ISBN: 978-3-95450-191-5 doi:10.18429/JACoW-SRF2017-TUPB009

HIGH-FREQUENCY SRF CAVITIES*

T. Oseroff†, D. L. Hall, M. Liepe, and J. T. Maniscalco
CLASSE, Cornell University, Ithaca, NY, 14853, USA

Abstract

Historically, the frequency of superconducting RF cavities has been limited by cryogenic power dissipation increasing rapidly with frequency, due to the BCS surface resistance having a quadratic dependence on frequency. Now, new SRF surfaces using doped niobium and compound superconductors like Nb3Sn can drastically reduce the BCS part of the surface resistance. The temperature independent part of the surface resistance (residual resistance) can therefore become dominant, and has its own, different frequency dependence. We have developed a model to analyze cryogenic cooling power requirements for SRF cavities as function of operating frequency, temperature, and trapped flux to evaluate the impact of the novel low-loss SRF surfaces on the questions of optimal operating frequency and frequency limit. We show that high-frequency SRF cavities now become a realistic option for future SRF driven accelerators. As the transverse cavity size decreases inversely with respect to its resonant frequency, such high-frequency SRF cavities could greatly reduce cost.

INTRODUCTION

SRF cavity dimensions scale inversely with resonant frequency. Thus if cavity designs were increased beyond the standard 0.5 - 1.3 GHz range, it could potentially decrease the power dissipated in the surface because of the smaller surface area. However, the power dissipated in the cavity wall also scales with the total surface resistance of the cavity. The surface resistance of the classic SRF material choice, Niobium, is dominated by BCS component which has a quadratic dependence on frequency. This means that going to higher frequencies offers no benefits for Nb. New candidates for SRF materials, Nb_3Sn and N-doped Nb, have a much smaller BCS resistance allowing for the residual resistance to have a significant contribution. If the residual resistance is large enough compared to the BCS resistance and has a small enough dependence on frequency then it is possible the total power dissipated could decrease at higher frequencies. For this reason it is worth investigating the potential benefits of N-doped Nb and Nb_3Sn at frequencies beyond the standard range.

POWER

The power dissipated in a cavity of length L can be put in terms of the shunt impedance R_a and the accelerating voltage V_c.

$$\begin{aligned} P_{diss} &= \frac{V_c^2}{R_a} \\ &= \frac{V_c^2}{\left(\frac{R_a}{Q}\right)_{total} Q} \\ &= \frac{E_{acc}^2 L^2}{\left(\frac{R_a}{Q}\right)_{cell} \left(\frac{L}{\lambda/2}\right) \left(\frac{G}{R_s}\right)} \\ &= \frac{E_{acc}^2 L^2 R_s}{\left(\frac{R_a}{Q}\right)_{cell} \left(\frac{2Lf}{c}\right) G} \end{aligned}$$

In the third line E_{acc} is the accelerating field, the ratio of shunt impedance to quality factor (R_a/Q) in a cell is independent of surface resistance and can be calculated for a given geometry, $L = N\lambda/2$ (where N is the number of cells in the structure) for π-mode operation, and the quality factor is $Q = \frac{G}{R_s}$ where G is the geometry factor of the cavity and R_s is the surface resistance. Thus the ratio of power dissipated to length of the structure is

$$\frac{P_{diss}}{L} = \frac{E_{acc}^2 c}{2\left(\frac{R_a}{Q}\right)_{cell} G} \left(\frac{R_{BCS}}{f} + \frac{R_0}{f}\right) \tag{1}$$

Where the total resistance has been split into its BCS and residual components. From Eq. (1) it is clear that the term resulting from $R_{BCS} \propto f^2$ can only increase the dissipated power with frequency. The residual term, however, could serve to decrease with frequency if $R_0 \propto f^\alpha$ where $\alpha < 1$.

The cryogenic power required to maintain the bath temperature is found by

$$P_{cryo} = P_{diss} \times \text{Efficiency}$$

The efficiency used is shown in Fig. 1. There is an optimal temperature, where the balance between R_{BCS} increasing with temperature and the efficiency decreasing with temperature is satisfied.

RESISTANCE

There has been little experimental work with N-doped and Nb_3Sn cavities at frequencies above 1.3 GHz. As seen in Eq. (1) the frequency dependence of the surface resistance plays a critical role in determining the viability of cavities at higher frequencies. This means that to more accurately estimate the performance of these cavities at high frequencies it is necessary to call upon untested theoretical predictions.

* This work was supported by the U.S. National Science Foundation under Award PHY-1549132, the Center for Bright Beams
† teo26@cornell.edu

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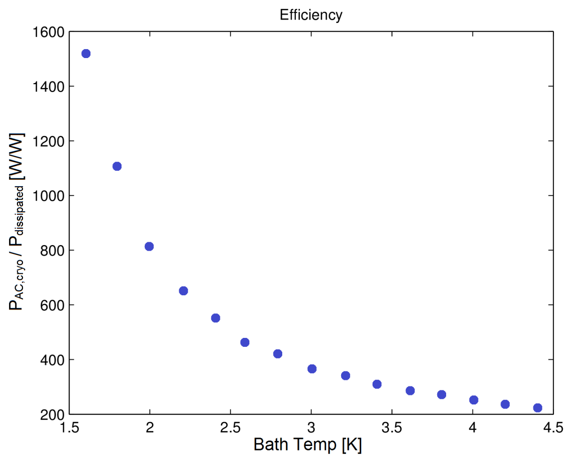

Figure 1: Cryogenics AC power required to maintain bath temperature per watt dissipated at cryogenic temperature vs bath temperature.

BCS Resistance

The BCS resistance for Nb_3Sn is a much smaller contribution than the residual resistance and is calculated from BCS theory using the SRIMP program [1]. At temperatures considered in this work its contribution is less important than the residual resistance.

For N-doped cavities an effect known as anti-Q-slope is observed and is defined as an increase in the quality factor (or a decrease in the surface resistance) with respect to the accelerating field in the cavity. Gurevich has proposed a theory [2] that attempts to model the anti-Q-slope behavior. The theory involves a calculation for the temperature of the quasiparticles on the inner surface of the cavity relative to the bath temperature on the outer wall.

$$T - T_0 = \frac{1}{2}\alpha' H_a^2 R_{BCS}(H_a, T, f) \tag{2}$$

Where T is the quasiparticle temperature at the cavity surface, T_0 is the bath temperature, α' is a parameter dependent on the heat transfer properties of the material, and H_a is the applied field. This temperature rise is called quasiparticle overheating. Work performed by Maniscalco [3] indicates that this theory does fit experimental data for heavily doped cavities, and finds there is a linear relationship between the overheating parameter, α', and the mean free path. It must be noted that Eq. (2) is derived assuming the overheating is linear and the approximation fails at high fields and frequencies.

Residual Resistance

It is assumed that the dominant source of residual resistance is that resulting from the oscillations of flux vortices trapped by defects in the material. For N-doping this residual resistance is thought to scale with the square-root of frequency [4] which is consistent with theories of strong pinning in Nb [5] - where the vortex is pinned by a defect and is unable to escape from that point.

The situation is more complicated with Nb_3Sn where the residual resistance has been observed to scale linearly with accelerating field [6]. The theory of strong pinning presented by Gurevich, depending on how far beneath the RF surface the pinning occurs, predicts a more complicated frequency dependence ranging from square-root to quadratic. However, this theory fails to account for the observed dependence on accelerating field.

The residual resistance will of course depend on how much trapped flux is present on the sample during the cooling process. For Nb_3Sn there is also an extra dependence on the thermal gradient present on the cavity, where the boundary between the different materials causes thermal currents that produce flux [6]. The exact dependencies have been measured at 1.3 GHz. For this work, the experimentally measured trapped field, thermal gradient, and field dependencies at 1.3 GHz are used with a relative frequency dependence as estimated by the before-mentioned theories.

For N-doped Nb,

$$R_{0,N-doped} = \left(\frac{R_0}{B_{trapped}}\right) B_{trapped} \left(\frac{f}{1.3\,\text{GHz}}\right)^{1/2}$$

Here the ratio $R_0/B_{trapped}$ depends on mean free path and experimentally determined values [7] are used. For Nb_3Sn [8],

$$\begin{aligned} R_{0,\text{Nb}_3\text{Sn}} = &\left[\left(0.47\frac{\text{n}\Omega}{\text{mG}} + 0.02\frac{\text{n}\Omega}{(\text{mG})(\text{mT})}B_{RF}\right)B_{trapped}\right. \\ &+\left.\left(2.9\frac{\text{n}\Omega}{\text{K/m}} + 0.13\frac{\text{n}\Omega}{(\text{K/m})\text{mT}}B_{RF}\right)\frac{\Delta T}{\Delta L}\right] \\ &\times\left(\frac{f}{1.3\,\text{GHz}}\right)^{\alpha} \end{aligned}$$

where α is the yet unknown frequency dependence of the residual resistance

RESULTS

The cryogenic power requirement is calculated using the previously stated models for temperatures 1.6 - 2.4 K for N-doped and 1.6 - 4.5 K for Nb_3Sn, frequencies 1 - 4 GHz, and accelerating fields 0 - 25 MV/m. A trapped field of 2 mG and a thermal gradient of 0.1 K/m during cool-down are assumed. Material parameters are shown in Table 1.

Table 1: Material Parameters

	Doped Nb	Nb_3Sn
T_c [K]	9.2	18
$\frac{\Delta}{K_b T}$	1.933	2.4
λ_L [nm]	38	89
ξ_0 [nm]	38	11
l [nm]	4	3.25

In N-doped Nb the residual and BCS resistances contribute similarly to the total resistance and the residual resistance due to trapped flux scales with the square-root of

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frequency. By Eq. (1), this serves to decrease cryogenic power requirements as frequency increases (in the considered frequency range). However, Gurevich's theory on anti-Q-slope introduced a frequency dependent temperature seen in Eq. (2). The BCS resistance now increases much more dramatically with frequency at high fields than the expected quadratic dependence. The only way N-doped cavities could maintain or increase performance at higher frequencies is if the overheating parameter, α', is small. This condition is met for heavily doped cavities where the mean free path is small. However, the standard BCS resistance has a dependence on mean free path that minimizes near half of the BCS coherence length [4]. This indicates that the optimal doping level must be shifted for different frequencies to balance the increased overheating with the mean free path dependence of the BCS resistance. There is a third mean free path dependence; that of the residual resistance [7]. In Fig. 2 the balance between these three dependencies is shown. If the mean free path, l, is too small (l = 0.1 nm in the plot), then the power requirements start high and increase quickly with frequency. If the mean free path is too large (l = 19 nm - ignoring overheating this would be an optimal mean free path) then the power dissipated will very quickly increase with frequency due to increasing overheating. For mean free paths below 4 nm a fit [7] of a theory by Gurevich is used to estimate flux sensitivity. This theory predicts a very low sensitivity for low mean free paths ($l \leq 1$ nm), as is evident for the corresponding curves in Fig. 2. In this work the optimal mean free path found was 1 nm. Note that when lines level off in Fig. 2 it is a nonphysical flaw in the simulation resulting from the linear assumption of Eq. (2). Fig. 2 shows that heavily N-doped cavities are feasible at higher frequencies. The overheating does cause the total power requirements to increase, but it does not increase enough that the benefits of a smaller cavity are insignificant.

Each frequency and field is minimized at a particular choice of temperature, due to the balance between cryogenic power efficiency shown in Fig. 1 and the temperature dependence of the BCS resistance. For N-doping, where the BCS resistance is very important, this results in a minimum temperature on the low end of the simulated range. Modern cryomodules generally do not go below 1.8 K, so the results are shown allowing the minimum temperature to drop to 1.6 K as well as the more realistic 1.8 K. Results from simulations using the above models are shown in Fig. 3. In the top right of the figures the simulation fails due to the overheating calculation becoming too large for the linear approximation.

For Nb_3Sn the results are preliminary due to the lack of knowledge of the frequency dependence of the residual resistance due to trapped flux. In Fig. 4 the simulation results are shown for the cases of residual resistance scaling with the square-root and the square of frequency. As presented, this work indicates that it would be worthwhile to experimentally investigate the frequency dependence of Nb_3Sn, as it could potentially decrease the power requirements. That being said, this study does ignore some potentially important effects

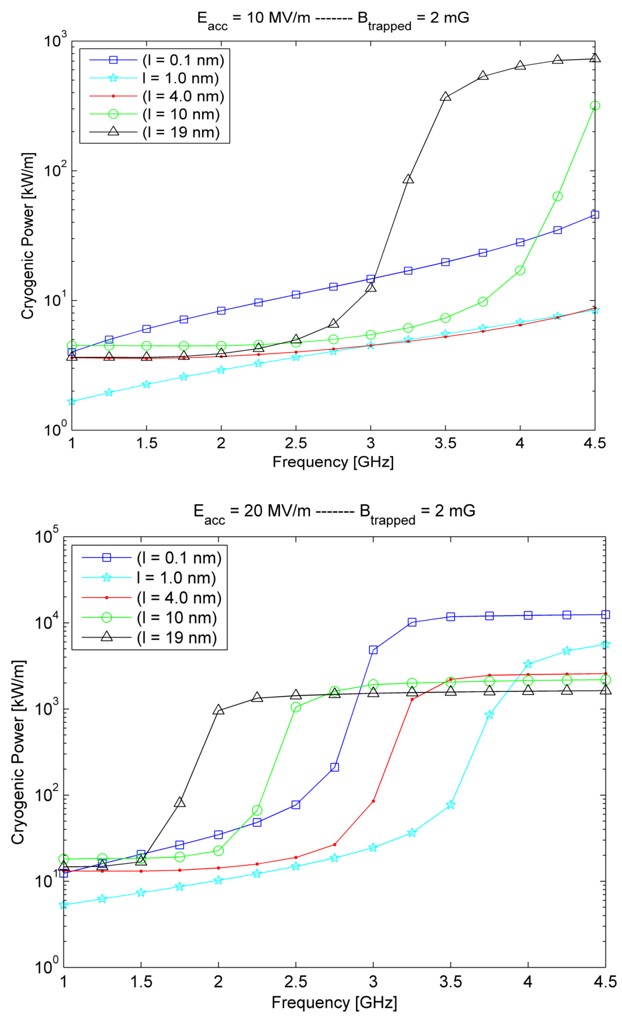

Figure 2: Cryogenic power requirements shown for N-doped Nb of various mean free paths at 10 MV/m (top) and 20 MV/m (bottom). Each point is shown at the temperature that minimized the power assuming a minimum bath temperature of 1.8 K.

that could limit Nb_3Sn at higher frequencies such as the dependence of thermal feedback on frequency.

CONCLUSION

The question this work attempts to answer is whether or not N-doped Nb and/or Nb_3Sn cavities would receive any cost benefits from operating at a higher frequency than the standard 0.5 - 1.3 GHz range. To answer this, the cryogenic power required to maintain cavity temperature was simulated for a variety of accelerating fields and frequencies.

For N-doped Nb simulations followed a theory describing a potential mechanism for the anti-Q-slope of N-doped Nb that also involves a frequency-dependent overheating of the quasiparticles at the RF surface of the cavity. This serves to counteract the expected benefit from the square-root frequency dependence of the residual resistance due to trapped flux. The amount of overheating is linearly proportional to

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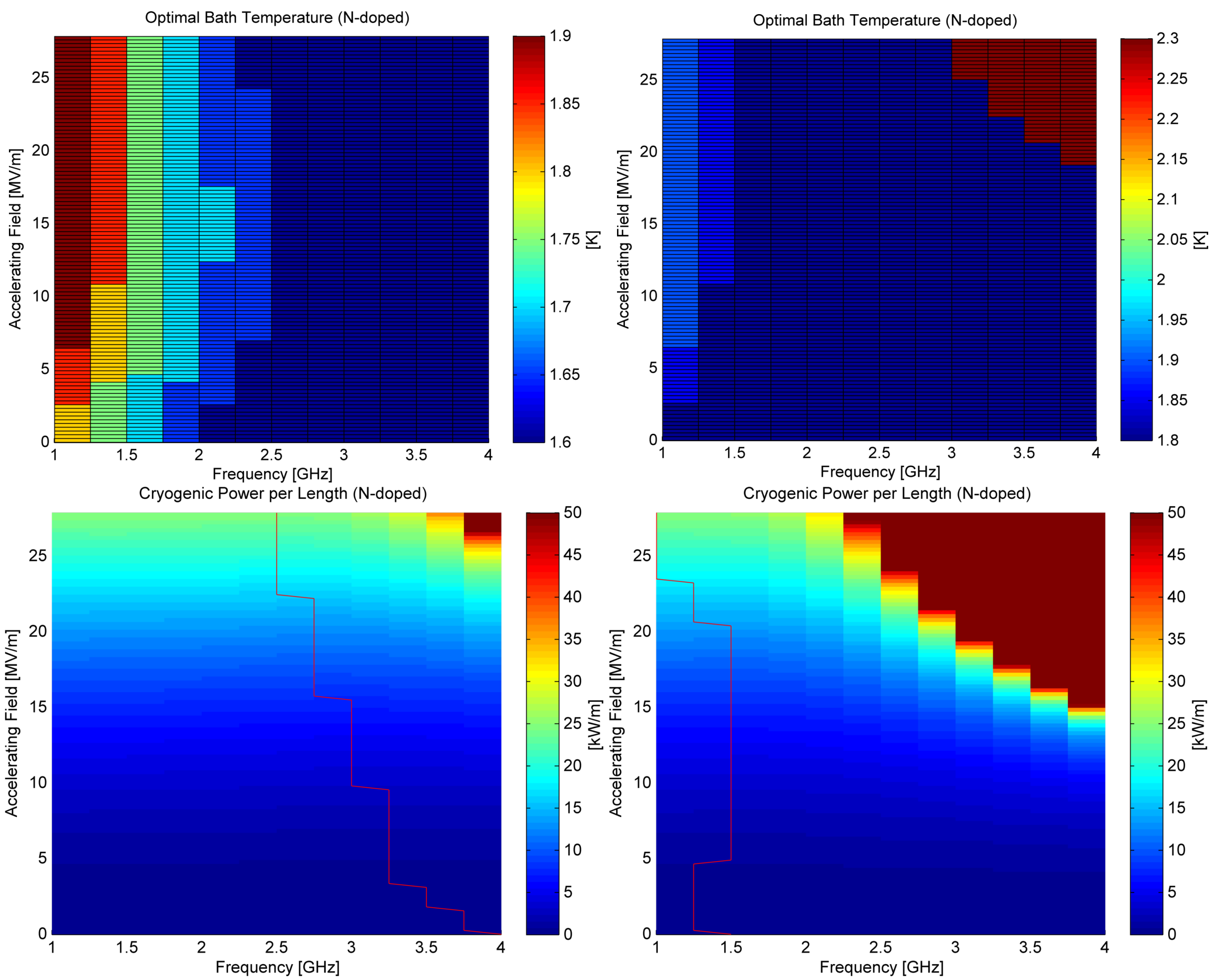

Figure 3: Cryogenic power loss as a function of frequency and accelerating field (bottom) shown at the temperature that minimizes this quantity for each point (top) for N-doped Nb with mean free path of $l = 4$ nm. Shown with minimum allowable bath temperature of 1.6 K (left) and 1.8 K (right). The frequency of the minimum cryogenic power requirement is shown for a given field by the red line.

the mean free path while the classical BCS resistance has a dependence on mean free path that minimizes at a mean free path equal to half of the BCS coherence length. Thus an optimal mean free path for higher fields and frequencies would be well below this expected value where the overheating does not overwhelm the quasiparticle temperature. In this work we found that the cryogenic power requirement at medium fields does depend rather weakly on frequency for a properly chosen mean free path. It should therefore be possible to efficiently operate N-doped high-frequency cavities while making use of other benefits of a high frequency cavity such as reduced materials cost. It is also worth noting that operating at a lower temperature than is easily feasible, like 1.6 K, provides much better performance with higher frequencies.

For Nb_3Sn it is difficult to make a solid claim due to the lack of experimental and theoretical work at higher frequencies; both of which are currently underway at Cornell. With this in mind, the models considered in this work predict that the power requirements of Nb_3Sn cavities will be feasible at higher frequencies. Unless there are overlooked frequency-dependent performance limiting phonomena, Nb_3Sn cavities may benefit from operating at higher fequencies.

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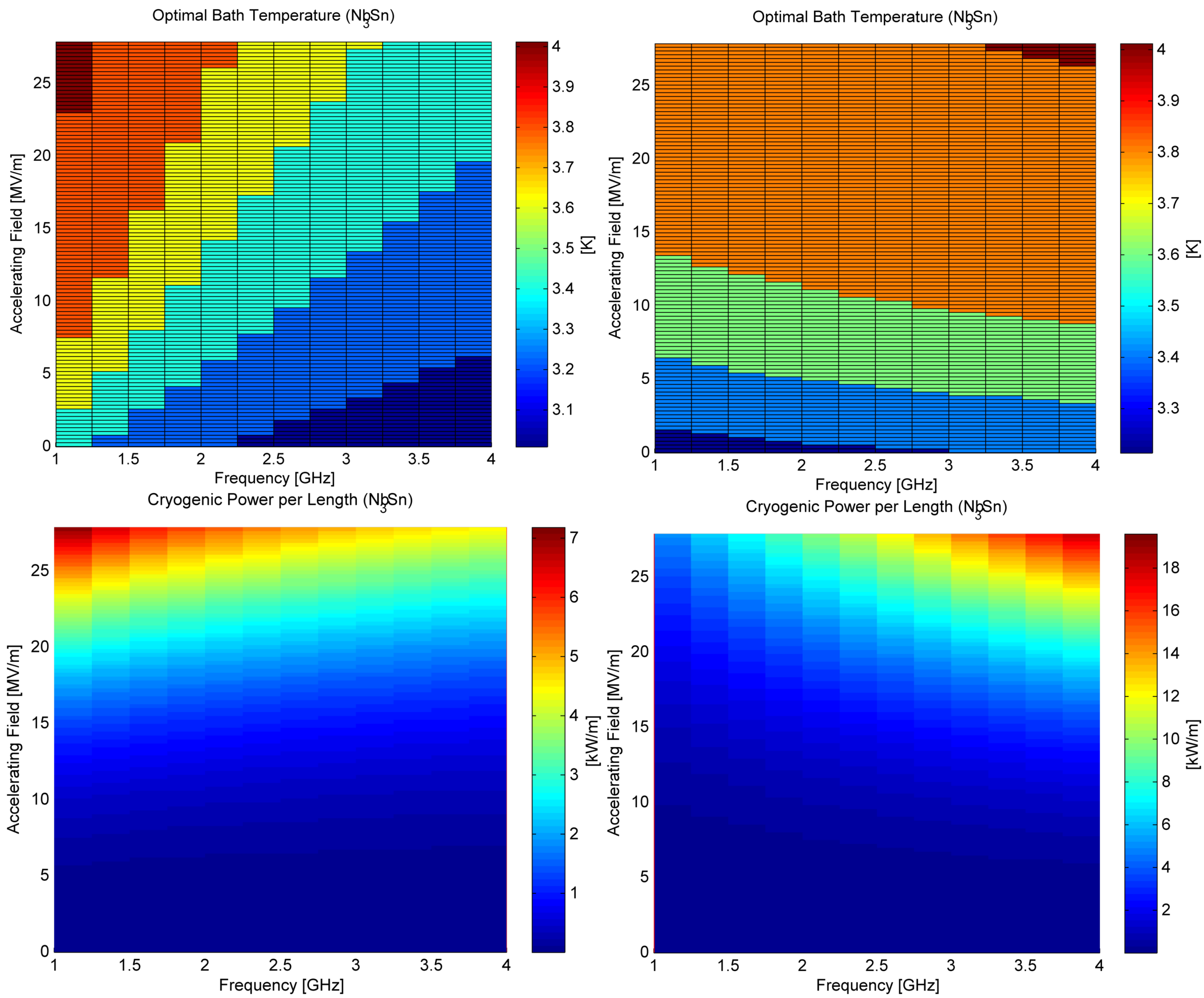

Figure 4: Cryogenic power loss as a function of frequency and accelerating field (bottom) shown at the temperature that minimizes this quantity for each point (top) for Nb_3Sn. The right column displays residual resistance frequency dependence of $R_0 \propto \sqrt{f}$. The left column displays residual resistance frequency dependence of $R_0 \propto f^2$.

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