AG and G. Ciovati, cond-mat/0710-1231

Dynamics of vortex penetration, jumpwise instabilities and high-field surface resistance

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based on the work with Gigi Ciovati, JLab

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Outline

- Best cavities (KEK, JLab, Cornell) have already reached the breakdown fields H_p close to H_c, for which penetration of vortices is inevitable. How many vortices can be tolerated?

- Dynamics, timescales and dissipation due to penetration and exit of single vortices. Supersound vortex penetration and jumpwise instabilities in strong rf fields.

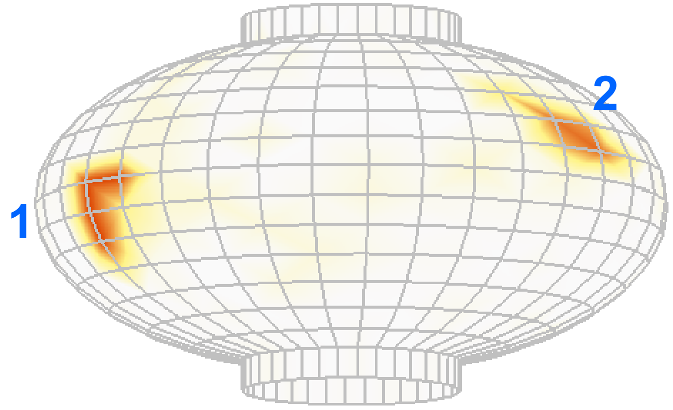
- Dissipation due to trapped vortices: contribution to the residual resistance and vortex hotspots.

- Are hotspots in Nb cavities due to trapped/penetrating vortices or surface defects?

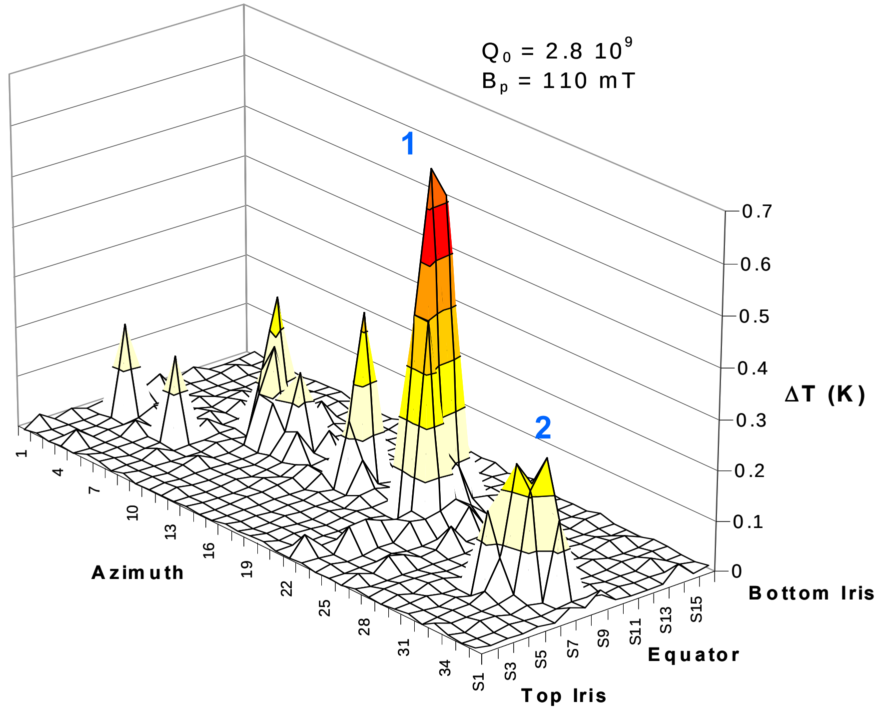
<u>Possible ways of reducing vortex dissipation:</u>

- Annealing vortex hotspots by rf cycling
- Removing vortex hotspots by thermal gradients
- Delaying vortex penetration by thin film multilayer coating

Thermal Maps (Cornell and J-Lab)



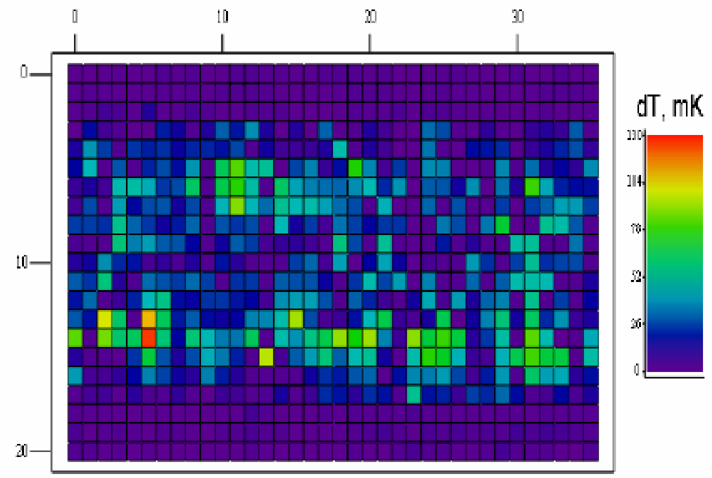
G. Ciovati - JLab



Thermometer array reveals hotspots, which ignite cavity breakdown

What are behind these hotspots?

1. **Locally enhanced BCS resistance**
2. **Penetrating single vortices**
3. **Trapped vortices**

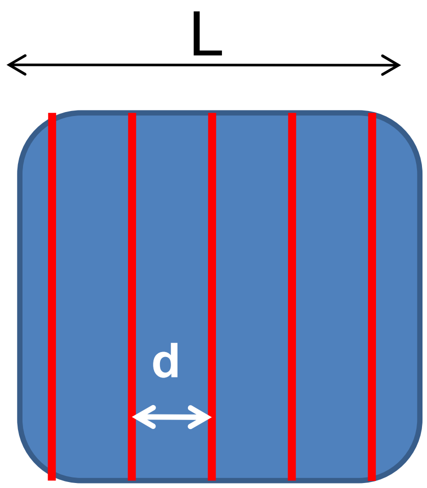


Vortex hotspots can be ameliorated: cheap ways of boosting the cavity performance

Problem of vortex dissipation

- Energy dissipated by a vortex segment of length L moving in and out of the hot spot: $q = 2\phi_0 H_a L$ where ϕ_0 = 2×10^{-15} Wb is the magnetic flux quantum

- How many vortices spaced by d can produce dissipation exceeding the low-field BCS limit?



$$L^2 R_s H_a^2 / 2 \cong 2 f \phi_0 H_a L^2 / d \qquad \Rightarrow \qquad d \cong \frac{4\phi_0 \mu_0 f}{R_s B_a}$$

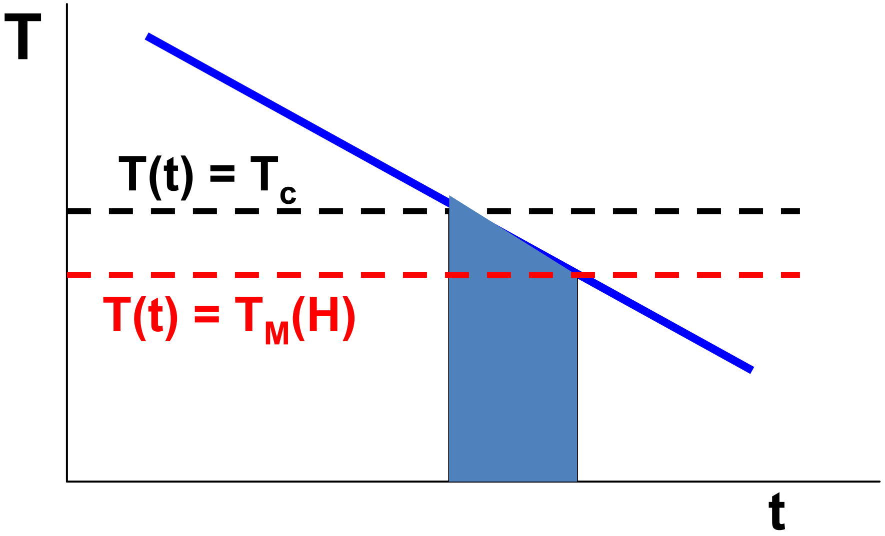
- For R_s = 20 nΩ, B_a = 100 mT and f = 2GHz, we obtain

$$\boxed{d \approx 1\text{cm}}$$

- Even few penetrating vortices can produce localized dissipation comparable to that of the BSC surface resistance

- Single vortices trapped during the cavity cooldown can produce sparse hotspots on the Meissner background

How do vortices get in a superconductor?



1. Field cooling a in the Earth field:

- Crossing the Meissner temperature line $T_M(H_i)$:

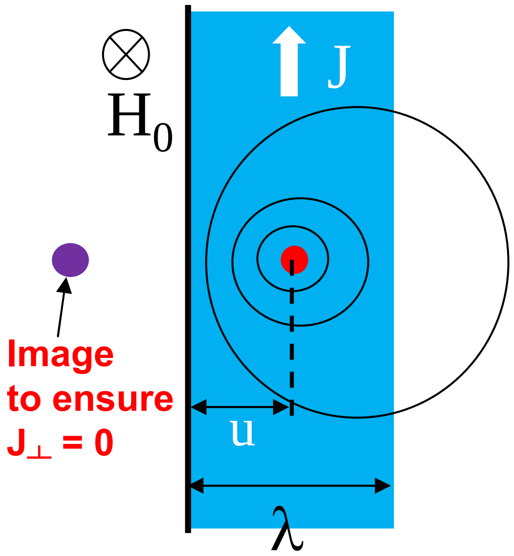
$$H_i = H_{c1}(1 - T_M/T_c)$$

- Most of the initial vortex density B_i/ϕ_0 is expelled during the cavity cooldown, as $H_i \ll H_{c1}$, but some of them get trapped by crystalline defects.

2. Vortices generated by thermoelectric currents $J = -\alpha_T \nabla T \sim 10^2 - 10^3$ A/cm^2 during crossing the Meissner line. Works even if the Earth field is completely shielded.

3. Direct vortex penetration through the regions where the magnetic surface barrier is locally suppressed by defects

Surface barrier for vortices in a magnetic field

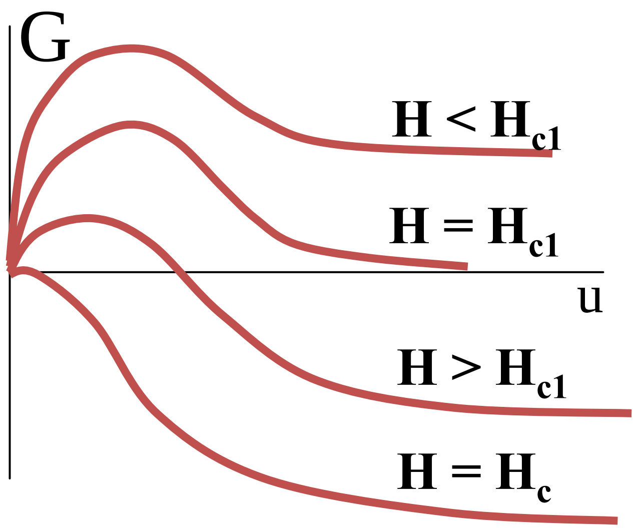


- Meissner currents push the vortex in the bulk
- Attraction to the antivortex image pushes the vortex out

Thermodynamic potential G(u) as a function of the position u:

$$G(b) = \phi_0[H_0 e^{-u/\lambda} - 0.5H_v(2u) + H_{c1} - H_0]$$

Meissner **Image**



- Vortices have to overcome the surface barrier even at $H > H_{c1}$ (Bean & Livingston, 1964)
- Surface barrier disappears at the overheating field $H \sim H_c > H_{c1}$ as the surface J reaches the depairing current density $J_d \sim H_c/\lambda$

Rf dissipation due to vortex penetration through the oscillating surface barrier in rf field

Penetration of vortices through the oscillating surface barrier for κ >> 1

$$\eta \dot{u} = \frac{\phi_0 H_0}{\lambda} e^{-u/\lambda} \sin \omega t - \frac{\phi_0^2}{2\pi\mu_0\lambda^3} K_1\left(\frac{2\sqrt{u^2+\xi^2}}{\lambda}\right)$$

$$\eta = \frac{\phi_0 B_{c2}}{\rho_n}$$

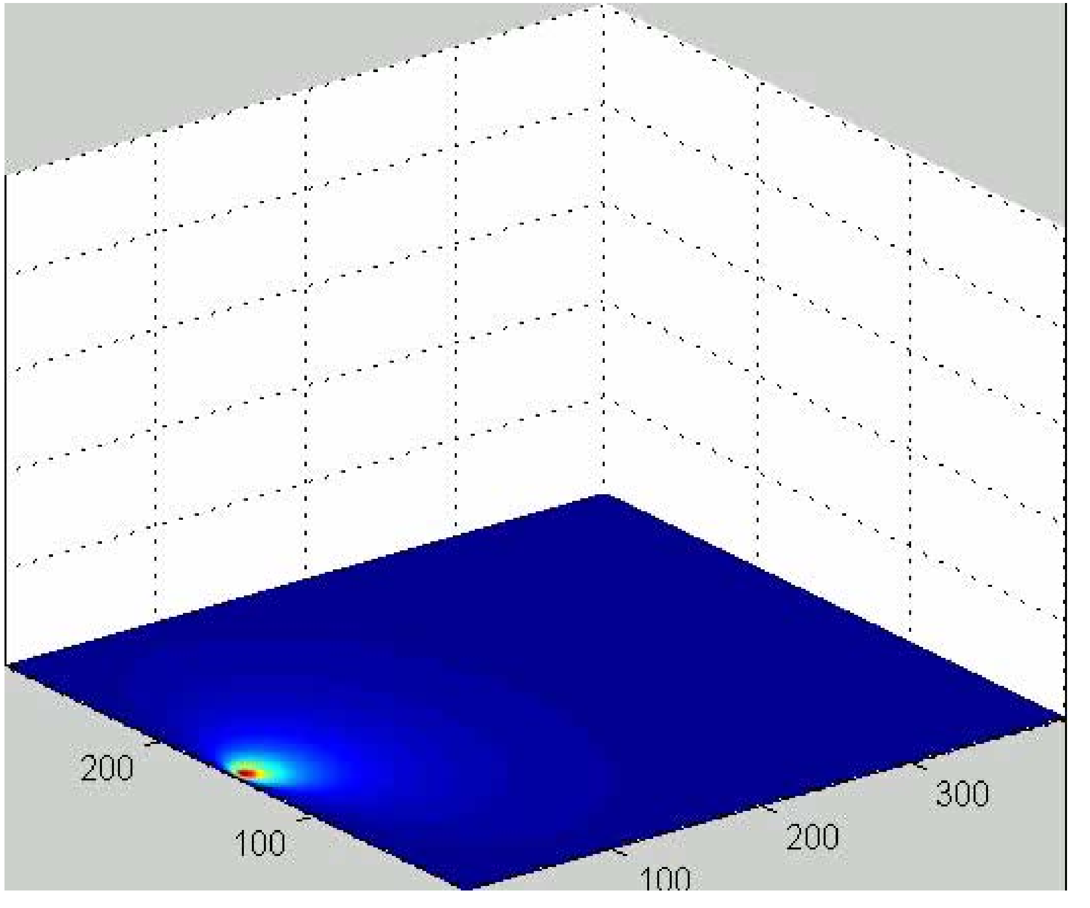
- Onset of vortex penetration
  $B_v \approx \phi_0/4\pi\lambda\xi = 0.71B_c$

- Vortex time constant for Nb_3Sn

  $\tau = \mu_0\lambda^2 B_{c2}/B_v\rho_n \approx 1.6\times10^{-12}$ s << 1μs

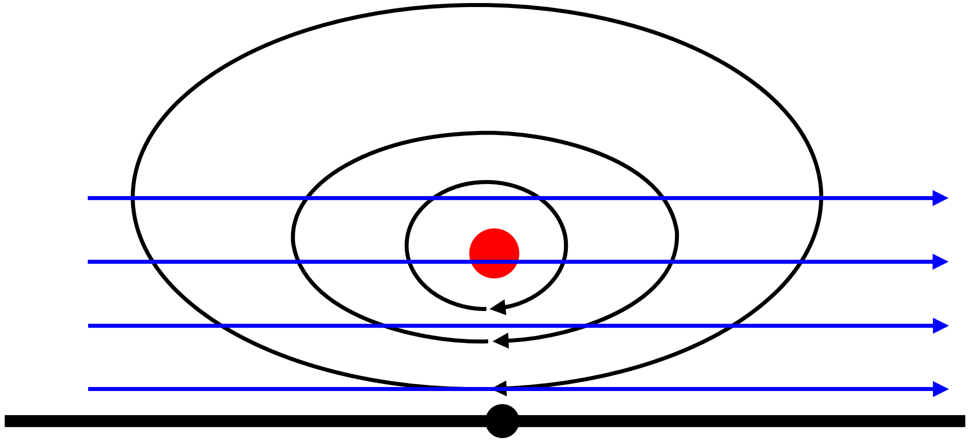
$$v_m \approx \frac{\rho_n \xi}{2\mu_0\lambda^2}$$

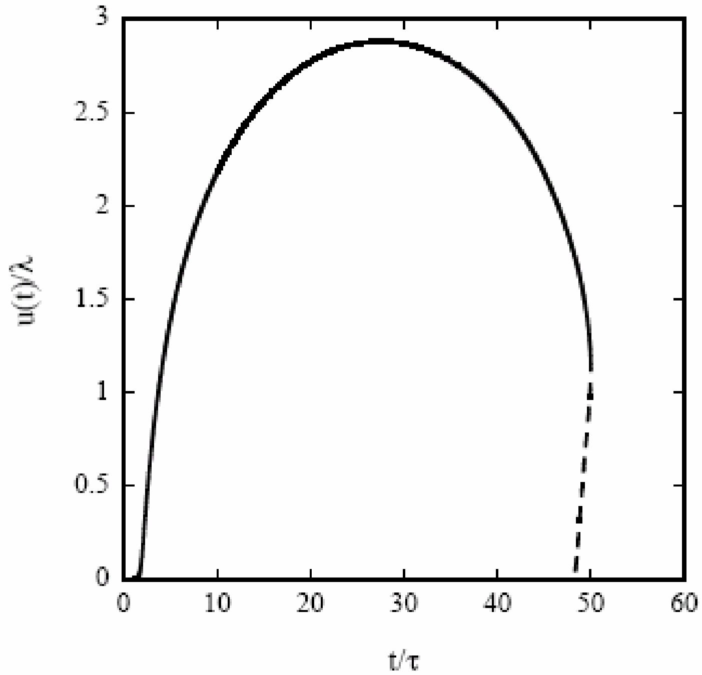
max. velocity



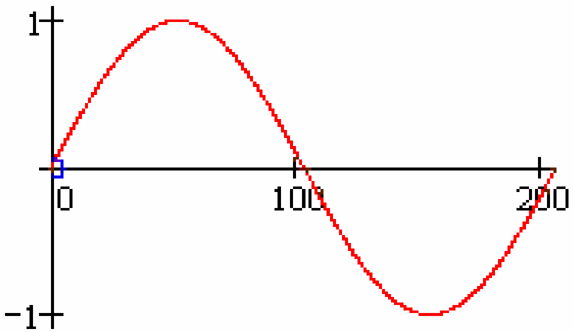
- For Nb: $v_m \sim 10$ km/s is greater than the speed of sound !

Vortex-antivortex penetration and annihilation



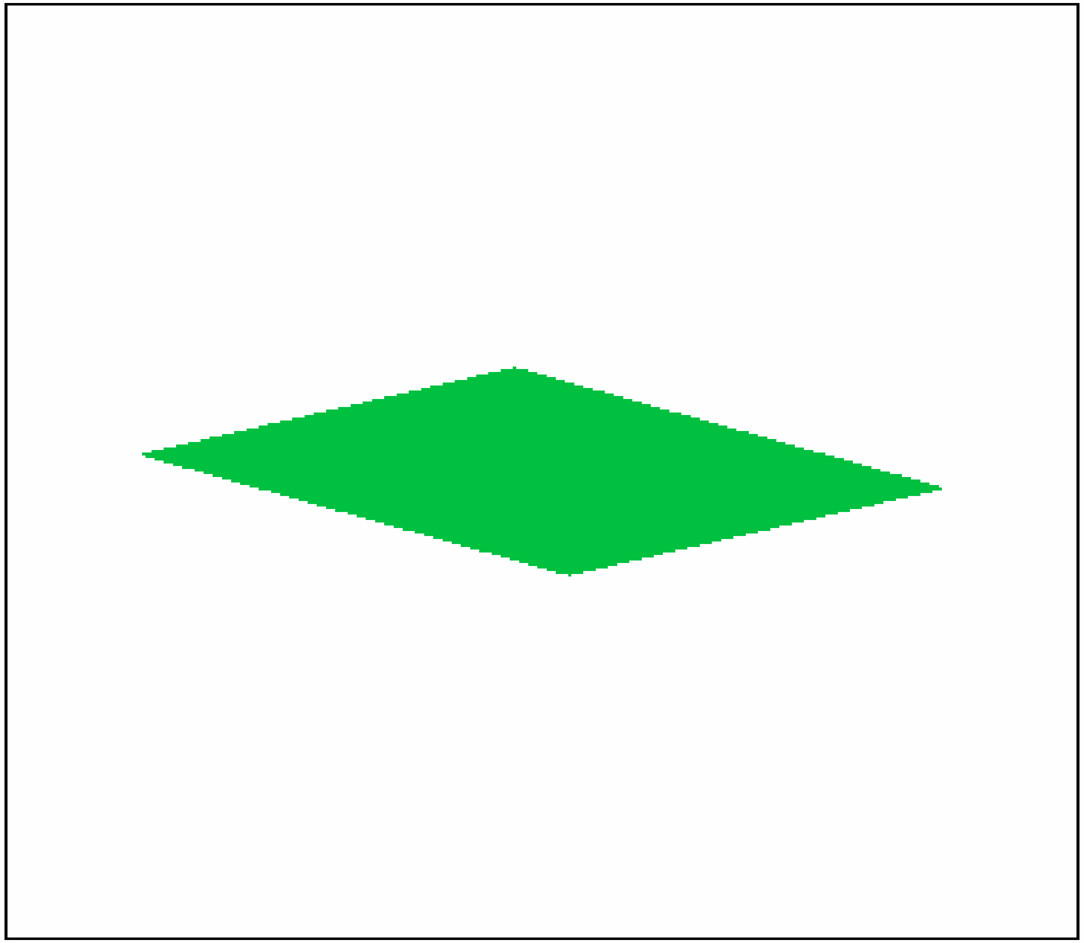


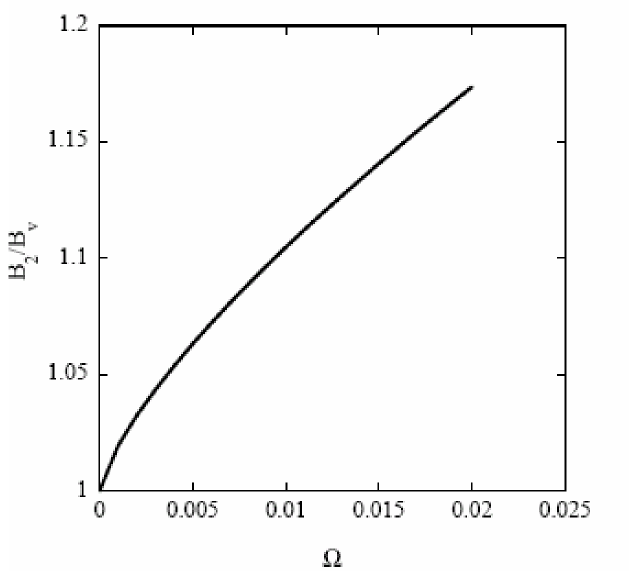
- Vortex penetrates when the instantaneous Meissner J(t) at the surface reaches $J_d = H_v/\lambda$
- Penetration of vortices suppresses the pairbreaking instability
- As the vortex moves out, the surface J(t) reaches – J_d before the vortex exits.
- Penetration of an antivortex
- Annihilation of vortex and antivortex



Dynamics of V-A annihilation

- Annihilation occurs when vortex moves out and antivortex moves in.
- Then a new antivortex penetrates and everything repeats with A $\rightarrow$ V



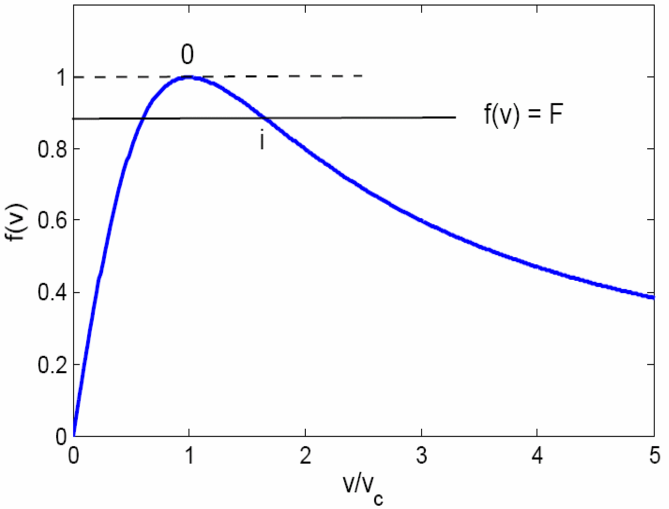


Second vortex penetration field $H_v < H < H_2$

Instability at high vortex velocities

- Modified viscous drag force for high velocities
  Larkin and Ovchinnikov, 1970

$$f = \frac{\eta v}{1+(v/v_c)^2}, \qquad v_c \approx 0.6\left(\frac{l v_F}{\tau_e}\right)^{1/2}$$



- l is the mean free path, τ_e is the energy relaxation time . For T = 2K , ℓ = 10 nm, and v_F = 300 km/s, the critical velocity $v_c \sim$ 1 km/s. The LO instability has been observed on many superconductors, including Nb.

- Vortex core size decreases as v increases for the LO diffusion mechanism, but increases for the overheating mechanism

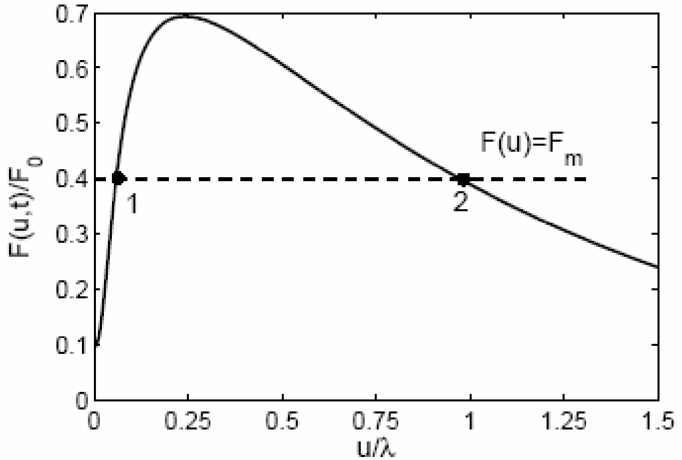
- Maximum viscous force $F_m = \eta v_c/2$. Jump-wise vortex penetration and exit

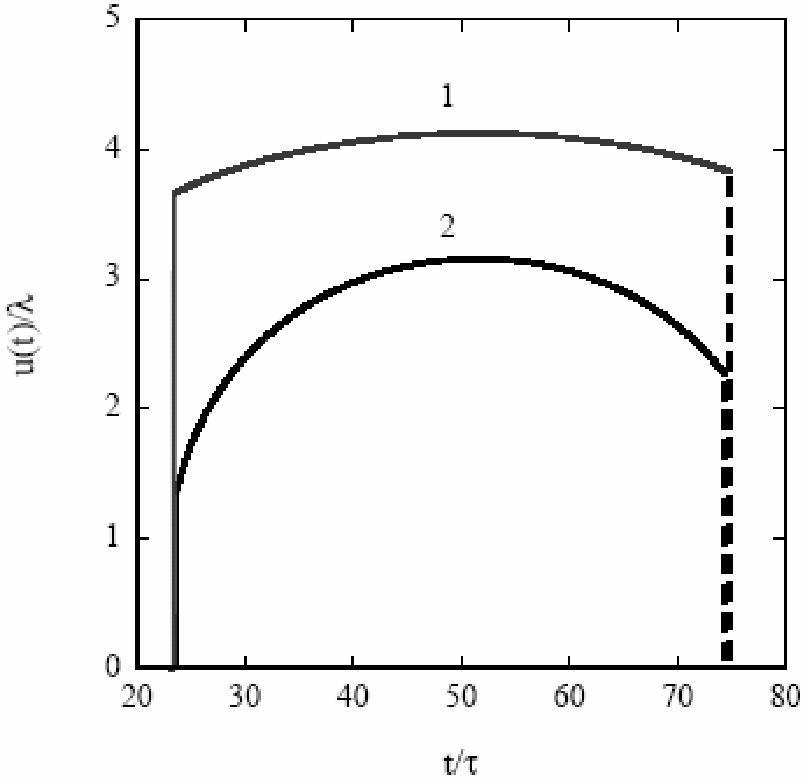
$$\frac{\eta \dot{u}}{1+\dot{u}^2/v_c^2} = \frac{\phi_0 H_0}{\lambda} e^{-u/\lambda} \sin \omega t - \frac{\phi_0^2}{2\pi\mu_0\lambda^3} K_1\left(\frac{2\sqrt{u^2+\xi^2}}{\lambda}\right)$$

Jumpwise vortex penetration

$$\dot{u} = \frac{v_c F_m}{F(u,t)}\left[1 \pm \sqrt{1 - \frac{F^2(u,t)}{F_m^2}}\right]$$

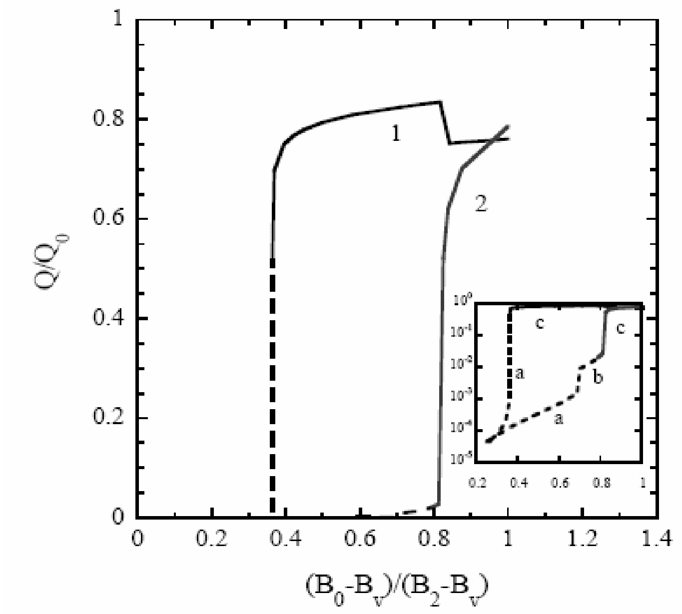
Force profile

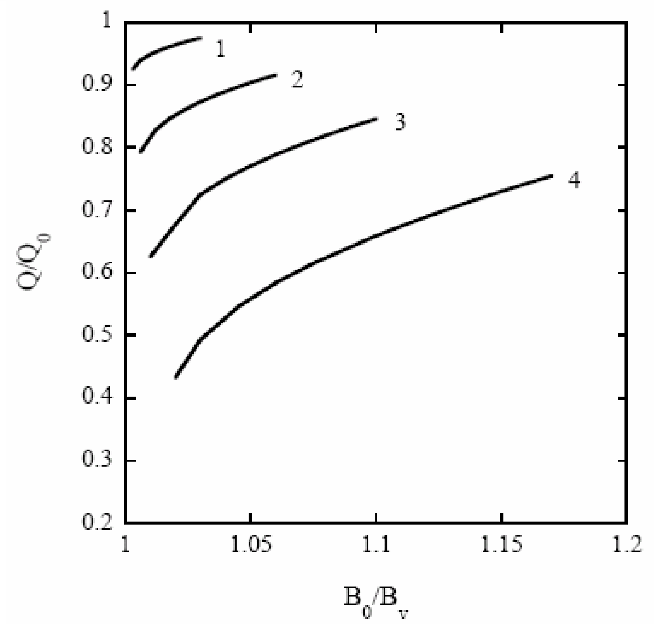




- Vortex rapidly accelerates and then jumps by the distance ~ λ.
- Supersonic jump velocities ~ λΔ/ħ
- Radiation of sound and electromagnetic spikes
- Jumpwise antivortex penetration and annihilation

Dissipated power





- **Dissipated power**

$$Q = \frac{\omega}{2\pi}\oint \eta \dot{u}^2 dt$$

- **Maximum power for H = H_2**

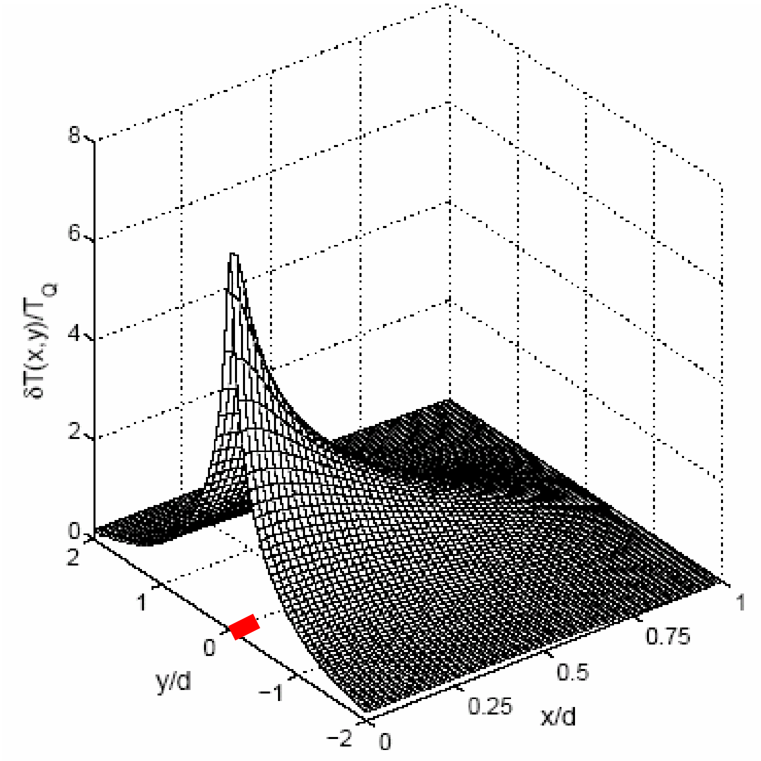
$$Q_0 = \frac{2\omega\phi_0 B_v}{\pi\mu_0}$$

- **No LO instability and H ≈ H_v:**

$$Q = Q_0\left(\frac{H_0^2 - H_v^2}{H_v^2}\right)^{\Omega}, \qquad \Omega \cong 4\omega\tau\kappa^{-2/3}$$

Temperature around the oscillating vortex

Hotspot produced by oscillating vortex:



- Integral equation for the temperature of oscillating vortex
- Steady-state T(x,y) averaged over rf oscillations:

$$\delta T(\vec{r}) = \frac{Q}{2\pi k} \ln \frac{\cosh(\pi y/2d) + \cos(\pi x/2d)}{\cosh(\pi y/2d) - \cos(\pi x/2d)}, \quad |r| >> r_0$$

$$\delta T_m = \frac{Q}{\pi k} \ln \frac{4d}{\pi r_0}, \qquad |\vec{r}| < r_0$$

- For Nb: with k = 10 W/mK, f = 2 GHz, $Q \approx 4B_v\phi_o/\mu_o$ d = 3mm, r_0 = 100 nm, B_v = 150 mT, we get

$\delta T_m \approx 0.6$K

- For Nb_3Sn, k = 10^{-2} W/mK, so vortex hotspots are much stronger, $\mathbf{\delta T_m \approx T_c}$

Breakdown field with hotspots

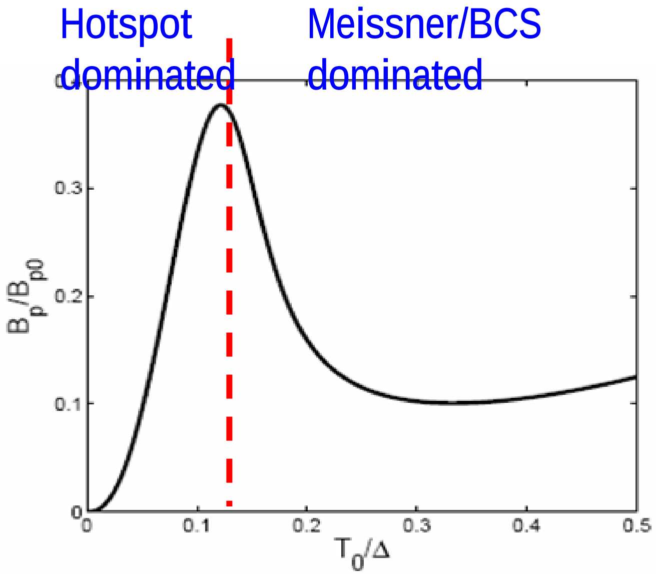


FIG. 16: The temperature dependence of $B_p(T_0)$ calculated from Eq. (81) for $R_{bcs}(T_0) = R_{n0}(\Delta/T_0)\exp(-\Delta/T_0)$, $R_i/eR_{n0} = 2 \times 10^{-5}$, $\alpha_\theta = \alpha' T_0^s$, $s = 3$, and $B_{p0}^2 = 2\mu_0^2\alpha' T_0^{s+1}/R_{n0}$.

Hotspots ignite thermal runaway at lower temperatures

Thermal feedback model with overlapping hotspots:

$$\frac{H^2}{2}\left[R_0 \exp\left[\frac{(T-T_0)\Delta}{k_B T_0^2}\right] + R_i\right] = (T-T_0)\alpha_\theta$$

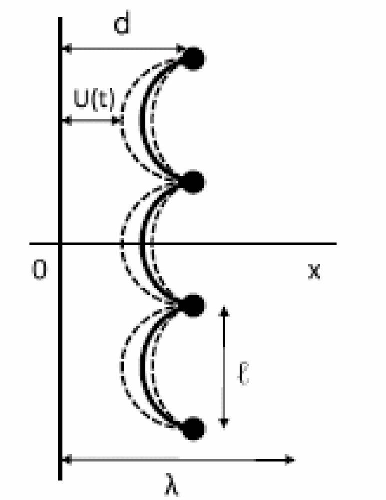
$R_0(T_0)$ is the BCS resistance, R_i is due to hotspots, and $\alpha_\theta = \kappa\alpha_K =/(d\alpha_K + \kappa)$

Equation for the breakdown field

$$\frac{R_0 H_p^2 e\Delta}{2k_B\alpha_\theta T_0^2}\exp\left(\frac{R_i H_p^2 e\Delta}{2k_B\alpha_\theta T_0^2}\right) = 1$$

$$H_{p0} = \left(\frac{2k_B\alpha_\theta T_0^2}{R_0 e\Delta}\right)^{1/2}$$

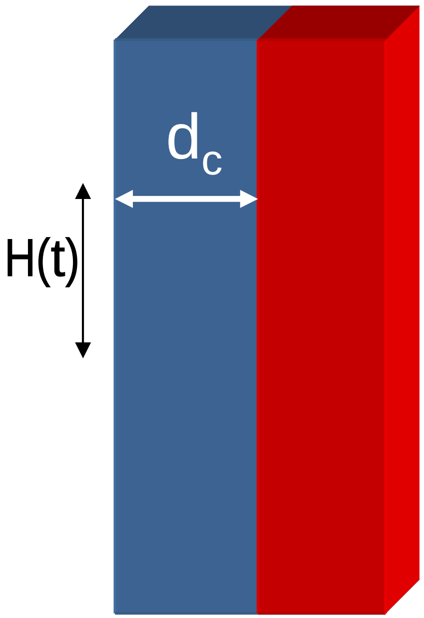
Effect of pinning



- Vortex trapped at the surface by defects spaced by ℓ

- Vibrations of the pinned vortex under the rf field

$$\eta\dot{u} = \varepsilon u'' + \frac{\phi_0 B_0}{\mu_0\lambda} e^{-u/\lambda} \sin\omega t - \frac{\phi_0^2}{2\pi\mu_0\lambda^3} K_1\left(\frac{2u}{\lambda}\right) + \sum_n f_p(u - n\ell)$$



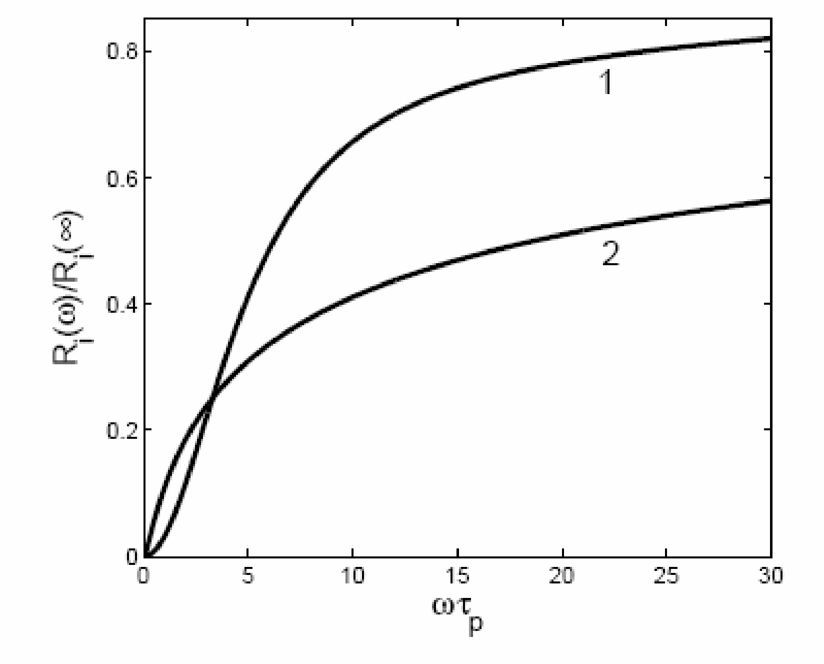
- Vortex-free layer of thickness $d_c \sim \lambda$ at the surface

$$\frac{\phi_0^2}{2\pi\mu_0\lambda^3} K_1\left(\frac{2d_c}{\lambda}\right) = \frac{f_p}{\ell}, \qquad \varepsilon \cong \frac{\phi_0^2}{4\pi\mu_0\lambda^2} \ln\frac{\lambda}{\xi}$$

- Pinning time constant: $\tau_p = \dfrac{\eta\ell^2}{2\varepsilon} = \dfrac{\tau\kappa}{\ln\kappa}\left(\dfrac{\ell}{\lambda}\right)^2$

$\tau_p \sim 10^{-8}$ s for $\ell \sim 4\ \mu$m

Surface resistance due to pinned vortices



- Full frequency dependence:

$$R_i = \frac{\phi_0^2 \langle e^{-2d/\lambda} \rangle}{\lambda^2 \eta a}\left[1 - \frac{\sinh\sqrt{\omega\tau_p} + \sin\sqrt{\omega\tau_p}}{\sqrt{\omega\tau_p}\,(\cosh\sqrt{\omega\tau_p} + \cos\sqrt{\omega\tau_p})}\right]$$

- Low-frequency surface resistance:

$$R_i = \frac{(\omega\tau_p)^2}{30} R_{i0}, \qquad R_{i0} \approx \frac{\rho_n B_i}{\lambda B_{c2}}$$

$a = (\phi_0/B_i)^{1/2}$

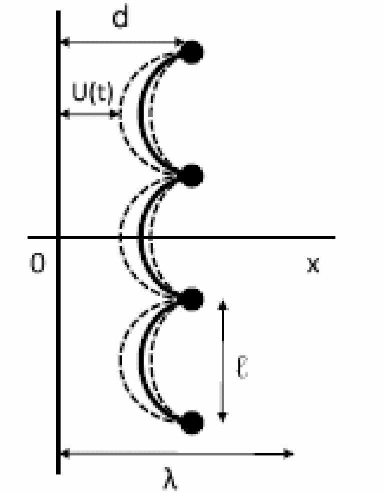
For Nb, R_{oi} = 2.5 μΩ for B_i = 0.4 Oe. Pinning strongly reduces R_i

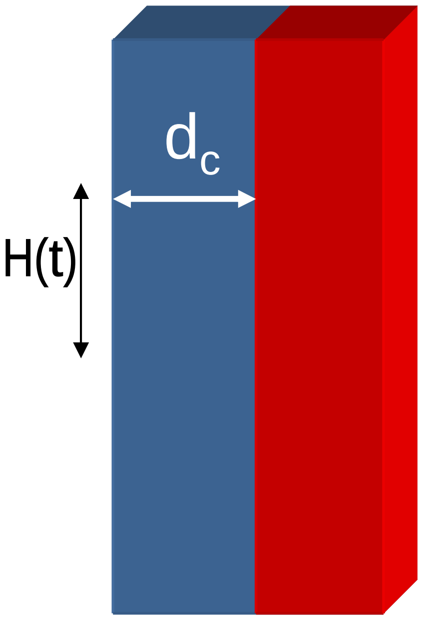
- Vortex hotspots provide a temperature independent residual resistance extremely sensitive to the position of the pinned vortex (see also M. Rabinowitz, JAP 42, 88 (1971)).

- Quadratic frequency dependence at $\omega\tau_p \ll 1$: same as the BCS resistance

- Averaging over distribution of pinned segments produces $R_i \propto \omega^\alpha$ with $\alpha \sim$ **0.5-0.8** for intermediate frequencies

Vortex free layer in the rf field





- Vortex-free layer of thickness d_c in rf field for weak pinning:

$$\frac{\phi_0^2}{4\pi\mu_0\lambda^3}\sqrt{\frac{\pi\lambda}{d_c}}\exp\left(-\frac{2d_c}{\lambda}\right)\left[1+\frac{B_0^2}{B_\phi^2}\right]=\frac{f_p}{\ell},$$

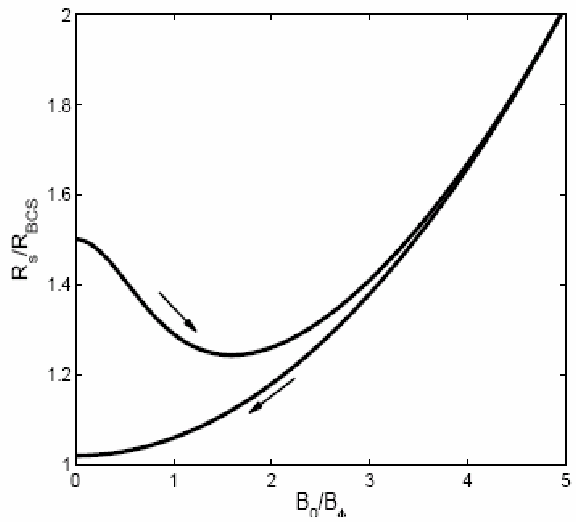
$$B_\phi=\frac{\phi_0}{\pi\lambda\ell}\sqrt{\frac{3\pi\lambda\ln\kappa}{d_c}},\qquad B_\phi << B_c$$

The RF field increases the attraction force of the vortex to the surface, pushing some of pinned vortices out.

$R_i(B)$ decreases as the field increases

$$R_i(H)=\frac{R_i(0)}{1+B_0^2/B_\phi^2}$$

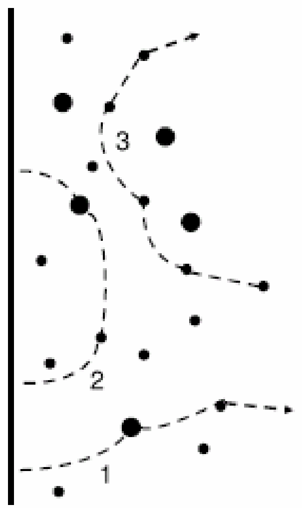
Low-field surface resistance and the rf annealing of pinned vortices



- **Nonlinear BCS + residual vortex hotspots**

$$R_s = R_{bcs}\left(1+\gamma\frac{H^2}{H_c^2}\right)+\frac{R_i(0)}{1+H^2/H_\phi^2}$$

- **Hysteretic field dependence**



- **Cycling the rf field can push some of trapped vortices out**
- **Decrease the surface resistance by the rf field cycling**

Eliminating vortex hotspots by thermal gradients

- Thermal force acting on the vortex:

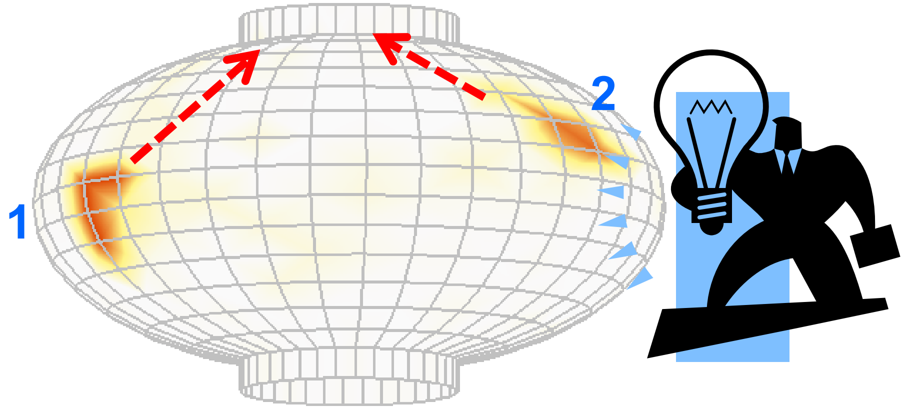
$$f_T = -s^*(T)\nabla T, \qquad s^* = -\phi_0 \frac{\partial H_{c1}}{\partial T}$$

- The condition: $f_T > J_c\phi_0$ gives the critical gradient, which can depin trapped vortices:

$$|\nabla T|_c = \frac{J_c}{|\partial H_{c1}/\partial T|} \simeq \frac{J_c \mu_0 T_c^2}{2B_{c1}T}$$

Taking B_{c1} = 0.15T, J_c = 1kA/cm^2 and T = 4.2K for clean Nb yields

$$|\nabla T|_c \approx 0.8 \text{ K/mm}$$



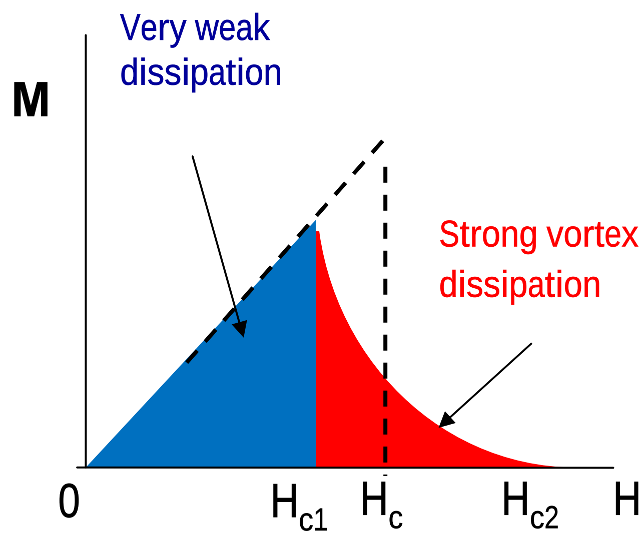
Pushing hotspots toward the orifice by external heaters

Any change of thermal maps after applying local heaters indicates that some of the hotspots are due to trapped vortices rather than local variations of the BCS resistance

Enhanced H_{c1} and surface barrier in films

Enhancement of the lower critical field in thin films with d < λ (Abrikosov, 1964)

$$H_{c1} = \frac{2\phi_0}{\pi d^2}\left(\ln\frac{d}{\xi} - 0.07\right)$$

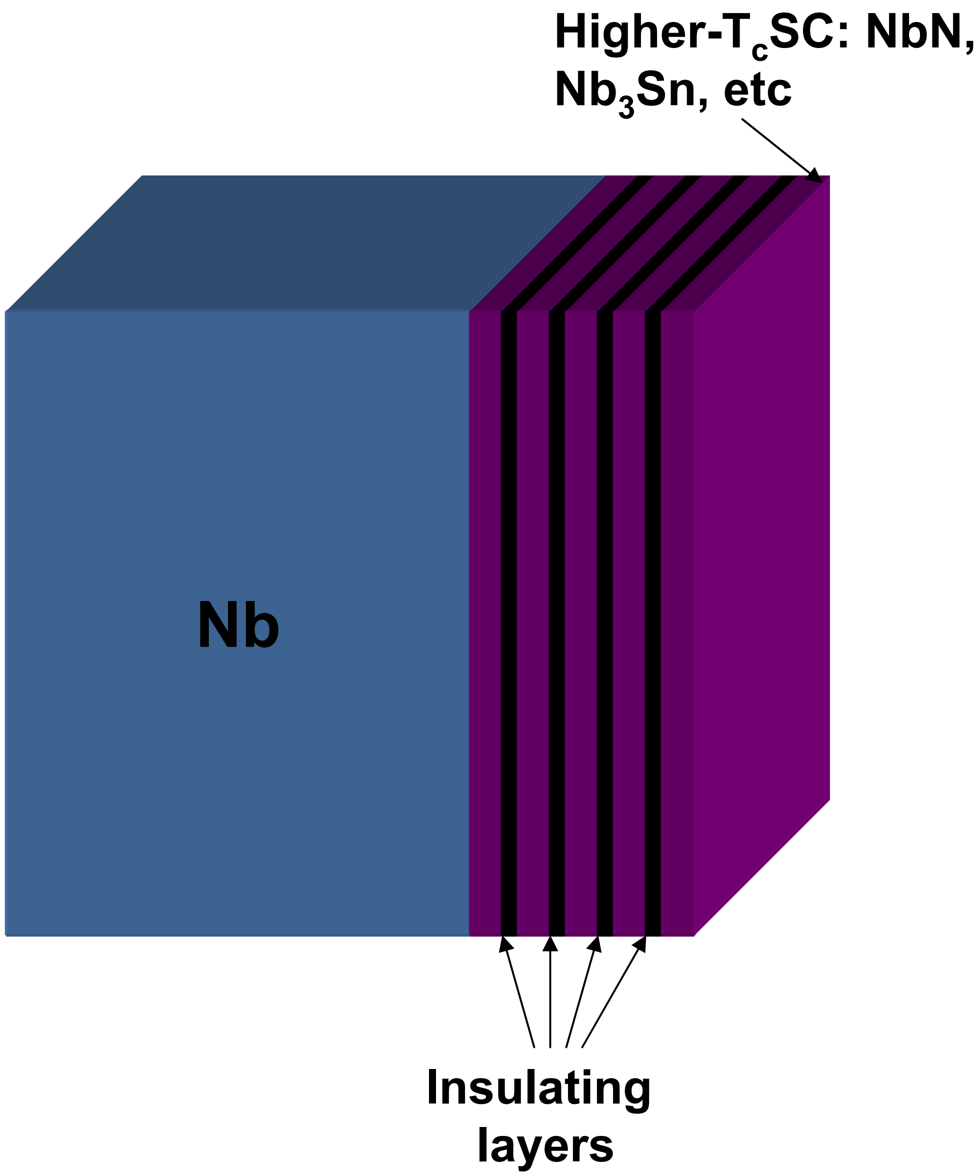


Field at which the surface barrier disappears

$$H_s = \frac{\phi_0}{2\pi d\xi}$$

Example: NbN (ξ = 5nm) film with d = 20 nm has H_{c1} = 4.2T, and H_s = 6.37T, Much better than H_c = 0.18T for Nb

How one can increase H_{c1} in SC cavities with T_c > 9.2K? Gurevich, Appl. Phys. Lett. 88, 012511 (2006)



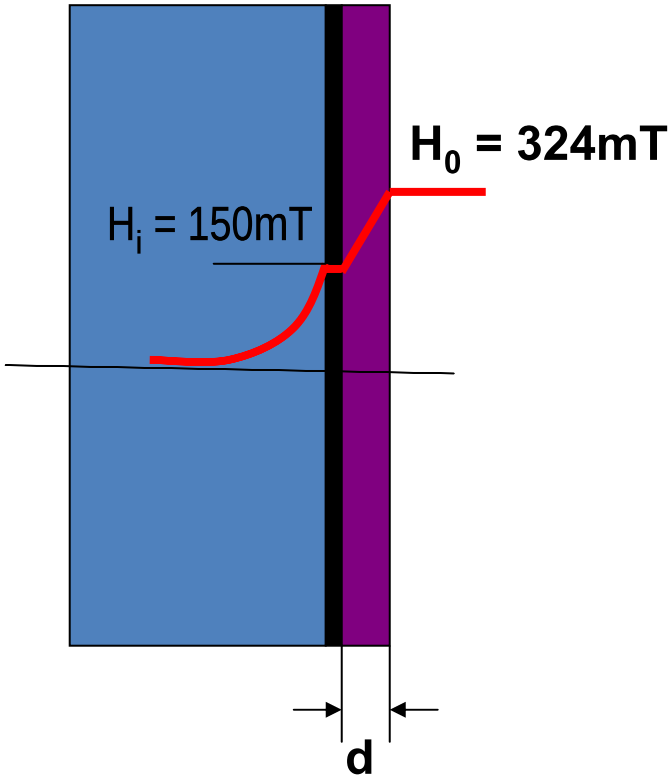
Multilayer coating of SC cavities: alternating SC and insulating layers with $d < \lambda$

Higher T_c thin layers provide magnetic screening of the bulk SC cavity (Nb) without vortex penetration

No open ends for the cavity geometry to prevent flux leaks in the insulating layers

The breakdown field could be increased up to H_c of the coating: 540 mT for Nb_3Sn

A minimalistic solution



A Nb cavity coated by a single Nb_3Sn layer of thickness d = 50nm and a dielectric layer in between

If the Nb cavity can withstand H_i = 150mT, then the external field can be as high as

$$H_0 = H_i \exp(d/\lambda_0) =$$
$$150 \exp(50/65) = 324\ mT$$

Lower critical field for the Nb_3Sn layer with d = 50 nm and ξ = 3nm: H_{c1} = 1.4T is much higher than H_c

A single layer coating more than doubles the breakdown field with no vortex penetration, enabling E_{acc}~ 100 MV/m

Global surface resistance

$$\widetilde{R}_s = (1 - e^{-2L/\lambda}) R_0 + e^{-2L/\lambda} R_b$$

Nb_3Sn coating of thickness L = 50 nm, $R_{Nb3Sn}(2K) \sim 0.1 R_{Nb}$

$$\widetilde{R}_s \cong 0.3 R_{Nb}$$

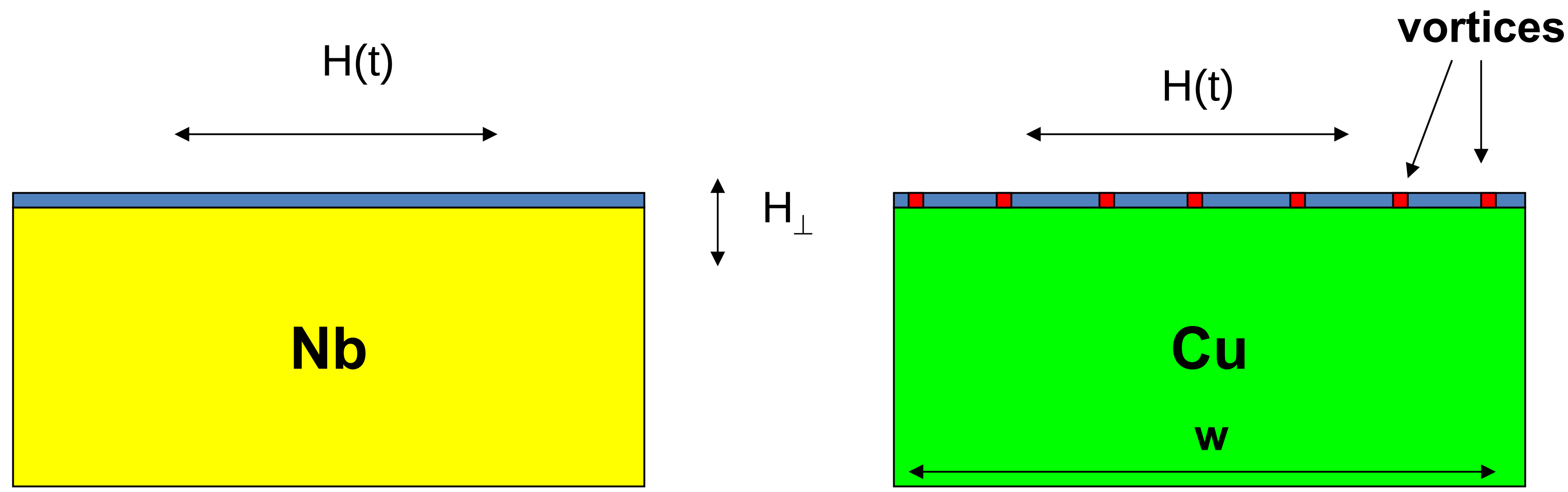
Screen the surface of Nb cavities by multilayers with lower surface resistance

Maximum breakdown field is limited by the higher H_c of the coating layer:

$$\boxed{H_{sh} = kH_c, \qquad k \cong 1}$$

$k \approx 0.9$ for $\lambda >> \xi$ and T = 0

Why is Nb_3Sn on a Nb cavity much better than Nb_3Sn on a Cu cavity?



Nb_3Sn/Nb cavity is much better protected against vortices produced by small transverse stray fields $H_\perp$ than Nb_3Sn/Cu cavity

Meissner state persists up to $H_\perp < H_{c1}^{(Nb)}$

Meissner state is destroyed for small $H_\perp < (d/w)H_{c1}^{(Nb_3Sn)} \ll H_{c1}^{(Nb_3Sn)}$ due to large demagnetization factor $w/d \sim 10^3\text{-}10^5$

Conclusions

- Dissipation due to the supersonic jumpwise penetration of single vortices produces hotspots on the cavity surface

- Hotspots due to trapped vortices produced by the Earth field or thermoelectric currents during to the cavity cooldown: residual resistance, decrease of R_s at low fields.

New possibilities to improve the cavity performance

- RF anealing of R_s by the rf field cycling: nonmonotonic hysteretic field dependence of Rs(H) at $H \ll H_c$.

- Eliminating vortex hotspots by external thermal gradients

- Multilayer S-I-S-I-S coating may break the Nb monopoly taking advantage of superconductors with much higher H_c without the penalty of lower H_{c1}

- Strong reduction of the BCS resistance because of using SC layers with higher Δ (Nb_3Sn, NbN, etc)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How high is 2 GHZ for Nb?

Hierarchy of time scales in a superconductor:

1. Relaxation of the SC condensate: $f_\Delta = \Delta/h = 400$ GHz (quasi-static)

2. Energy relaxation of thermally-activated normal electrons on phonons:

$$f_e = f_0 \left(\frac{\pi T}{T_c}\right)^{1/2} \left(\frac{2\Delta}{k_B T_c}\right)^{5/2} \exp\left(-\frac{\Delta}{k_B T}\right)$$

Kaplan et al, PRB 14, 8454 (1976)

- TESLA single cell cavity (f = 1.3 GHz, T = 2K, f_0 = 6.7 GHz), $f_e \approx 0.03$ GHz << f
  Nonequilibrium electrons overheating strongly increases R_s.

- Quasi-static pairbreaking strongly accelerates electron thermalization:

$$f_e(H) \cong f_e(0) \exp\left(\frac{H\Delta}{k_B T H_p}\right), \qquad H_p \approx 0.9 H_c$$

H = 90 mT, f_e = 2.7 GHz > f

3. Vortex time scales include 1 and 2 + times of vortex penetration