## NONLINEAR EFFECTS IN THE He–FILLED SUPERCONDUCTING RF SYSTEMS\*

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## Abstract

Thermally induced nonlinear phenomena in superconducting niobium microstrip resonators filled by liquid helium are studied. With increasing input power a resonance curve of the resonator revealed the nonlinear behaviour and its form depended on the rate and direction of the microwave generator frequency sweep. The found nonlinear resonance is explained by thermally induced variations of the helium dielectric permittivity caused by the microwave power losses in superconductor. A number of interesting manifestations of this thermal instability have been observed including the parametric pulse generation of the monochromatic microwave signal, generation of acoustic pulses, stimulated Mandelshtam-Brillouin scattering like effect.

## **1 INTRODUCTION**

Thermal breakdown or quench is one of the main limitations to achieve reliably high gradients in superconducting RF cavities. Despite the fact that thermal instability analysis show that the maximum magnetic field on uniform defect-free surfaces should be higher than 150 mT, most experimental results on actual cavities exhibit quenches around 80 mT [1] with very localised heating produced by small micron-size defects. In future high field accelerator applications of superconducting niobium cavities a problem of heat sinking by liquid helium may be significant. In this connection we studied thermally induced nonlinear phenomena in superconducting niobium microstrip resonators filled by liquid helium.

#### **2 EXPERIMENTAL TECHNIQUES**

Measurements were done in the half-wave mode of microstrip resonator at 8 GHz. Fig. 1 shows a shielded symmetric microstrip resonator, which consists of niobium foil with a thickness of 15  $\mu$ m sandwiched between two dielectric disks and forming a one-half wavelength strip. The  $\lambda/2$  strip together with gap coupling microstrip transmission lines were assembled inside a cylindrical Nb cavity to form a shielded microstrip resonator. Microstrip lines coupled to and extracted electromagnetic energy within the cavity and were connected with a waveguide interface via microstrip-probe-into-waveguide transitions. Critical temperatures of superconducting niobium materials were of 9.2 K. The assembly was mounted in the helium bath of a cryostat.

The boiling suppresser designed in the Cryogenic Department of LPI was used to prevent from helium boiling in the temperature range from 2.1 to 4.3 K. Sapphire or teflon with relative dielectric constants of 10 and 2, respectively, were used as the dielectric laminae of a microstrip resonator. The residual volume of a resonant cavity was full of helium. In the teflon-niobium resonators plastic teflon disks could be pressed slightly inside the cylindrical Nb cavity to reduce a volume occupied by helium.



Figure 1: Layout of the microstrip niobium resonator assembly. Left bottom: view of cylindrical cavity in the Nb shielding without a cap (shown at the right bottom) with inserted niobium foil sandwiched between two dielectric disks (shown at the left top) and forming a one-half wavelength strip. Right top: cross-section of the foil-disk sandwich.

An C4-27 spectrum analyser was used to measure the microwave transmission spectra of the resonator at frequencies around 8 GHz. A microwave system with a 10 mW klystron generator  $\Gamma$ 4-83 or an BWT sweepgenerator PK2-28 was used as a microwave input power source. To investigate the transient behaviours, the output power was measured as a function of time without changing the cavity arrangement. This enabled us to study the nonlinear dynamics of the electromagnetic response of the cavity filled by helium. An C1-90 oscilloscope was used to measure the transient signal of the RF detector.

## **3 RESULTS AND DISCUSSIONS**

## 3.1 Nonlinear behaviour of the helium-filled superconducting microstrip resonator

The primary manifestations of resonator nonlinearity are reduction in Q, a shift in resonant frequency and an asymmetry of resonance curve. It has been found that the

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superconducting microstrip resonators filled with liquid helium exhibit highly nonlinear behaviour in resonance curve. The nonlinearity was revealed as asymmetry of the resonance curve seen at the spectrum analyser display during the slow sweep of an input power frequency. Moreover, a shape of the asymmetric resonance curve depended on the frequency-sweep direction. The nonlinear effects appear at input power levels exceeding the order of tenths of mW. Such behaviours can be seen in Figs. 2a to 2d in which several resonant curves of the resonator for both up and down frequency-sweeps are shown for resonance frequency and liquid-helium temperature values ranging from 7.9 GHz to 8.6 GHz, and 4.26 K to 4.32 K, respectively. A sweep of the input frequency was performed by use of the frequency modulation with the controllable depth and rate provided by a low frequency oscillator.



Figure 2: Transmitted power versus frequency increment  $f-f_0$  for different resonance frequency  $f_0$  and helium bath temperature  $T_{He}$  values measured for both up (solid line) and down (dotted line) frequency sweeps with different sweeping rates. A value of the sweeping rate is characterised by the time derivative of a detuning parameter  $\xi = (f-f_0)/f_0$ . (a)– 8.6 GHz and 4.26 K,  $|\partial \xi_1/\partial t|$ ; (b)– 7.9 GHz and 4.28 K,  $|\partial \xi_2/\partial t|$ : (c)– 7.9 GHz and 4.32 K,  $|\partial \xi_3/\partial t|$ ; (d)– 7.9 GHz and 4.31 K,  $|\partial \xi_4/\partial t|$ , where the following inequalities are held:

$$\left|\frac{\partial \xi_1}{\partial t}\right| > \left|\frac{\partial \xi_2}{\partial t}\right| > \left|\frac{\partial \xi_3}{\partial t}\right| > \left|\frac{\partial \xi_4}{\partial t}\right|.$$

## 3.2 Thermal detuning model of nonlinear resonance and helium heating effects

The found in Fig. 2 nonlinear behaviour of the superconducting microstrip resonator is provided by the fact that a resonant frequency of the resonator  $f_p$  depends on the liquid helium dielectric permittivity  $\varepsilon_{He}$ . As a result it should depend on the temperature [2].

The maximum RF electric and magnetic fields  $|\mathbf{E}|$  and  $|\mathbf{H}|$  reach of rather high values near the superconductor surface in high-Q cavities even at low levels of the input

power. The microwave power losses per unit area are estimated as

$$P \cong R_{HF} \frac{H^2}{2},\tag{1}$$

where  $R_{HF}$  is the surface resistance of superconductor, H is equal to the absolute value  $|\mathbf{H}|$  at surface of superconductor.

Heat flux generated by the RF field has to be conducted through the niobium wall, niobium surface and the helium layer in resonator cavity to be removed by the helium bath of cryostat. Since the thermal conductivity of superconductor and the thermal conductivity of liquid helium have finite values, the locale and global heating of the superconducting surface and of the adjacent layer of liquid helium is possible. Helium heating should change the resonant frequency of microstrip resonator because of the temperature variation of helium permittivity  $\varepsilon_{He}$ . If temperature varies in a small range  $\Delta T \cong 1.5$  K, the liquid helium permittivity versus temperature  $\varepsilon_{He}(T)$  can be expressed as

$$\mathcal{E}_{He} \cong -\beta T + \gamma, \tag{2}$$

where the constant coefficients  $\beta$  and  $\gamma$  are defined from experiment [2] ( $\beta$  and  $\gamma$  are positive).

The inverse square of the resonant frequency as a function of the helium permittivity  $f_p^{-2}=\phi(\epsilon_{He})$  can be written as follows [2]

$$\frac{1}{f_p^2} \cong \frac{1}{f_0^2} (1 + c_1 \mathcal{E}_{He})$$
(3)

for a small interval of temperature variations  $\Delta T \cong 1$  K. Here the constant coefficient  $c_1$  is defined from experiment [2] ( $c_1>0$ ). For our resonators the value of  $c_1$  was usually low ( $c_1\cong 0.1\div 0.2$ ). That results in a low value of the product  $c_1\epsilon_{He}$ . From eqs. (2) and (3) it follows

$$f_p^2 \cong f_0^2 (1 - c_1 \varepsilon_{He}) \cong f_0^2 (1 - c_1 \gamma + c_1 \beta T) .$$
 (4)

The equation of thermal conductivity for the considered system can be written as

$$Q = k\Delta T = k(T - T_0), \tag{5}$$

where T is the temperature of liquid helium near the surface of superconductor,  $T_0$  the helium bath temperature, k the thermal conductivity, which is defined experimentally and takes into account the thermal conductivity of both superconductor and liquid helium as well, as the system geometry.

On the other hand from the eq. (1) heat flux can be reexpressed

$$\Delta Q \cong P \cong R_{HF} \frac{H^2}{2} \tag{6}$$

and from eqs. (6), (5) and (4) one can deduce

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$$f_p^2 \cong f_0^2 (1 - c_1 \gamma + c_1 \beta T_0 + \frac{c_1 \beta R_{HF}}{2k} H^2).$$
 (7)

The last expression shows that the resonant frequency  $f_p$  depends *nonlinearly* on the oscillation amplitude H of magnetic field. Eq. (7) for the resonant frequency of the helium-filled superconducting microstrip resonator indicates an appearance of the nonlinear resonance. The term

 $\frac{c_1 \beta R_{HF}}{2k} H^2$  causes the nonlinear resonance and a value

of the coefficient  $c_1\beta R_{HF}/2k$  determines degree of the nonlinearity of resonance.

Since the resonant frequency increases with rise of the oscillation amplitude according to the eq. (7), the resonance curve plotted in the "output power vs. frequency" co-ordinates bends to the higher frequencies as it is seen in Fig. 2d. Moreover, a resonance curve of the resonator revealed the dependence of its form on the rate and direction of the microwave generator frequency sweep. It can be explained by the following two considerations. The resonant frequency changes to the higher values with increasing temperature. 1) If a frequency of the input power generator changes from higher to lower values, it moves across the resonant peak region more quickly because of an upper shift of the resonant frequency. It can be expressed by following: if  $\Delta F/\Delta t$  is the sweep rate of generator frequency and  $\Delta f_p/\Delta t$  the speed of change of the resonant frequency of resonator, so the total speed of the generator frequency movement across the resonance curve will be

$$\left(\frac{\Delta F}{\Delta t}\right)_{+} = \frac{\Delta F}{\Delta t} + \frac{\Delta f_{p}}{\Delta t} \,. \tag{8}$$

2) If a frequency of the input power generator changes from lower to higher values, similarly to eq. (8) it can be written

$$\left(\frac{\Delta F}{\Delta t}\right)_{-} = \frac{\Delta F}{\Delta t} - \frac{\Delta f_{p}}{\Delta t}.$$
(9)

Hence in the second case according to the eq. (9) frequency of the input power generator will spend a longer period of time in the region of peak on the resonance curve than it will be in the first case described by eq. (8). As a result the superconducting microstrip resonator in the second case will be heated stronger and its resonant frequency will increase up to the higher value during the generator frequency movement from down to up than it will be in the alternative case when the generator frequency sweeps from higher to lower values. Thus the results presented above indicate that the observed nonlinearity requires incorporation of the helium heating effects.

# *3.3 Parametric generation of monochromatic RF pulses*

Even higher degree of nonlinearity has been found for the microistrip resonator, the cavity of which was tightly occupied by teflon (effective partial volume filled by helium [2] is equal to  $v \cong 0.03$ ). The pronounced hysteresis behaviour can be seen in Fig. 3a, in which the results of measurements of the transmitted power versus generator frequency are shown for the very slow (manually) both up and down frequency sweeps. The quite narrow transmitted power transient pulse arises during frequency moving down. Measurements of the spectral width of the pulse signal were restricted by the spectral resolution of our experimental units (order of 10 kHz). The kinetics of the observed pulse is shown in Fig 3b that demonstrates exponential growth and decay of the transmitted power. A value of lifetime of the exponential growth and decay was estimated from this measurement as 30 msec. It should be noted that the measured value of a decay time in the case of linear resonance would correspond to the effective quality factor order of  $2 \times 10^9$ .



Figure 3: a) Transmitted power versus frequency for the very slow (manually) both up and down changes of the generator frequency. Arrows show the frequency sweep direction. Solid lines correspond to steady state signal; dotted lines correspond to the unstable transient signal. b) Kinetics of the transmitted power pulse shown near the frequency 8.649 GHz in Fig 3 a.

Taking into account the results presented above we can suppose that the exponential pulse generation of the monochromatic microwave signal can be explained by the parametric effect caused by thermally induced variations of the helium dielectric permittivity. We suppose that the monochromatic microwave signal arises due to the fast liquid-gas phase transition of helium. The estimations show that the helium vapour bubble exponentially grows and disappears in the thin layer surrounding the niobium strip between teflon disks that results in the observed transient spike of the transmitted power.

## 3.4 Generation of acoustic pulses

In the teflon resonator case, in addition to the effect presented in the previous section, the generation of acoustic pulses was detected as one more manifestation of this thermal instability. During the helium vapour pumping in course of the reducing the liquid helium temperature in the cryostat the generation of periodical acoustic pulses was observed. This effect was especially pronounced at temperatures slightly above the temperature of  $\lambda$ -point of helium (T=2.17 K). Such behaviours can be seen in Figs. 4a in which several resonant curves of the resonator during the generation of the acoustic pulses are shown. Fig. 4a demonstrates the periodically varied intensity of the resonant peak. To investigate these behaviours, the output power was measured as a function of time without changing the cavity arrangement. The results of these measurements are shown in Fig. 4b. From this figure we can conclude that the acoustic pulses correlate with the periodical variations of the resonator transmittivity. The period of oscillations in these measurements is estimated to be about 100 msec. It correlates with the frequency of acoustic pulses, which is about 10 Hz, and, if we take into account the  $|\sin t|$ -like functional dependence of the curve in Fig. 4b, with the decay time value of 30 msec for the exponential transient signal observed in the previous section.



Figure 4: a) Transmitted power versus frequency during the generation of the acoustic pulses. Solid lines correspond to the upper and lower bounds of varied signal; dotted lines correspond to the unstable periodically varied signal. Note the FWHM is defined by the spectral resolution of measuring spectrum analyser. b) Kinetics of peak of the transmitted power shown near the frequency 8.625 GHz in Fig 4 a.

#### 3.5 Nonlinear effects in resonator transmittivity

In measurements of the temperature dependence of resonant curve we have found the unusual relation between the peak transmitted power and the full width at the half maximum (FWHM) of the resonant curve. Such behaviour can be seen in Fig.5 in which the peak transmitted power and the FWHM of the resonant curve are shown for a sapphire microstrip resonator in dependence on temperature varied between 2.3 and 8.5 K both in liquid (closed symbols) and in vapour (open symbols) helium. The measurements were done with the FM input signal of an BWT sweep-generator. The FM depth was about 2 MHz. From Fig. 5 we can see that in the course of cooling in liquid helium the peak transmitted power increases at temperatures below 3.66 K despite the unchanged value of the FWHM. The observed growth of the transmitted power correlates with the appearance double-peak resonant curve at temperatures below 3.66 K as it is illustrated by marks in Fig. 5. This double-peak curve is a manifestation of the nonlinear resonance measured with the FM input power (cf. Fig.2). Note that in the helium vapour at the same temperatures the nonlinear resonance does not appear and no growth of the transmitted power is observed. From this figure we can suppose that at lower temperature of liquid helium, when the nonlinear resonance is established, the nonlinear effect similar to the stimulated Mandelshtam-Brillouin light scattering may results in the observed growth of the transmitted power. The frequency modulation of the input power and helium thermal instability may result in appearance of the second resonance mode (cf. the second peak on the resonance curve in Fig. 2), which in turn would increase the transmitted power.



Figure 5: Peak transmitted power (rectangles) and the FWHM (circles) of the resonant curve for a sapphire microstrip resonator in dependence on temperature varied between 2.3 and 8.5 K both in liquid (closed symbols) and in vapour (open symbols) helium. The measurements were done with the FM input signal of an BWT sweep-generator. The FM depth was about 2 MHz. Single and double-peak resonance marks indicate the appearance of the nonlinear resonance at temperatures of liquid helium below 3.66 K. Arrows show the direction of temperature variations.

## **4 CONCLUSIONS**

Thermally induced nonlinear phenomena in superconducting niobium-on-sapphire (niobium-on-teflon) microstrip resonators filled by liquid helium are studied. It was found that with increasing input power over approximately 0.1 mW a resonance curve of the resonator revealed the nonlinear behaviour and its form depended on the rate and direction of the microwave generator frequency sweep. The found nonlinear resonance is explained by thermally induced variations of the helium dielectric permittivity caused by the microwave power losses in superconductor. A number of interesting manifestations of this thermal instability have been observed including the parametric pulse generation of the monochromatic microwave signal, generation of acoustic pulses, stimulated Mandelshtam-Brillouin scattering like effect.

#### **5 REFERENCES**

- H. Safa, "Statistical analysis of the quench fields in SCRF cavities", Proc. of the 8<sup>th</sup> Workshop on RF Superconductivity, Abano Terme (Padova), Italy, Oct. 1997, V. Palmieri, A. Lombardi Eds., p.503-509.
- [2] V. A. Dravin, A. L. Karuzskii, A. E. Krapivka, N. A. Osipova, A. V. Perestoronin, "Measurements of the effective electrodynamical parameters of Nb microstrip resonator," Proc. of the 9<sup>th</sup> Workshop on RF Superconductivity, Santa Fe, USA, Nov. 1999, p.190-194..