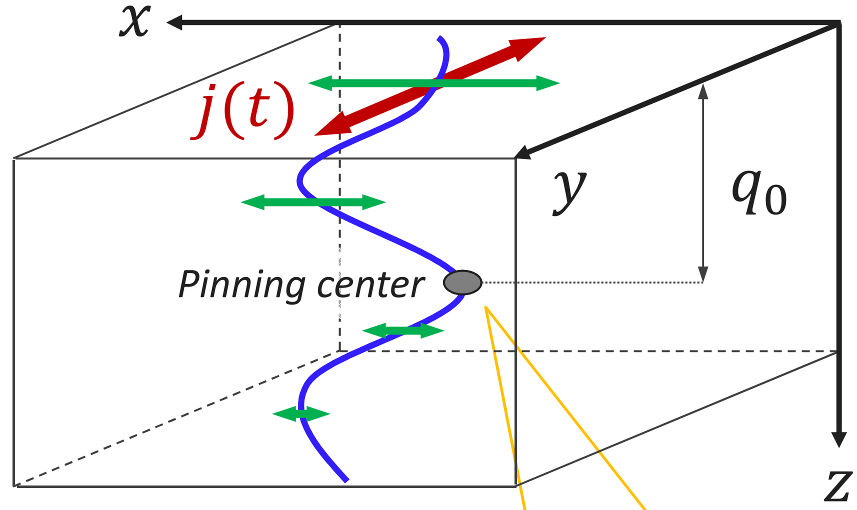
- The *pinning center* is *localized at* the *depth* q_0 from the RF surface

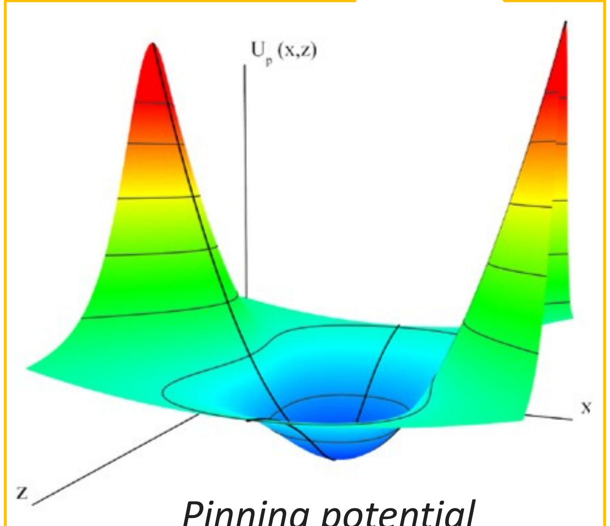
- The *pinning potential must be 2D*:

$$U_p(x,z) = -\frac{U_0\xi^2}{\xi^2 + x^2 + (z-q_0)^2}$$

Parabolic approx. along x

$$\approx -\frac{U_0\xi^2}{\xi^2 + (z-q_0)^2} + \frac{U_0\xi^2}{[\xi^2 + (z-q_0)^2]^2}x^2$$





Pinning potential

M. Checchin *et al.*, Supercond. Sci. Technol. **30**, 034003 (2017)

Motion equation of a vortex

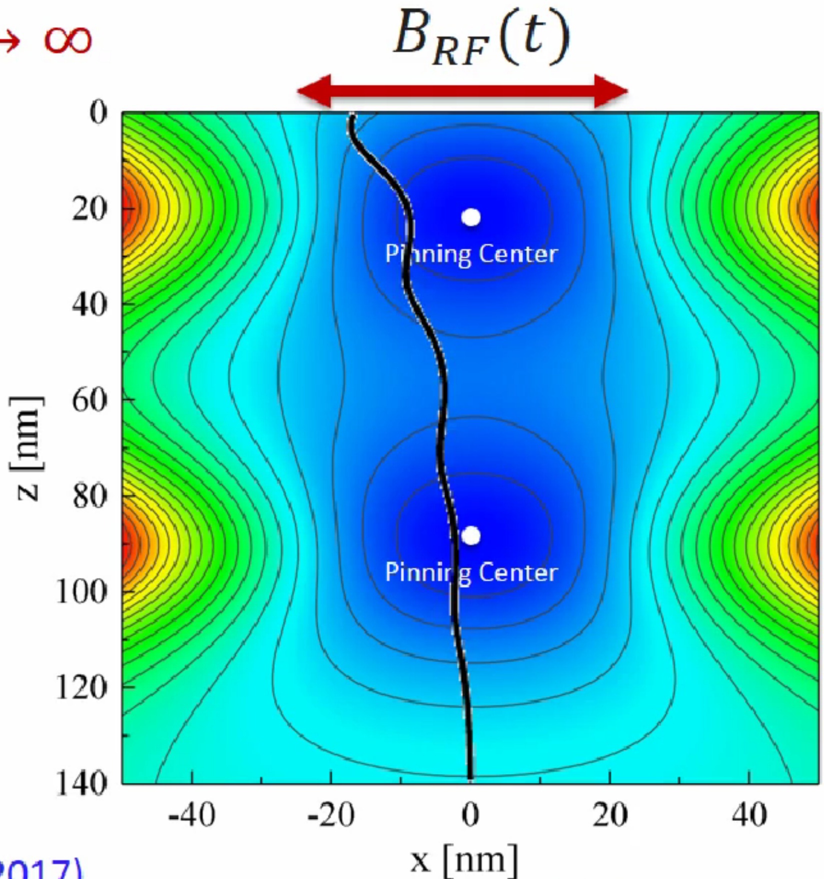
The motion equation has form:

$$M(l)\ddot{x} + \eta(l)\dot{x} + p(z,l)x = j_0\phi_0\sin(\theta)e^{i\omega t - z/\lambda(l)}$$

2D pinning force!

The solution is valid from $z = 0$ to $z \rightarrow \infty$

- The pinning potential assumed is a 2D Lorentzian function
  ⇒ *parabolic approximation along x*
- The pinning constant $p(z,l)$ is depth-dependent
  ⇒ *flexible vortex line*
- Multiple pinning centers per flux line can be considered



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Trapped Flux Sensitivity

—S—

Vortex surface impedance

The *complex resistivity* of the vortex line follows from the calculation of the apparent power (active plus reactive power) :

$$\rho(z,l) = \rho_1 + i\rho_2 = \frac{\phi_0^2 \sin^2(\theta)}{\pi\xi_0^2[(p - M\omega^2)^2 + (\eta\omega)^2]}[\eta\omega + i(p - M\omega^2)]$$

The vortex surface impedance (using the classic definition of Z) is then:

$$Z(l) = \frac{\pi\xi_0^2 B}{\phi_0} \int_0^{q_0^\vee} \int_{U_{0_0}^\wedge}^{U_{0_0}^\vee} \cdots \int_0^{q_n^\vee} \int_{U_{0_n}^\wedge}^{U_{0_n}^\vee} \frac{\prod_{i=0}^{n} \Gamma(q_i)\Lambda\left(U_{0_i}\right)}{\int_0^L \frac{e^{-z/\lambda}}{\rho(z,l)} dz} dU_{0_0} dq_0 \cdots dU_{0_n} dq_n$$

Number of vortices B/B_v

Vortex impedance weighted over normal distributions of pinning positions and strengths

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Parameters affecting the sensitivity

The sensitivity S can be tuned acting on several parameters:

1. Mean free path—l
2. Frequency—f
3. Pinning center depth—q_0
4. Pinning center strength—U_0

1. Mean-free-path dependence

SEE ALSO:
M. Martinello, TUYAA02

- Small l – *pinning regime* $\eta \ll p$:

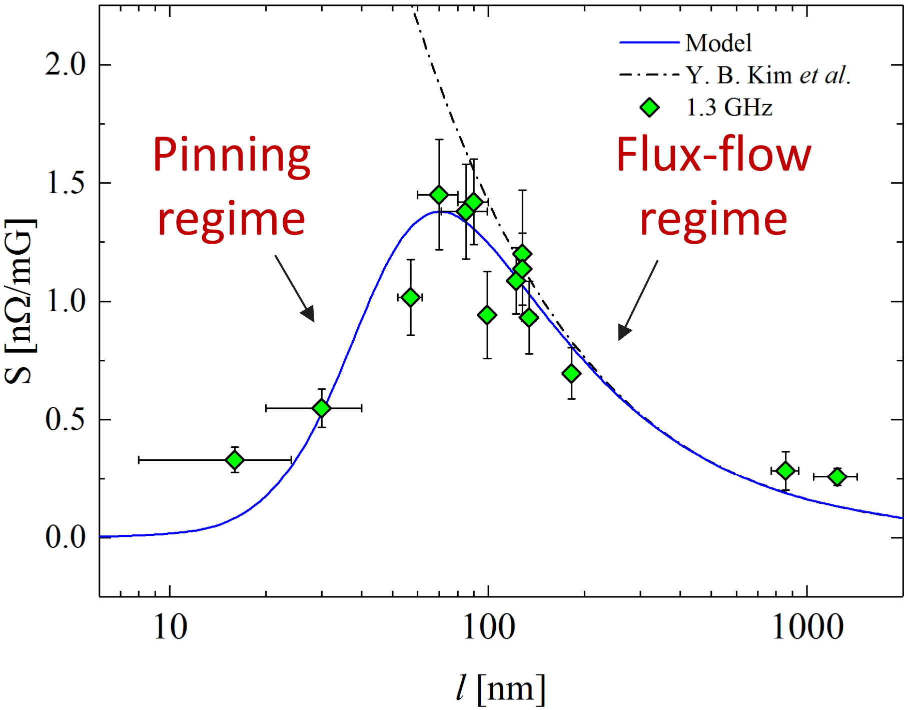
$$\rho_1(l, U_0) \approx \frac{\eta(l)}{p(l, U_0)^2}$$

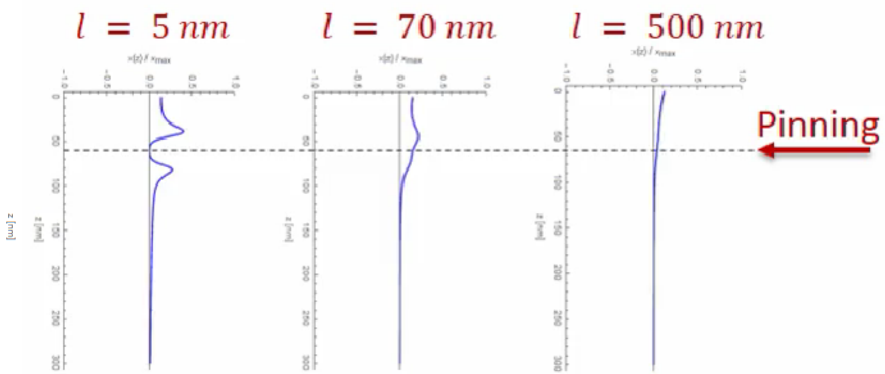
ρ_1 increases with l and ω^2, decreases with the increasing of U_0

- Large l – *flux-flow regime* $\eta \gg p$:

$$\rho_1(l) \approx \frac{1}{\eta(l)}$$

ρ_1 decreases with l, independent on ω and U_0





M. Checchin *et al.*, Supercond. Sci. Technol. **30**, 034003 (2017)
Data: M. Martinello *et al.*, App. Phys. Lett. **109**, 062601 (2016)

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2. Frequency dependence

- Small l – <u>*pinning regime*</u> $\eta \ll p$:

$$\rho_1(\omega) \approx \omega^2$$

ρ_1 *increases with* ω^2

- Large l – <u>*flux-flow regime*</u> $\eta \gg p$:

$$\rho_1 = constant$$

ρ_1 *independent on* ω

- The higher f the higher the sensitivity peak

- *Lower frequencies* are favorable to *minimize the sensitivity*

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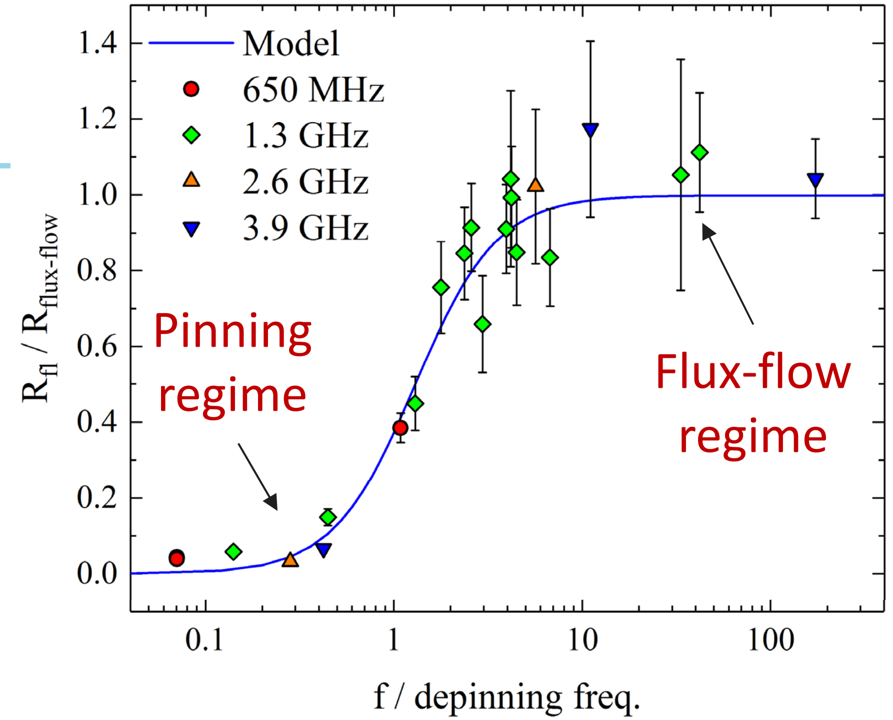
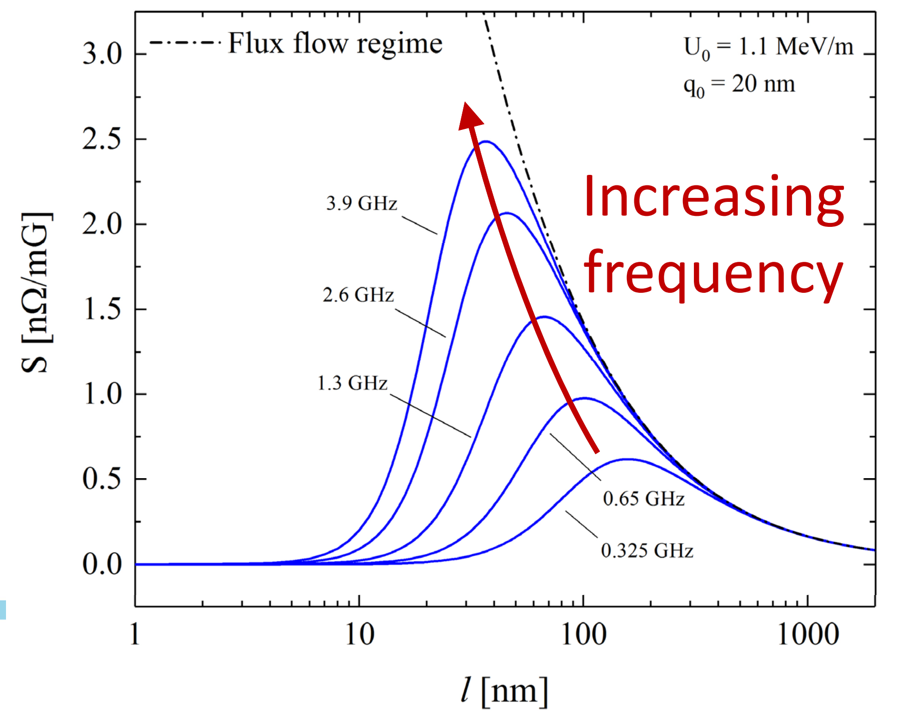
Data: M. Martinello *et al.*, App. Phys. Lett. **109**, 062601 (2016)

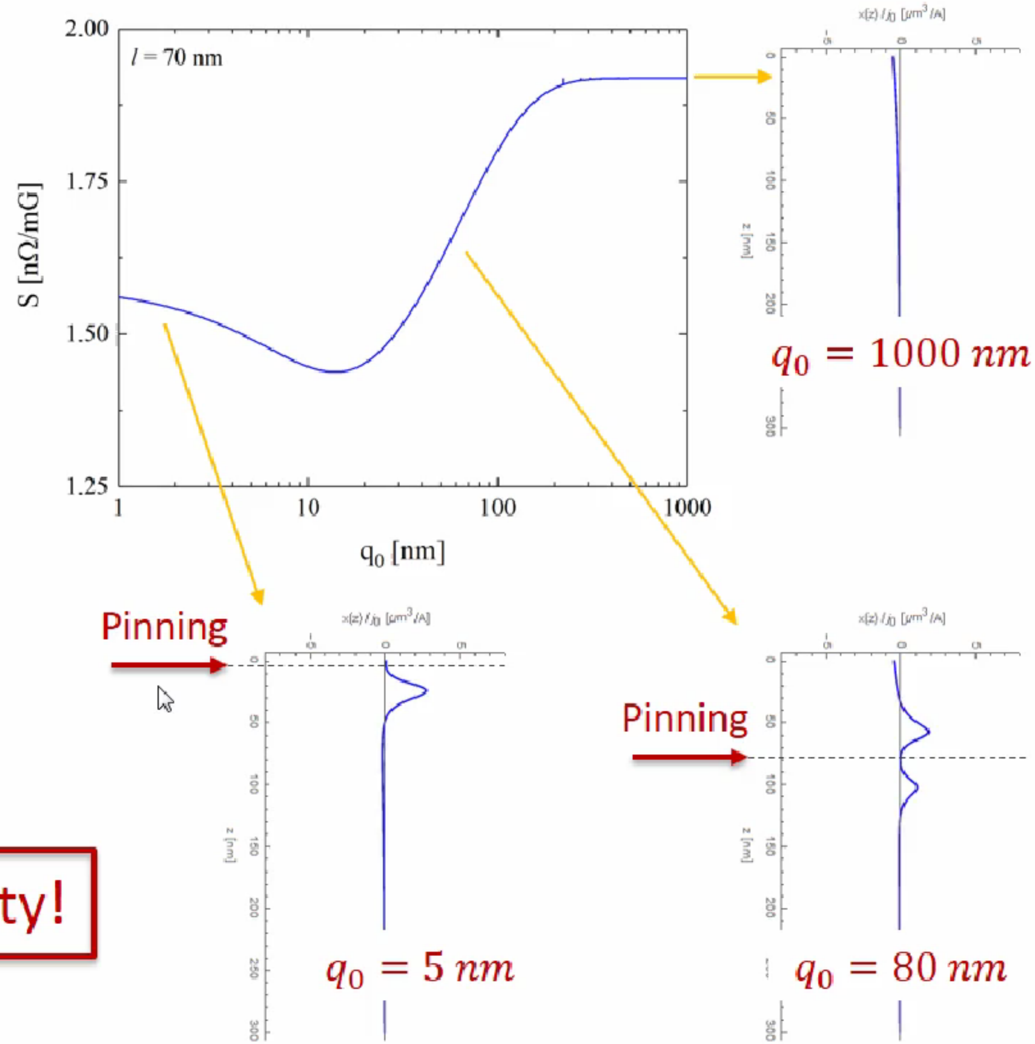




3. Pinning center depth dependence

- The pinning site distance from the surface q_0 determines the resistance
- For instance, if $l = 70\ nm$:
  - $q_0 \cong 15\ nm \Rightarrow$ *sensitivity is the lowest*
  - $q_0 > 400\ nm \Rightarrow$ *constant sensitivity*
  - *bulk pinning does not affect the vortex oscillation*!

$\Rightarrow$ S is a near-surface property!



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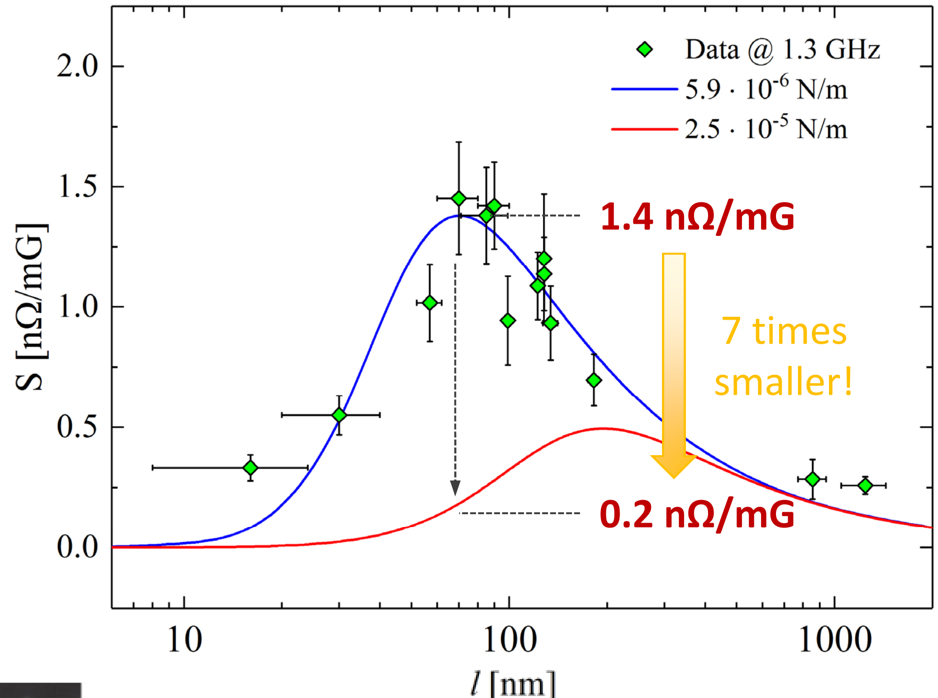
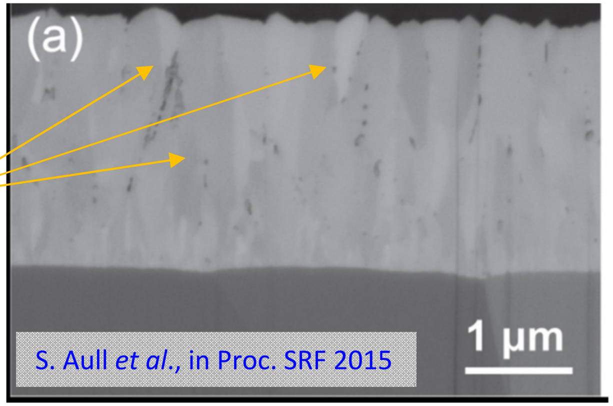
SEE ALSO:
S. Aull, WEXA06

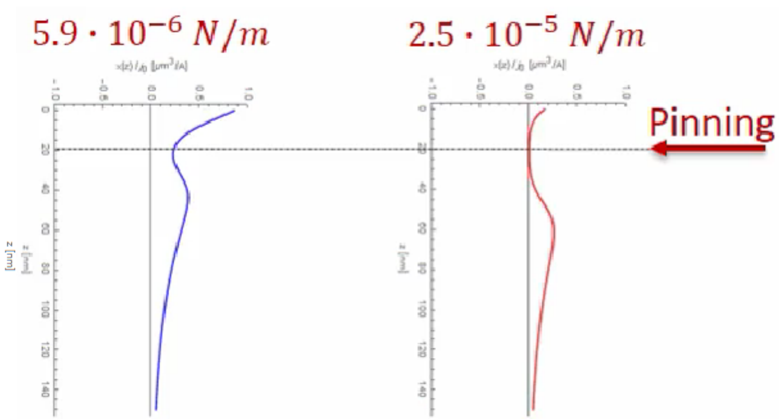
4. Pinning center strength dependence

- The *higher* the *pinning center strength*, the *higher* the *pinning force* and the *more constrained* the *oscillation*
- Dirtier or more defective materials (e.g. *sputtered Nb thin films*) have larger pinning strength



Porosities and defects in a Nb thin film enhance the pinning force





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Conclusions

- Trapping Efficiency—$\eta_t(\nabla T, J_c)$—<u>bulk property</u>:
  - Defined by the *balance of pinning and Gibbs free energy gradient* at the interface between Meissner and mixed state
  - Well described by a statistical argumentation

- Trapped Flux Sensitivity—$S(l, \omega, q_0, U_0)$—<u>near-surface property</u>:

  a. *<u>Two regimes of dissipation</u>*:
     - Small l : *pinning regime*, vortices constrained by pinning
       —S increases if $l\uparrow$, $\omega\uparrow$ and $U_0\downarrow$
     - Large l : *flux-flow regime* , vortex motion counteracted by viscosity
       —S decreases if $l\uparrow$, but independent on ω and U_0

  b. *<u>Pinning depth dependence</u>*:
     - Pinning site distance from the surface q_0 determines the resistance
     - *Bulk pinning does not affect the sensitivity*

  c. *<u>Pinning strength dependence</u>*:
     - The larger U_0, the smaller the sensitivity

Thank you for your attention