Microwave suppression of nonlinear surface resistance and extended Q(H) rise in alloyed Nb cavities

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Supported by DOE under grant No. DE-SC0010081

International Workshop on Vortex Matter in Superconductors. VORTEX 2015, El Escorial, Spain, May 10-15, 2015

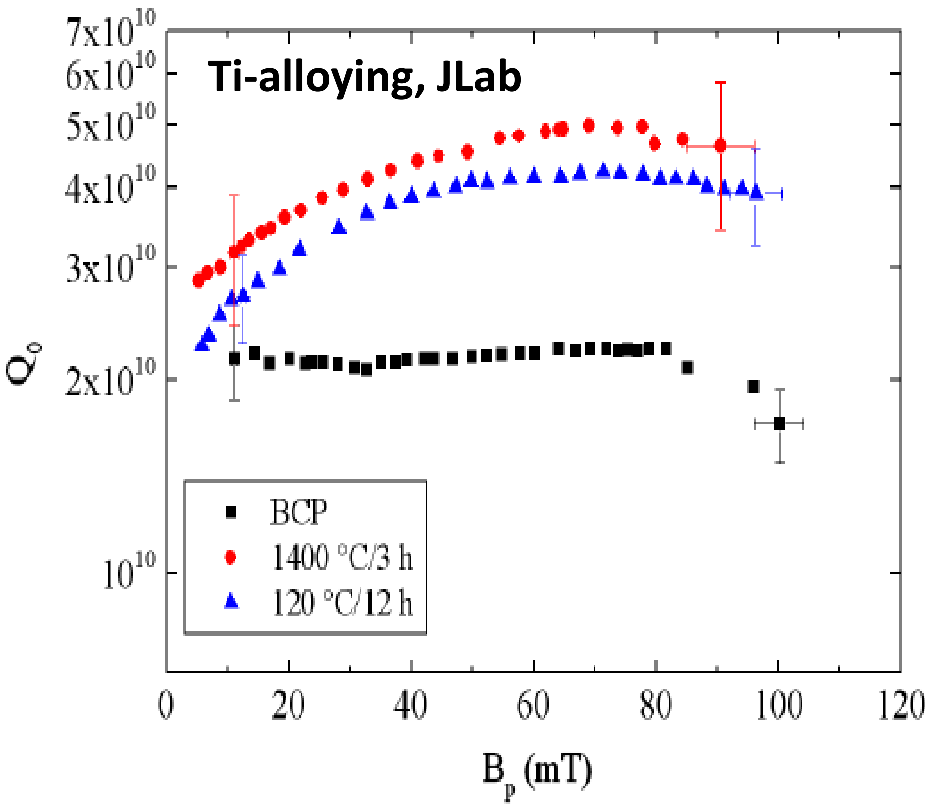
Outline

- What is behind the extended Q(H) rise in Ti or N -doped Nb cavities and what makes them special: special defects, processing, new SRF physics or else?
- Counterintuitive suppression of surface resistance by the rf field extending to 100-120 mT – half of the superheating field of Nb!
- Effect comes from the tell-known broadening of gap peaks in the density of states by strong dc or rf currents in the BCS theory.
- Quick fixes of the low-field Mattis-Bardeen theory are inadequate. Recently $R_s(H)$ has been derived from the dynamic nonequilibrium BCS theory which describes experiment very well. **A.Gurevich, PRL 113, 087001 (2014).**
- Materials conditions: sharp peaks in the density of states and fewer subgap states: apparently N or Ti-allowing does the job.
- How far can it be extended and can negative Q-slopes be fully reversed without reducing the breakdown field? If so can it be extended beyond the superheating field of Nb?
- S-I optimal Nb dirty layer may extend the Q-rise above the superheating field of Nb

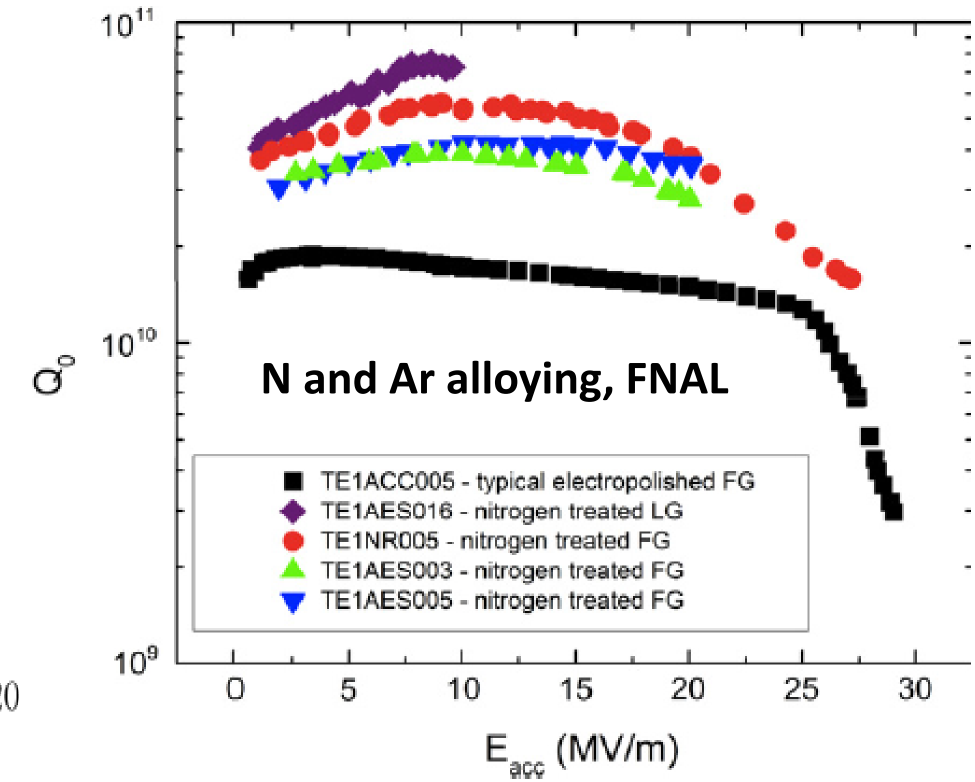
  A.Gurevich, AIP Advances, 5, 087001 (2015).

Extended microwave enhancement of Q(H)

P. Dhakal et al, Phys Rev. ST-AB 16, 042001 (2013)



A. Grasselino et al, SUST 26, 102001 (2013)



- Dirty layer due to diffusion of N or Ti into a few μm thick layer (>> λ = 40 nm) at the surface

- Decrease of $R_s(B)$ up to **B ≈ 0.5B_c: microwave suppression of surface resistance**

Microwave suppression of R_s is not unique to Nb cavities

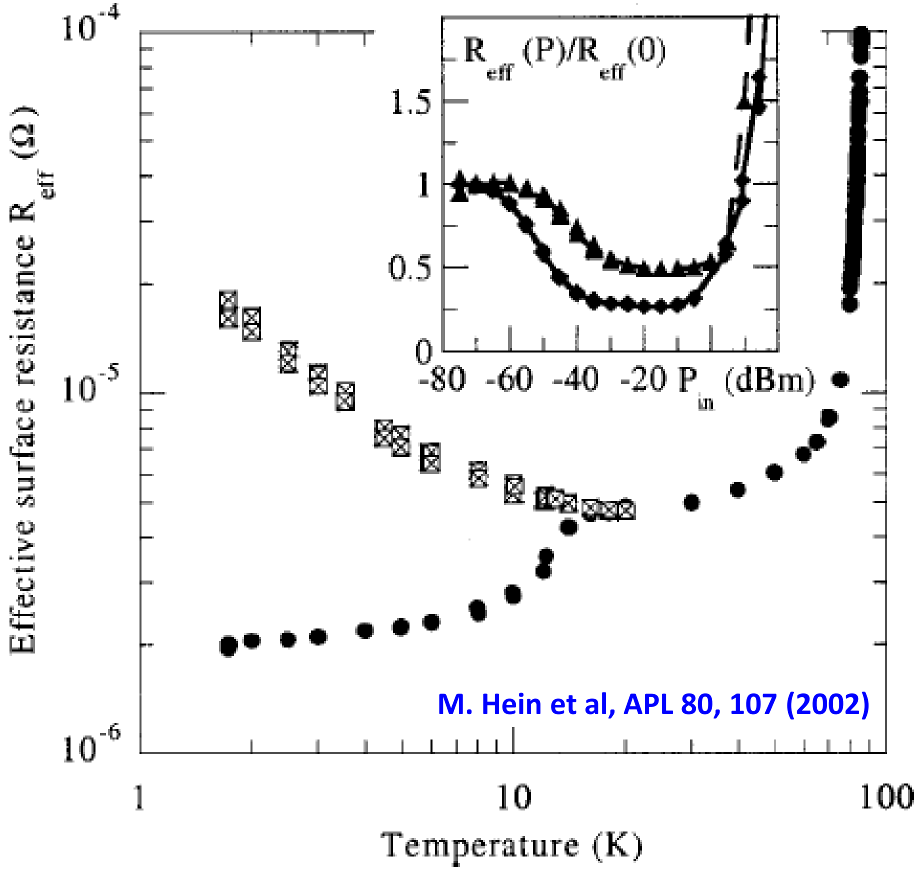
350 nm thick $YBa_2Cu_3O_7$ or Nb striplines on MgO substrates: minimum in R_s at 10 kV/m

but no minimum on sapphire or $LaAlO_3$ …? Surface states or nonlinear dielectric losses in MgO?

Recent observations on 60 nm Al striplines @ 5.5 GHz and 0.2 K
P.J. deVisser et al, PRL 112, 047004 (2014)

Earlier observations on Pb and other LTS

A.B. Pippard, Proc. Roy. Soc. 203, 210 (1950)
M. Spiewak, Phys. Rev. 113, 1479 (1959)
Yu.V. Sharvin and V.F. Gantmakher, JETP 12, 866 (1961)
R.L. Richards, Phys. Rev. 126, 912 (1962)
R.T. Lewis, Phys. Rev. 134, A1 (1964)
J.F. Koch and C.C. Kuo, Phys. Rev. 164, 618 (1967)
S. Sridhar and J.E. Mercerau, PRB 43, 203 (1996)



Suppression of $R_s(H)$ by dc or rf field can be:
1. significant but also very sample dependent
2. masked by high radiation and vortex losses
3. Affected by substrates

What could make $R_s(H)$ to decrease with field?

Mattis-Bardeen formula for the surface resistance in the dirty limit:

$$R_s = \mu_0^2 \sigma_n \lambda^3 \omega^2 \frac{\Delta}{kT} \ln\left(\frac{9kT}{4\hbar\omega}\right) e^{-\Delta/kT}$$

In BCS the product **$\lambda^3\Delta$** is inversely proportional to the superfluid density , **Δ^2** so:

$$R_s \propto \Delta^{-2} \exp(-\Delta/kT)$$

Here Δ decreases with H because currents break Cooper pairs. Thus, according to the conventional wisdom, $R_s(H)$ should increases with H proportionally to the density of thermally-dissociated electrons

Can the microwave suppression of $R_s(H)$ come from a field-dependent m.f.p (not in BCS) or the log. Factor (often hidden in the A factor in $R_s = (A\omega^2/T)\exp(-\Delta/kT)$?

$$\ln\left(\frac{9kT}{4\hbar\omega}\right) \rightarrow \quad ?$$

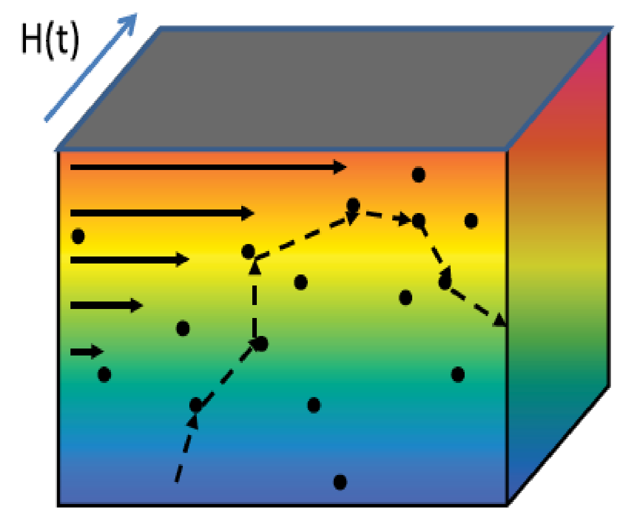
Mattis-Bardeen theory in the dirty limit

In the dirty limit of $l < \xi$, the relation between the current density **J**(**r**) and the electric field **E**(**r**) becomes local:

$$J(r,\omega) = [\sigma_1(\omega) - i\sigma_2(\omega)]E(r,\omega), \qquad R_s(\omega) = \frac{1}{2}\mu_0^2\omega^2\lambda^3\sigma_1(\omega)$$

The reactive conductivity $\sigma_2 = 1/\omega\mu_0\lambda^2$ accounts for the Meissner effect, and the small dissipative conductivity σ_1 results from thermally-dissociated quasiparticles at $h\omega << kT$

$$\sigma_1 = \frac{2\sigma_n}{kT}\int_\Delta^\infty \frac{(\epsilon^2+\Delta^2+\hbar\omega)e^{-\epsilon/kT}d\epsilon}{\sqrt{\epsilon^2-\Delta^2}\sqrt{(\epsilon+\hbar\omega)^2-\Delta^2}}$$



At $h\omega \rightarrow 0$, the integral diverges. At finite $h\omega$, the divergence is cut off giving the logarithmic factor

$$\simeq \ln(9kT/4\hbar\omega)$$

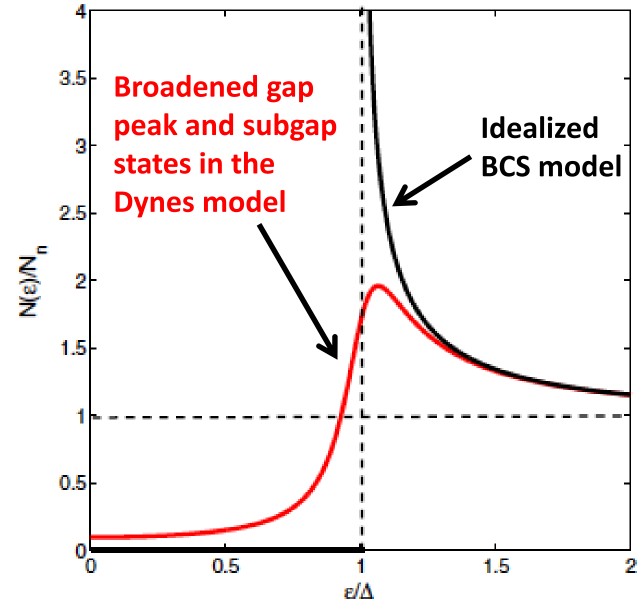
Broadening of gap peaks in N(ε) reduces R_s

At 1-2 GHz and 2K, we have $hf/kT \approx 10^{-2}$, so R_s simplifies to:

$$R_s \simeq 2\sigma_n \mu_0^2 \omega^2 \lambda^3 \frac{\Delta}{kT} \int_\Delta^\infty N(\epsilon) N(\epsilon + \hbar\omega) e^{-\epsilon/kT} d\epsilon$$

Density of states with broadened peaks:

$$N(\epsilon) = \mathrm{Re} \frac{\epsilon - i\gamma}{\sqrt{(\epsilon - i\gamma)^2) - \Delta^2}}$$

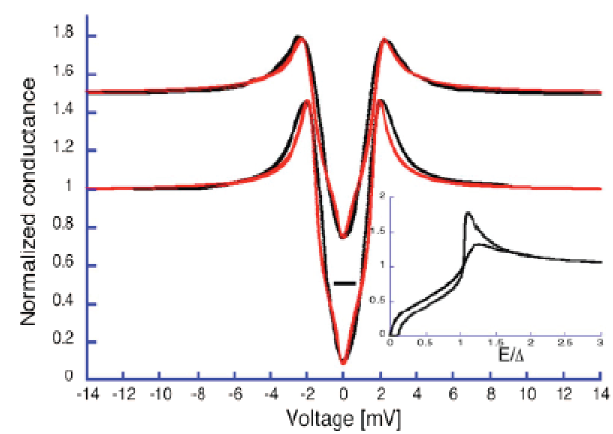


- The log. term in R_s results from the singularity in the BCS density of states ($\gamma = 0$) at $\varepsilon = \Delta$

- No singularity in N(ε) extracted from tunneling exps. which reveal finite N(ε) at $\varepsilon < \Delta$ (subgap states)

$$\ln \frac{9kT}{4\hbar\omega} \to \ln \frac{kT}{\gamma}$$

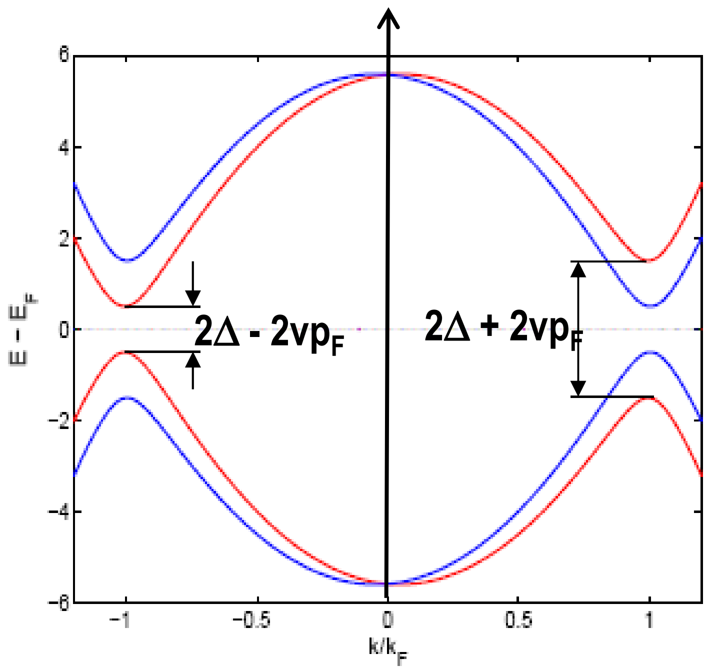
Small broadening of gap peaks in N(ε) can reduce R_s by 4-5 times



T. Proslier et al, APL 92, 212505 (2008)

Current-induced broadening of N(ε) in clean limit

J. Bardeen, Rev. Mod. Phys. 34, 667 (1962)



Rocking "tilted" electron spectrum in the current-carrying rf state, $J(t) = J_0\cos\omega t$

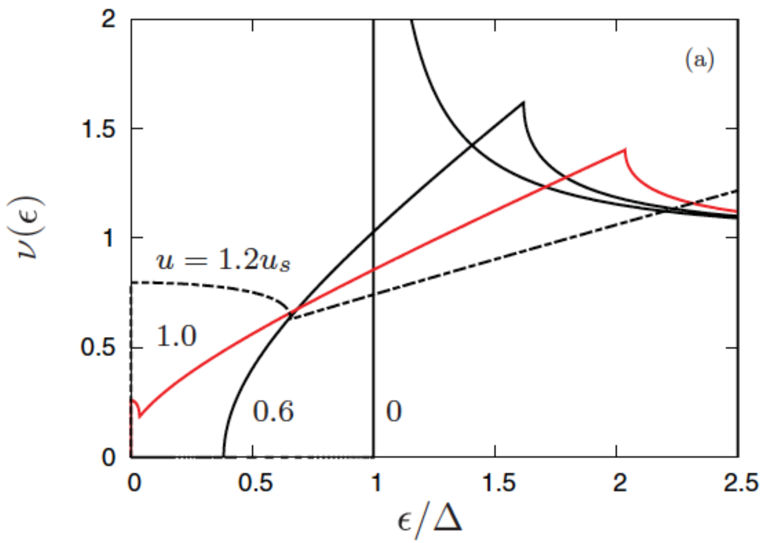
$$E(p) = \pm\sqrt{\Delta^2 + (p^2/2m - E_F)^2} \pm \vec{p}_F\vec{v}_s(t)$$

Superfluid velocity $\mathbf{v_s(t) = J/n_s e}$

At J > 0, the energy gap ε_g is no longer equal to Δ

- **Reduction of the energy gap** $\varepsilon_g = \Delta - p_F|v_s|$ increases the density of thermally-dissociated normal electrons, which supposedly increases

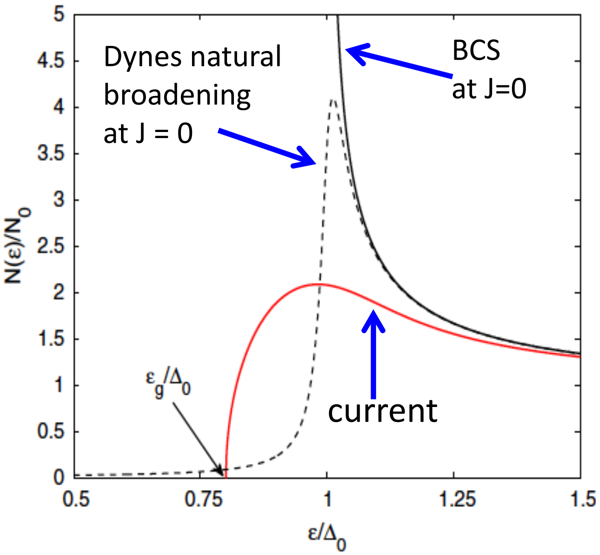
$$R_s \propto \exp[-\epsilon_g(H)/k_B T] \quad ??$$



- Critical pairbreaking velocity $v_c = \Delta/p_F$

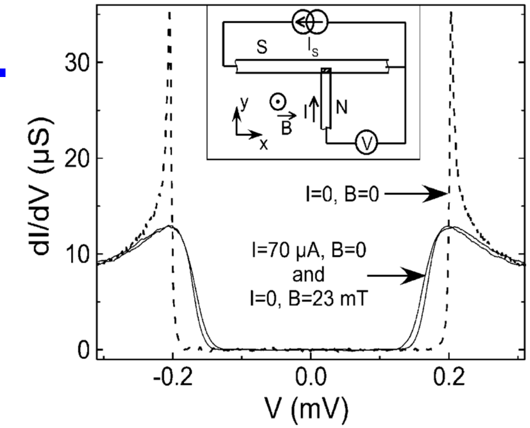
Current-induced broadening of N(ε)

Current-induced broadening in dirty limit



Broadening of the gap peaks in N(ε) by current was calculated some 50 years ago:

K. Maki, Prog. Theor. Phys. 29, 10; 333 (1963);
in Superconductivity, part 2, ed. R.D. Parks, 1967;
P. Fulde, Phys Rev. 137, A783 (1965)



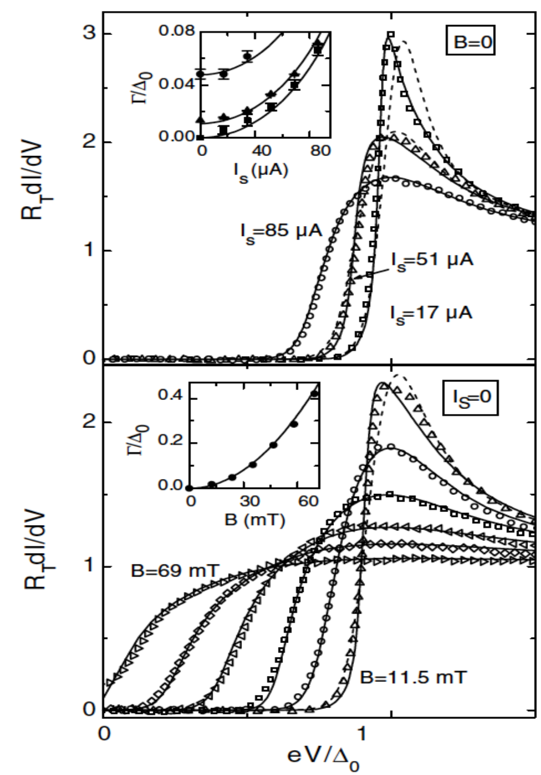
N(ε) oscillates between black (H=0) and red ($H=H_a$) curves

$$\Delta - \epsilon_g(H) \sim (H/H_c)^{4/3}\Delta \qquad \ln\frac{kT}{\gamma} \to \ln\frac{kT}{\Delta - \epsilon_g}$$

If **$H > H_c(\gamma/\Delta)^{3/4}$** the current-induced broadening takes over:

$$R_s \sim \mu_0^2\omega^2\lambda^3\sigma_n e^{-\Delta/kT}\left[\ln\frac{H_c}{H} + C\right]$$

A poor man prediction of logarithmic decrease of $R_s(H)$



Tunnelinng measurements by Anthore, Pothier, and Esteve, on a 40 nm Al wire, PRL 90, 127001 (2003)

Sanity check from experiment

APPLIED PHYSICS LETTERS **104**, 092601 (2014)

Decrease of the surface resistance in superconducting niobium resonator cavities by the microwave field

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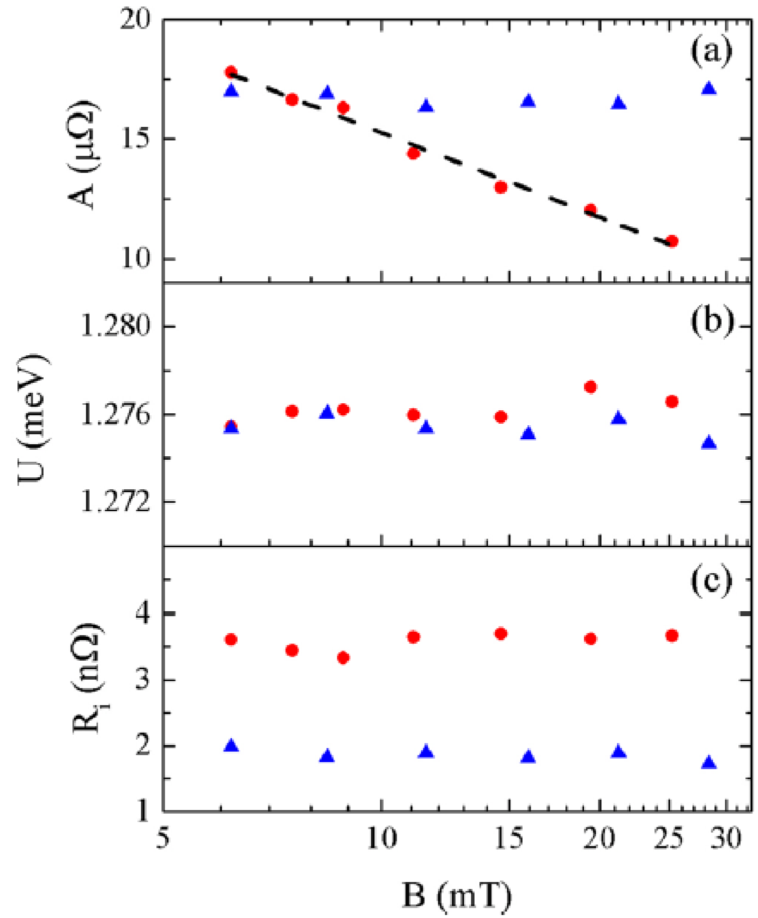
Arrhenius fit of the Ti-doped Jlab data using the generic formula:

$$R_s = A\exp(-U/kT) + R_i$$

where A(H), U(H) and R_i(H) were measured at different fields

Take into account rf heating to provide stable fit in a wide temperature range



Both U and R_i depend weakly on H, but A exhibits a logarithmic decrease, consistent with the above qualitative consideration

Can Mattis-Bardeen help us obtain $R_s(H)$ at high fields?

Linear response nonlocal relation for the current density **J** induced by a **weak** magnetic vector potential **A**

$$\mathbf{J}(\mathbf{r},\omega) = \frac{eN_0 v_F}{2\pi\phi_0}\int \frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{A}(\mathbf{r}')]}{R^4} I(\omega,R,T)e^{-R/\ell}d\mathbf{r}', \qquad \mathbf{R} = \mathbf{r} - \mathbf{r}'$$

- R_s is defined by the exponentially small, out-of-phase dissipative component of **J** which must be calculated with big care.
- I(ω,R,T) is given by a complicated MB integral which is entirely determined by the density of states and coherence factors of a **clean** superconductor.
- The effect of impurities is accounted by the factor **exp(-R/l)**

What if we substitute current-broadened N(ε,J) and the coherence factors into the M-B I(ω,R,T) to calculate R_s(H) at high fields? **B.P. Xiao, C.E. Reece, M.J. Kelly, Physica C490, 26 (2013)**

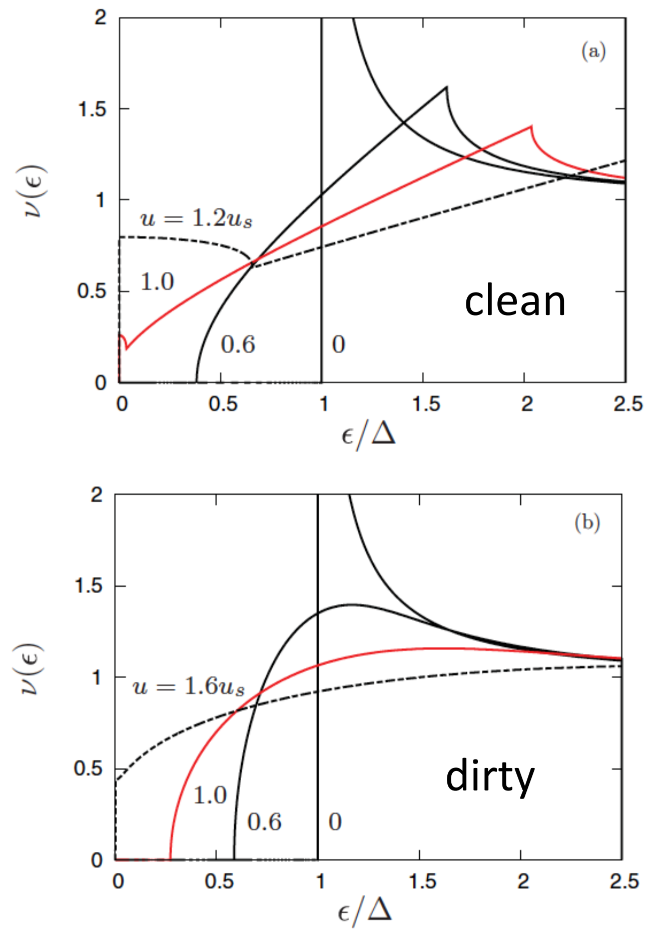
M-B theory cannot be used to calculate the nonlinear response by inserting field-dependent parameters in the linear conductivity σ_1

Why a quick fix of M-B does not work?

- The M-B factorization **I(ω,R,T)×exp(-R/l)** only works at very weak fields below few mT:

$$H < (\hbar\omega/\Delta)^{3/4} H_c$$

- At high fields current pairbreaking becomes intertwined with impurity scattering which affect both N(ε) and the coherent factors



N(ε) in the clean limit is very different from N(ε) in the dirty limit

Problems of calculation of R_s(H)

- Go back to BCS and derive R_s(H) at high fields
- Solve nonlinear dynamic BCS equations with strongly oscillating N(ε) and other parameters
- To account nonlinear current pairbreaking and kinetics of nonequilibrium quasiparticles at strong rf fields
- Much more complicated problem than the low-field M-B linear response

Nonequilibrium theory in the dirty limit

$$\partial_t \hat{\sigma}_z \check{G} + \partial_{t'} \check{G} \hat{\sigma}_z = D \check{\Pi} \cdot (\check{G} \cdot \check{\Pi} \check{G}) - [\hat{\Delta}, \check{G}]$$

A.I. Larkin and Yu.N. Ovchinnikov, In Nonequilibrium Superconductivity, (North Holland, 1986),
W. Belzig, et al Superlatt. & Microstruct. 25, 1251 (1999).
N.B. Kopnin, Theory of Nonequilibrium Superconductivity (2001).

time-dependent Usadel equation for 4x4 matrix Green's functions

$$\check{G} = \begin{pmatrix} \hat{G}^R & \hat{G}^K \\ 0 & \hat{G}^A \end{pmatrix}, \qquad \hat{G}^{R,A} = \begin{pmatrix} G^{R,A} & F^{R,A} \\ -F^{\dagger R,A} & -G^{R,A} \end{pmatrix} \qquad \hat{G}^K = \hat{G}^R \cdot \hat{f} - \hat{f} \cdot \hat{G}^A$$

$\hat{\Pi} = \nabla + i\pi \mathbf{A}(\mathbf{r},t)\hat{\sigma}_z/\phi_0$ A(r,t) is the magnetic vector potential, $D = \ell v_F/3$

nonlinear electromagnetic response for slow variations of A(r,t) at low frequencies hν << kT

$$\mathbf{J}(\mathbf{r},t) = \frac{\sigma_n}{2} \mathrm{Im} \int D(t,t',\mathbf{r}) \mathbf{A}(\mathbf{r},t') dt',$$

$$D(t,t') = \mathrm{Tr} \int \left[e^{i(\epsilon-\epsilon')(t-t')} \hat{G}_z^R(\epsilon',t_0) + e^{i(\epsilon'-\epsilon)(t-t')} \hat{G}_z^A(\epsilon',t_0) \right] \left[\hat{G}_z^R(\epsilon,t_0) - \hat{G}_z^A(\epsilon,t_0) \right] f(\epsilon,t_0) \frac{d\epsilon d\epsilon'}{(2\pi)^2}$$

D(t,t') explicitly depends on both A(r,t) and the impurity mean free path

Results

A.Gurevich, Phys. Rev. Lett 113, 087001 (2014).

Slow temporal and spatial variations of J(x,t):
$\lambda >> \xi$ and $h\omega << kT$.

Nonlinear conductivity as a function of rf amplitude:

$$\sigma_1(H_a) = \frac{2\sigma_n}{\pi}\int_0^{\pi/\omega} dt \int_{\epsilon_g(t)}^{\infty} [f(\epsilon, s) - f(\epsilon+\omega, s)] M d\epsilon,$$

$$M(\epsilon, \omega, s) = \cos v_\epsilon \cos v_{\epsilon+\omega} \cosh(u_\epsilon + u_{\epsilon+\omega}),$$

Temporal oscillations of spectral function M(t):

$$\sinh 2u = [(r + \epsilon\Delta s)^{1/3} - (r - \epsilon\Delta s)^{1/3}]/s,$$
$$r = [\epsilon^2\Delta^2 s^2 + (\epsilon^2 + s^2 - \Delta^2)^3/27]^{1/2},$$
$$\sin v = \left[-\Delta + (\Delta^2 - s^2\sinh^2 2u)^{1/2}\right]/2s\cosh u$$

rf driving parameter:

$$s(t) = e^{-2x/\lambda}\beta(t)\Delta_0,$$
$$\beta(t) = (H/2H_c)^2$$

Quasiparticle gap reduced by current

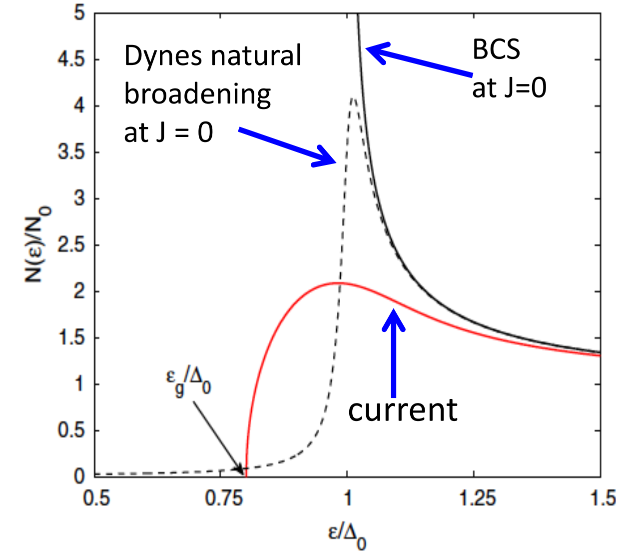
$$\epsilon_g = [\Delta^{2/3} - s^{2/3}]^{3/2}$$

$$\Delta = \Delta_0 - \pi s/4$$

Δ_0 is the SC gap at H =0

Nonequilibrium superconductivity

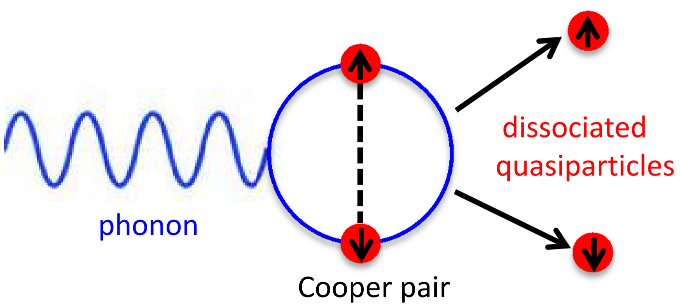
Oscillating rf field causes strongly oscillating density of states N(ε,t) which drives quasi-particles out of equilibrium



$$f(\epsilon) \neq \frac{1}{e^{\epsilon/kT} + 1}$$

Non Fermi-Dirac distribution function

Temporal oscillations of the quasiparticle density under strong rf fields



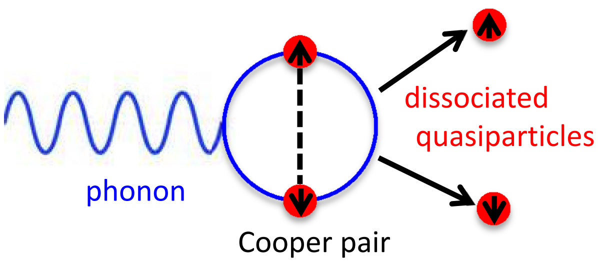
Do quasiparticles have enough time during the rf period (1 ns) to absorb a phonon and recombine at low T?

at hν << Δ the number of quasiparticles can only change due to dissociation and recombination of Cooper pairs caused by electron-phonon collisions

Oscillating or frozen density of quasiparticles?

RF power transfer mechanisms

phonon-assisted dissociation
and recombination of Cooper pairs

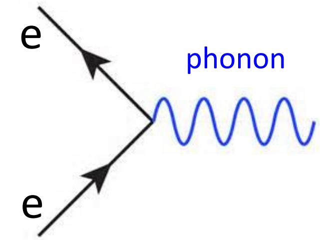


changes the number of quasiparticles

$$\tau_r = \tau_1 (T_c/T)^{1/2} e^{\Delta/T}$$

For Nb @ 2K, $\tau_r \cong$ 400 ns

inelastic scattering of quasiparticles
on thermal phonons



does not change the number
of quasiparticles

$$\tau_s = \tau_2 (T_c/T)^{7/2}$$

For Nb @ 2K, $\tau_s \cong$ 17 ns

Kaplan, Chi, Langenberg, Chang, Jafarey, Scalapino PRB 14, 4854 (1976)

- Both processes transfer the absorbed rf power from quasiparticles to the lattice, but because $(\tau_r, \tau_s) >> \nu^{-1} = 0.5$ ns (2GHz), electrons cannot effectively transfer power during the rf period: quasiparticles are overheated with respect to the lattice:

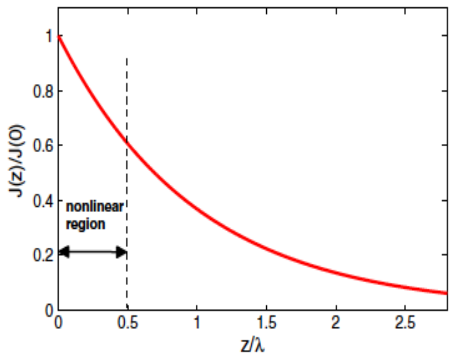
- Temperature of electrons T(H) is higher than temperature of the lattice

- frozen density of quasiparticles does not change during the rf period

Field-dependent surface resistance

Integration of the total rf power over the region of the rf field penetration yields the surface resistance:

$$R_s = \frac{\mu_0^2 \omega^2 \lambda^3}{2\beta_0} \int_0^{\beta_0} \sigma_1(\beta) d\beta, \qquad \beta_0 = \left(\frac{H_a}{2H_c}\right)^2$$



- R_s depends on the electron temperature T(H) which is determined self-consistently by the global power balance:

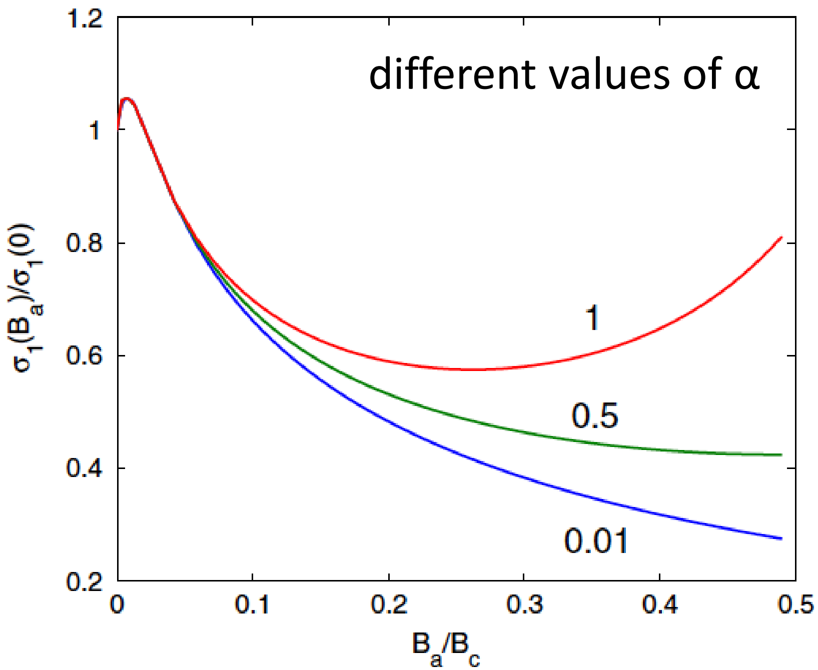
$$T - T_0 = \frac{\alpha T_0}{R_{s0}} \left(\frac{H_a}{H_c}\right)^2 R_s(H_a, T),$$

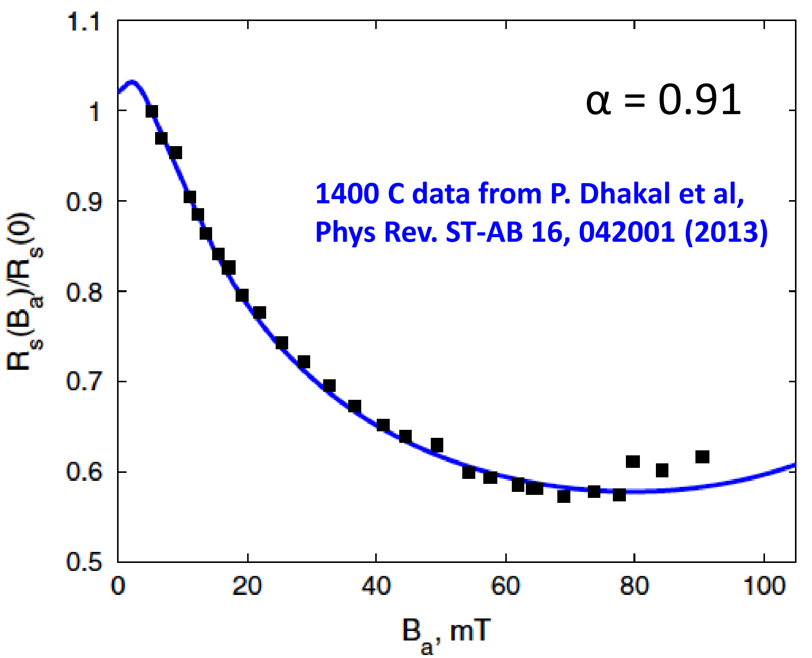
- α is determined by three different mechanisms of the rf power transfer

$$\alpha = \frac{R_{s0} B_c^2}{2\mu_0^2 T_0} \left(\frac{1}{Y} + \frac{d}{\kappa} + \frac{1}{h_K}\right)$$

electron overheating — diffusion of heat — Kapitza

which of the three is a bottleneck?



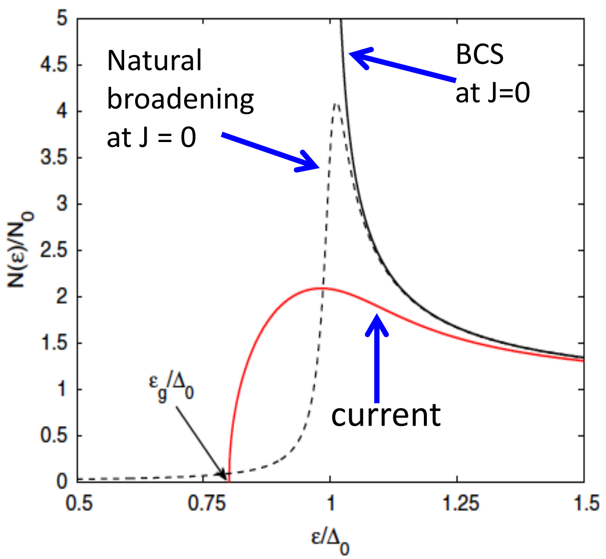


Good agreement with experiment

- Fast numerical code to calculate $R_s(H)$
- 3-4 fold microwave reduction of $\sigma_1(H)$ for weak overheating, $\alpha < 0.1$
- Of $\alpha = 0.91$ used to fit the data indicates significant electron overheating as only 9% comes from the phonon heat transfer.
- For $\alpha = 0.91$, the maximum overheating remains weak even at the breakdown field where $\Delta T \approx 0.17$ K
- Microwave suppression of R_s (H) is not a "new BCS": it can be derived from the dynamic BCS at low temperatures and frequencies hf << kT in the dirty limit

What makes it so dependent on the cavity treatment?

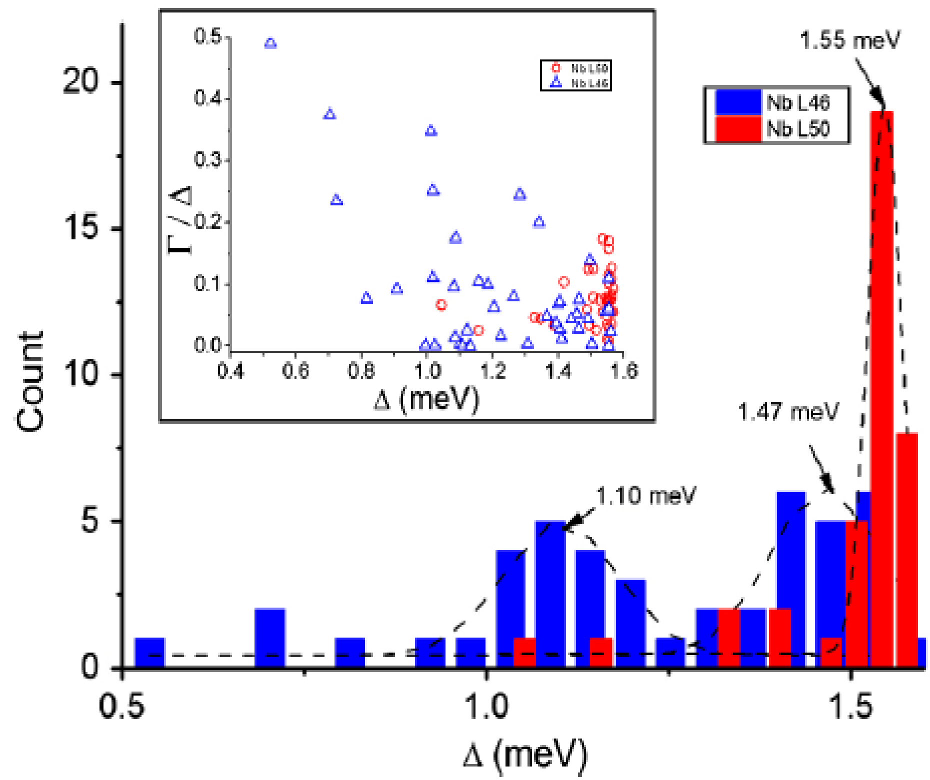
model



Current-induced broadening of N(ε) should be stronger than the "natural" broadening

effect of different treatments on the subgap states

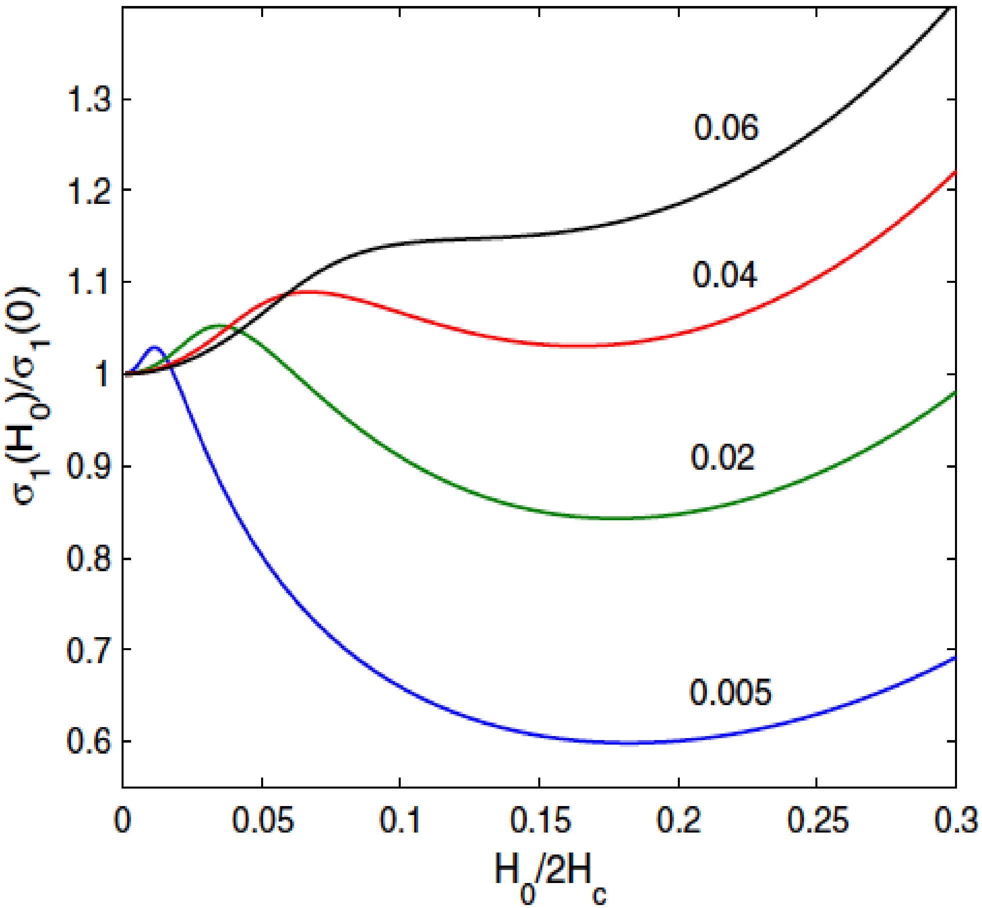
tunneling measurements



tunneling spectroscopy from P. Dhakal et al, Phys Rev. ST-AB 16, 042001 (2013)

Ti-alloyed high-T heat-treated Nb cavities with extended Q(H) rise do exhibit much sharper gap peaks in N(ε) and much less subgap states

Tuning R_s for weak rf field with parallel dc field

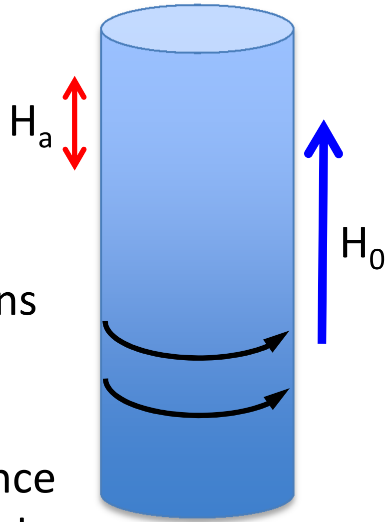


Calculated for different ratios of hν/Δ

AG, PRL 113, 087001 (2014).

Superimposed parallel strong dc and weak rf fields

$$H(t) = H_0 + H_a \sin \omega t$$



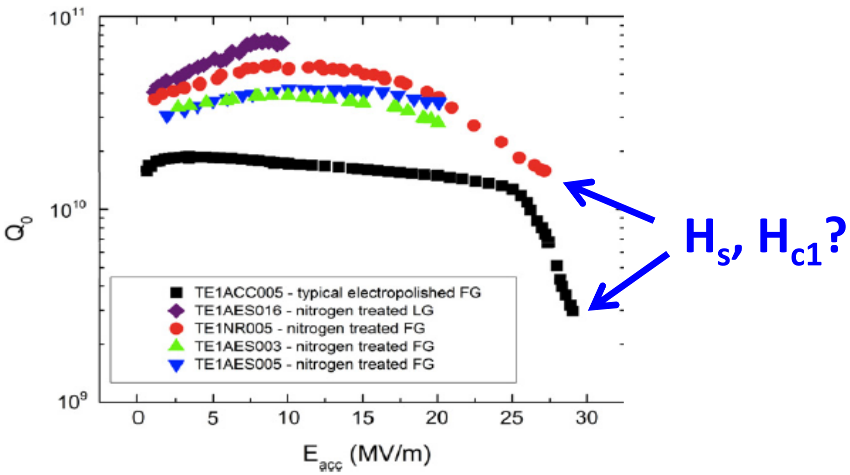
Probes the field suppression of R_s(H) under equilibrium conditions

Frequency dependence of R_s tuned by dc field

Earlier calculations of R_s(H) for a clean limit

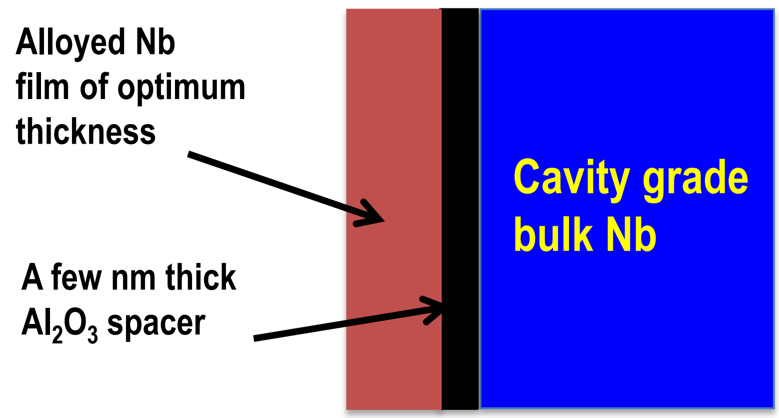
M.P. Garfunkel, Phys. Rev. 173, 516 (1968)

Can the extended Q(H) rise in N-doped Nb cavities be extended above the superheating field of Nb?



A. Grasselino et al, SUST 26, 102001 (2013)

Possible cure by optimized dirty layer?



A few μm thick dirty layer at the surface Increases the GL parameter κ = λ/ξ, and decreases both H_s and H_{c1}:

$$B_s \approx 1.2 B_c, \quad \kappa \approx 1 \quad \text{before}$$

$$B_s \approx 0.745 B_c, \quad \kappa \gg 1 \quad \text{after}$$

$$B_{c1} = \frac{\phi_0}{4\pi\lambda^2}\left(\ln\frac{\lambda}{\xi} + 0.5\right)$$

- To combine the **extended Q(B) rise** and **to reverse the reduction of H_{c1} and H_s**
- Use an optimized Nb bilayer to push the breakdown field above both superheating fields up to 250-300 mT

$$H_m \simeq (H_s^2 + H_{s0}^2)^{1/2}$$

A. Gurevich, AIP Advances, 5, 017112 (2015)

Conclusions

- Extended Q(H) rise occurs as N or Ti doping results in a dirty layer with non-suppressed T_c and Δ at the cavity surface. Penalty of reduced H_{c1} and H_s
- The microwave suppression of $R_s(H)$ is not unique to Nb cavities: it has been observed on many other materials.
- The effect results from the well-know current-induced broadening of the gap peaks in the density of states in the BCS theory at $h\nu << kT$.
- A theory of $R_s(H)$ was proposed based on the solution of time-dependent Usadel equations of nonequilibrium BCS superconductivity in the dirty limit. Dependence of $R_s(H)$ on the mean free path at high fields can be different from R_s at low fields
- Good agreement with experiments on Ti and N-alloyed Nb cavities
- Extended Q(H) rise is facilitated by sharper peaks in the density of states $N(\varepsilon)$ and fewer subgap states. Apparently it is what Ti or N-doping does.
- Possibility to extend the extended Q(H) rise beyond the superheating field of Nb by depositing optimized N-doped Nb multilayers