Calculating the field dependent surface resistance from Quality factor data

R. Kleindienst, S. Keckert, J. Knobloch, O. Kugeler
Helmholtz Zentrum Berlin

Abstract

The quality factor of an RF cavity and the surface resistance are typically related with a constant geometry factor. The implicit assumption made is that the surface resistance is field independent, which is however not observed experimentally in superconducting cavities. The approximation error due to this assumption becomes larger the less homogeneous the magnetic field distribution along the cavity walls is. In this paper we calculate the surface resistance error for different cavity types. Correction factors as well as a numerical method to correct for this error are presented.

Introduction

The quality factor Q_0 of an RF cavity relates the stored energy U with the energy dissipated per RF cycle. It is calculated by:

$$Q_0 = \frac{\omega U}{P_{\mathrm{Dis}}} = \frac{\omega \int_V |B|^2 \,\mathrm{d}v}{\mu_0 \int_S R_S \cdot |B|^2 \,\mathrm{d}s} \approx \frac{G}{R_S} \qquad (1)$$

where P_{Dis} is the dissipated power and R_S is the surface resistance. In the last term, the geometry factor G is introduced which directly links the quality factor with the surface resistance. This factor is independent of the material and of the size of the cavity and is calculated with:

$$G = \frac{\omega \int_V |B|^2 \cdot \mathrm{d}v}{\mu_0 \int_S |B|^2 \cdot \mathrm{d}s} \qquad (2)$$

Calculating $R_S^{\mathrm{meas}} = G/Q_0$ will return a mean surface resistance, which is only identical to the local material surface resistance $R_S(B)$ if it is field independent or if the field distribution on the cavity surface is uniform. The less homogenous the surface magnetic field is distributed, the larger the approximation error becomes. This effect is shown in Figures 1, where the hypothetically measured surface resistance R_S^{meas} is shown for various different cavity types.

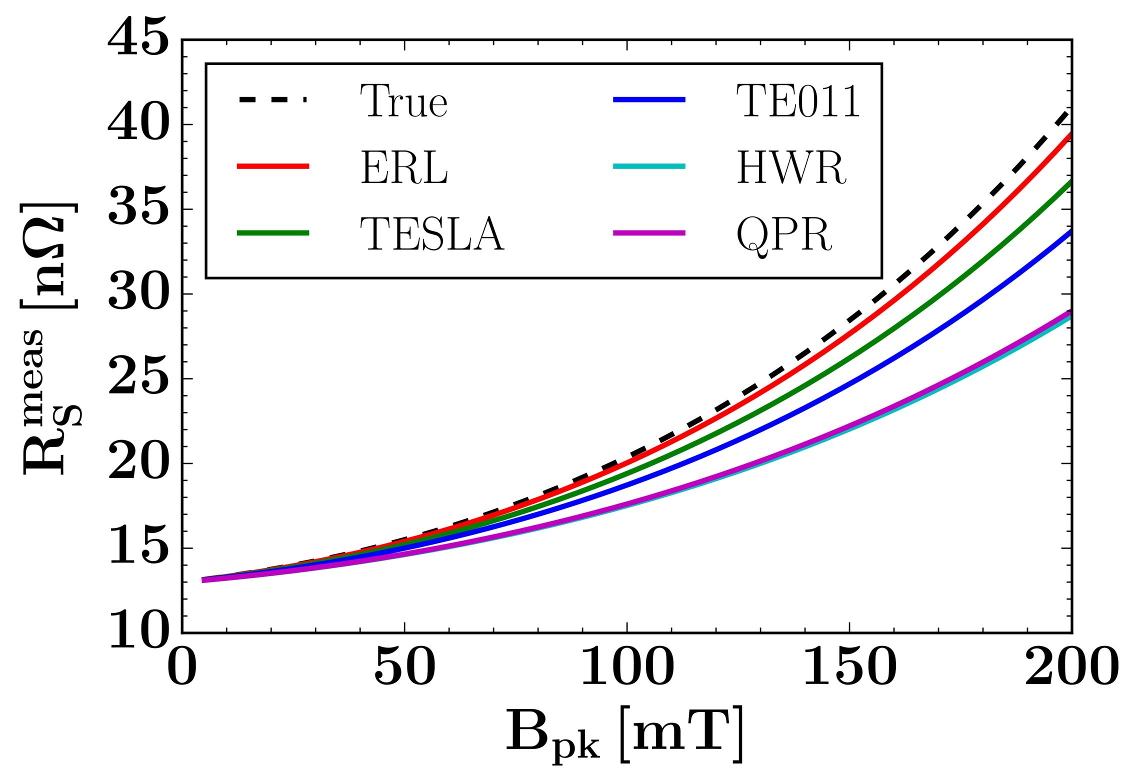

Fig. 1: Hypothetical measurement of the same material with different cavities. Shown in the dotted black line is the assumed surface resistance which has a quadratic and an exponential contribution. For cavities types with very inhomogenous surface magnetic fields, the error when calculating the surface resistance as $R_S^{\mathrm{meas}} = G/Q_0$ can be as large as 30%.

Correction Factors

So how does one correct for this problem? If one assumes a polynomial dependence of the surface resistance ($R_S(B) = R_0 + \sum_{i=1}^{\infty} \alpha_i B^i$) one can explicitly calculate the correction factors for each coefficient [1]:

$$\begin{aligned} R_S^{\mathrm{meas}} = \frac{G}{Q_0} &= \frac{\int_S (R_0 + \sum_{i=1}^{\infty} \alpha_i B^i)|B|^2 \,\mathrm{d}s}{\int_S |B|^2 \,\mathrm{d}s} \\ &= R_0 + \sum_{i=1}^{\infty} \alpha_i \cdot \frac{\int_S (|B|/B_{\mathrm{pk}})^{i+2} \,\mathrm{d}s}{\int_S (|B|/B_{\mathrm{pk}})^2 \,\mathrm{d}s} \cdot B_{\mathrm{pk}}^i \\ &= R_0 + \sum_{i=1}^{\infty} \alpha_i \cdot \beta_i \cdot B_{\mathrm{pk}}^i \end{aligned} \qquad (3)$$

The correction factors for the selected cavities are shown in Table 2. Calculating a correction factor does not work however if the surface resistance is exponential or of a other, non-polynomial form.

	β_1	β_2	β_3
TESLA Cavity	0.91	0.85	0.80
ERL Cavity	0.97	0.95	0.93
Half Wave Resonator	0.74	0.58	0.48
TE_{011} Host Cavity	0.84	0.74	0.67
Quadrupole Resonator	0.72	0.58	0.48

Fig. 2: Correction factors β_i, calculated with Equation 3 for several cavity types.

Numerical Method

Starting with the naive calculation of the surface resistance ($R_{S,0} = G/Q_0$), an expected quality factor is calculated, using Equation 1 and a exlicit calculation of the dissipated power P_{Dis}. A field dependent geometry factor is then computed and the surface resistance results are updated. The updated results are used to compute a new geometry factor, and so on.

$$\begin{aligned} R_{S,0}(B) &= \frac{G_0}{Q_0(B)} \\ G_1(B) = Q_{\mathrm{calc}}(R_{S,0}) \cdot R_{S,0} \quad &, \quad R_{S,1}(B) = \frac{G_1(B)}{Q_0(B)} \\ G_2(B) = Q_{\mathrm{calc}}(R_{S,1}) \cdot R_{S,1} \quad &, \quad R_{S,2}(B) = \frac{G_2(B)}{Q_0(B)} \\ &\vdots \end{aligned} \qquad (4)$$

Note that as the measurement data $Q_0(B)$ is discrete, one has to interpolate the intermedeate surface resistance results $R_{S,i}$ to be able to calculate the expected quality factor at each iteration. The application of these update rules is shown in Figure 3, using as an example the half wave resonator, modeled as coaxial transmission line shorted at both ends [2], and the same surface resistance functions as assumed previously. One can see that the algorithm converges towards the correct result within a few iterations.

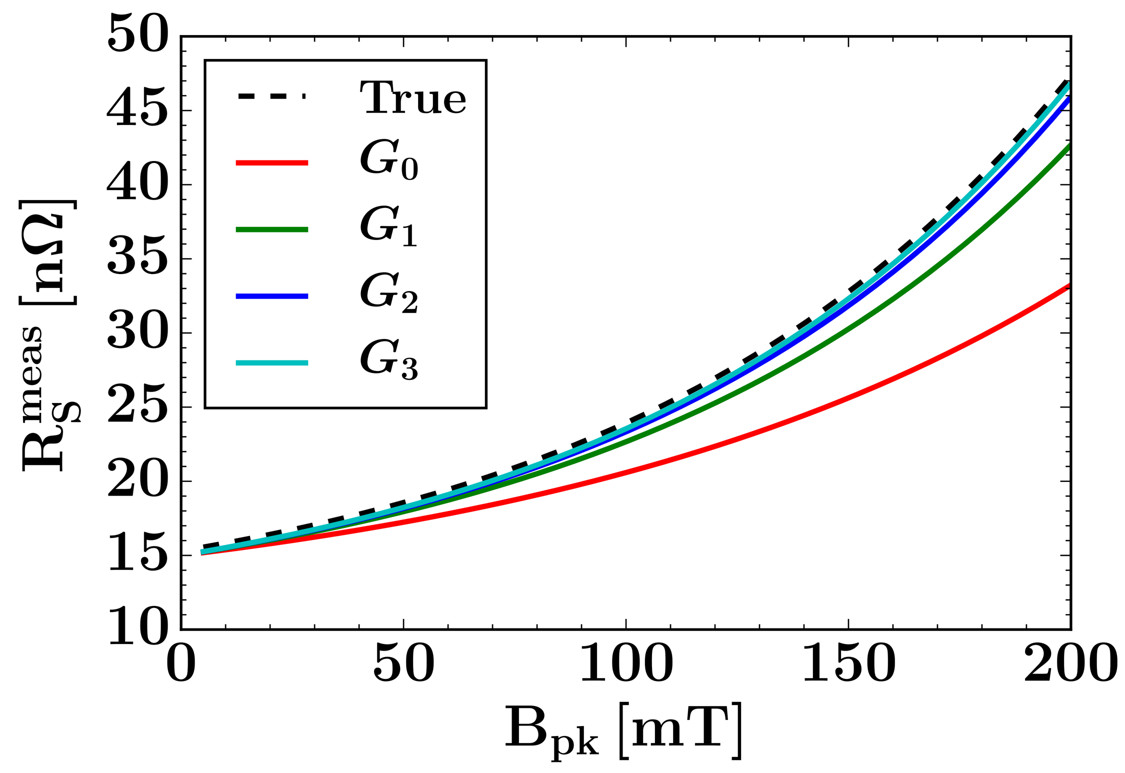

Fig. 3: Correction calculation for a Half Wave Resonator assuming a monotonically increasing surface resistance as used in Figure 1. The black dotted line indicates the 'true' surface resistance, the result obtained using a constant geometry factor is shown in red . After only a few iterations, the calculation converges towards the correct result.

Conclusion

We have shown that the approximation error caused by calculating the surface resistance directly from the geometry factor can be very significant for realistic scenarios. If the surface resistance follows a polynomial function, one can pre-compute correction factors. Furthermore a simple method was introduced that correctly calculates the surface resistance from Q_0-data without making assumptions about the underlying loss model.

Acknowledgement

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References

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