Abstract
The last experimental results obtained on a detector of small harmonic displacements, based on two coupled superconducting cavities, are presented. Starting from these results, and from a deeper understanding of the detector's working principles, new ideas for the development of a realistic gravitational waves detector, based on superconducting cavities, are discussed. The outline of the detector design and of its expected final sensitivity are also shown.

1 INTRODUCTION
In a series of papers it was studied how the effects due to the interaction between the gravitational and the electromagnetic fields could be used to detect gravitational waves [1]. The proposed detector exploits the energy transfer induced by the gravitational wave between two levels of an electromagnetic resonator, whose frequencies $\omega_s$ and $\omega_a$ are both much larger than the angular frequency $\Omega$ of the g.w. and satisfy the resonance condition $\omega_a - \omega_s = \Omega$. The interaction between the g.w. and the detector is characterized by a transfer of energy and of angular momentum. Since the elicity of the g.w. (i.e. the angular momentum along the direction of propagation) is 2, it can induce a transition between the two levels provided their angular momenta differ by 2; this can be achieved by putting the two cavities at right angle or by a suitable polarization of the electromagnetic field axis inside the resonator. In the scheme suggested by Bernard et al. the two levels are obtained by coupling two identical high frequency cavities; the angular frequency $\omega_s$ is the frequency of the level symmetrical in the fields of the two cavities, and $\omega_a$ is that of the antisymmetrical one. The frequency difference between the symmetric and the antisymmetric level is determined by the coupling and can be adjusted by a careful resonator design. Since the detector sensitivity is proportional to the square of the resonator quality factor, superconducting cavities should be used for maximum sensitivity.

The power transfer between the levels of a resonator made up of two pill-box cavities, mounted end-to-end and coupled by a small circular aperture in their common end wall, was checked in a series of experiments by Melissinos et al., where the perturbation of the resonator volume was induced by a piezoelectric crystal [2]. Recently the experiment was repeated by our group with an improved experimental set-up; we obtained an order of magnitude sensitivity to fractional deformations of the resonator length as small as $\delta l/l \approx 10^{-20}$ Hz$^{-1/2}$ [3].

In this report we shall briefly review the last experimental results obtained on the first prototype of the detector after the last Workshop on RF Superconductivity held in Santa Fe (NM) in 1999; details on the detector working principles and on the experimental set-up can be found in the proceedings of that workshop (see Figure 1) [4].

Figure 1. PACO cavity mounted on the test cryostat.

2 EXPERIMENTAL RESULTS
The electromagnetic properties of the detector were measured in a vertical cryostat after careful tuning of the two cavities. In fact in order to get maximum sensitivity we need to have two identical coupled cavities, or, in other words, a flat field distribution between the two cavities. The symmetric mode frequency was measured at 3.03 GHz and the mode separation was 1.38 MHz.

In order to suppress the noise coming from the symmetric mode at the detection frequency, the
transmission detection scheme, with two magic-tees, was used, as described elsewhere (see Figure 2) [3,4].

Figure 2. Schematic view of the transmission detection scheme with two magic-tees

In Figure 3 the signal from the Δ port of the output magic-tee is shown for an input power $P_i = 1$ W and no adjustments made on the phase and amplitudes of the rf signal entering and leaving the resonator. The overall attenuation of the symmetric mode is $R = -48$ dB.

After balancing the arms of the two magic-tees in order to launch the symmetrical mode at the cavity input and to pick up the antisymmetrical one at the cavity output, with 1 W (30 dBm) of power at the Σ port of the first magic-tee, $6.3 \times 10^{-15}$ W (-112 dBm) were detected at the Δ port of the second one, giving an overall attenuation of the symmetric mode of $R \approx -140$ dB (see Figure 4).

Figure 3. Transmission of the symmetric mode (no optimisation) measured at the Δ port of the output magic-tee (left peak) and antisymmetric mode excited by the piezo (right peak).

At a detection frequency of $\Omega/2\pi = 1$ MHz, the sensitivity of the system is quite independent from the value of $R$, because of the high cavity $Q_L$.

Nevertheless for lower frequencies, in a range $\Omega \leq 10$ kHz, where astrophysical sources of gravitational waves are expected to exist, this noise source can become dominant and the achieved rejection is fundamental in order to pursue the design of a working g.w. detector in the 1-10 kHz frequency range.

The cavity loaded quality factor was $Q_L = 10^9$ at 1.8 K, and the energy stored in the cavity with 10 W input power was approximately 1.8 J (limited by the maximum power delivered by the rf amplifier), with both the input and output ports critically coupled ($\beta_1 = \beta_2 = 1$).

To excite the antisymmetric mode a piezoelectric crystal (Physik Instrument PIC 140, with longitudinal piezoelectric coefficient $k_l = 2 \times 10^{-10}$ m/V) has been fixed to one cavity wall. A synthesized oscillator provided the driving signal to the crystal with a power output in the range 2-20 mW (3-13 dBm). The oscillator output was further attenuated to reduce the voltage applied to the piezo by a series of fixed attenuators and a variable attenuator (10 dB step). The oscillator frequency was carefully tuned to maximize the energy transfer between the cavity modes.

The signal emerging from the Δ port of the output magic-tee was amplified by a LNA (48 dB gain, 0.6 dB noise figure) working at room temperature, and fed into a spectrum analyzer. In Figure 3 an example of the parametric conversion process is shown.

The minimum detected noise signal level at the antisymmetric mode frequency, with no excitation coming from the piezo, was $P_{\text{out}}(\omega_a) = 5 \times 10^{-19}$ W in a bandwidth $\Delta f = 100$ Hz, giving a noise power spectral density $P_{\text{out}}(\omega_a) = 5 \times 10^{-21}$ W/Hz; the main contribution to this signal was the Johnson noise of the LNA used to amplify the signal picked from the Δ port of the output magic-tee.

Taking into account the input and output coupling coefficients system sensitivity is given by $h_{\text{min}} = 3 \times 10^{-20}$ Hz$^{1/2}$.

§ We point out that the input port is critically coupled to the symmetric mode, while the output port is critically coupled to the antisymmetric mode.
3 FUTURE TRENDS

The second phase of the PACO R&D program is focused on the development of a detector based on two spherical coupled cavities (see Figure 5). In order to approach the interesting frequency range for g.w. detection, the mode splitting (i.e. the detection frequency) will be $\omega_a - \omega_s \approx 10$ kHz. The internal radius of the spherical cavity will be $r = 100$ mm, corresponding to a frequency of the TE$_{011}$ mode $\omega \approx 2$ GHz. A tuning cell, or a superconducting bellow, will be inserted in the coupling tube between the two cavities, allowing to tune the coupling strength (i.e. the detection frequency) in a narrow range around the design value. The tuning sensitivity vs. the distance between the cells has been calculated and is shown in Figure 6.

The choice of spherical cells depends on several factors:

- From the point of view of the electromagnetic design the spherical cell has the highest geometrical factor, and so the highest quality factor, for a given surface resistance.

- From the mechanical point of view it is well known that a sphere has the highest interaction cross-section with a g.w. and that only a few mechanical modes of the sphere do interact with a gravitational perturbation (the quadrupolar ones – see Figure 7) [5].

The mechanical design is highly simplified if the spherical geometry is used since the deformation of the sphere is given by the superposition of just one or two normal modes of vibration and thus can be easily modeled. In fact the proposed detector act essentially as a standard g.w. resonant bar detector: the gravitational perturbation interacts with the mechanical structure of the resonator, deforming it. The e.m. stored inside the resonator is affected by the time-varying boundary conditions and a small quantity of energy is transferred from the initially excited e.m. mode to the initially empty one, provided the g.w. frequency equals the frequency difference of the two modes. A possible design of the detector makes use of both the mechanical resonance of the resonator structure, and the e.m. resonance. This can be accomplished if the detector is designed in order to have the fundamental mechanical mode frequency equal to the e.m. modes frequency difference $\omega_m = \omega_a - \omega_s$. The sensitivity of the system if this condition is satisfied is shown in Figure 8.

For the TE$_{011}$ mode of a sphere the geometric factor $G$ has a value $G = 850$ $\Omega$, while for a standard elliptical accelerating cavity the TM$_{010}$ mode has a value of $G = 250$ $\Omega$. Looking at the best reported values of quality factor of accelerating cavities, which typically lie in the range $10^{10} - 10^{11}$, we can extrapolate that the quality factor of the TE$_{011}$ mode of a spherical cavity can exceed $Q = 10^{11}$.
The spherical cells can be easily deformed in order to remove the unwanted e.m. modes degeneracy and to induce the field polarization suitable for g.w. detection (see Figure 9).

The interaction between the stored e.m. field and the time-varying boundary conditions is not trivial and depends both on how the boundary is deformed by the external perturbation and on the spatial distribution of the fields inside the resonator. It has been calculated that the optimal field spatial distribution is with the field axis of the two cavities orthogonal to each other. Different spatial distributions (e.g. with the field axis along the resonators’ axis) give a smaller effect or no effect at all.

The spherical shape can be easily used to investigate whether the niobium-on-copper technique could be useful for the detector final design.

The choice between bulk niobium or niobium-on-copper for the final detector design has not yet been made and is still under investigation. Both techniques present in principle advantages and drawbacks. A prototype of two coupled spherical cavities in bulk niobium will be built at CERN in 2002. A single cell, seamless, copper spherical cavity has been built at INFN-LNL by E. Palmieri and will be sputter coated at CERN (see Figure 10).

4 CONCLUSIONS

A first prototype of the detector, made up of two pill-box cavities, mounted end-to-end, has been built and successfully tested.

A detector based on two coupled spherical cavities is now being designed, and preliminary tests on normal conducting prototypes are being made. The planned timeline is as follows:

- In 2002 a bulk niobium detector (two spherical cavities, $\omega = 2$ GHz, $\Omega = 10$ kHz, fixed coupling) will be built at CERN;
- In 2003 a variable coupling detector will be built and tested.

If experimental results will be encouraging, by the end of 2003 a proposal for the realization of a g.w. detector, based on superconducting rf cavities will be made.

5 ACKNOWLEDGEMENTS

Several people gave a significant contribution to this work. We wish to thank S. Cuneo, from INFN-Genoa, for the development of the detector’s mechanical model and design and E. Palmieri, from INFN-LNL, for the realization of the single cell spherical cavity; E. Chianveri and R. Losito, from CERN-SL/CT, and C. Benvenuti and S. Calatroni, from CERN-EST/SM, for the commitment in the construction and testing of the superconducting PACO-2 cavities.

6 REFERENCES
