RESEARCH ON BEAM DYNAMICS OF A 2 GeV 6 MW ISOCHRONOUS FFA*

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Abstract

CIAE has proposed an innovative design for a 2 GeV, 6 MW isochronous FFA in 2019. This study aims to present the results of beam dynamics research, demonstrating the feasibility to accelerate the intense proton beam with the energy beyond 1 GeV limitation of isochronous cyclotrons. By introducing 1st - 3rd radial gradient of peak magnetic field to simulate the quadrupole to octupole component of the isochronous machine, three different lattice designs are obtained. Adjusting the radial gradient of the peak field allows an option to avoid or cross integer resonances. Various inherent and coupled resonances are investigated subsequently, with a focus on the destructive effects of the v_r =3 on the transverse phase space. Based on PIC method, we simulate the vortex motion caused by space charge in a large-scale alternating gradient field. Results indicated that the radial size of beam is ~ 10 mm, which is expected to be improved after considering the effects of neighboring bunches. Additionally, high-Q RF cavities and precession extraction further enlarge the turn separation to 30 mm, ensuring efficient beam extraction in the extraction region.

INTRODUCTION

High energy and high current proton accelerators are widely applied in frontier research fields such as nuclear physics and particle physics, national economic fields such as public health and advanced energy, and even national defence industry and national security [1][2]. Proton accelerator with an average beam power of 5-10 MW is the world's dream machine for more than 30 years [3][4]. Cyclotrons can provide continuous wave beam but are restrict by energy limitation ~1 GeV [5]. Beyond the traditional solutions for high intensity machines such as superconducting linac and rapid cycling synchrotron, CW FFA which is non-scaling and nonlinear was considered as an attractive proposal for several megawatts machine [6]. To utilize strong focusing optics and fixed frequency for CW acceleration, a 2 GeV/6 MW FFA concept design was proposed in 2019 [7]. In this machine, the F-D-F lattice design has been adopted to realize strong focusing. Each focusing and defocusing magnet has a third-order magnetic field gradient in the radial direction, which can achieve the effect of the dipole to octupole magnets, and thereby balance the isochronism and focusing. Based on this principle of adjusting gradient to provide strong focusing, higher-order nonlinear magnetic field components were added (such as quadrupole, hexapole, octupole, etc) to avoid important resonance crossings, resulting in a "radial local achromatic" effect. Three different lattice schemes (2019, 2020, MOIBA01

2022) are designed to achieve the goal of the isochronous acceleration for 2 GeV proton. The basic parameters of the three schemes are listed in Table 1. In this paper, beam dynamics of the 2 GeV/6 MW FFA is summarized for better comprehension of non-scaling and nonlinear FFA machines.

Table 1: Basic Parameters of 2 GeV CW FFA Machine

Parameters	Scheme 1	Scheme 2	Scheme 3
Extracted energy	2 GeV	2 GeV	2 GeV
Focusing magnet ra- dius*	23.3~26.8 m	18.2 ~20.9 m	17.6~19.4 m
Defocusing magnet ra- dius*	23.3~26.8 m	18.2 ~20.9 m	17.6~19.4 m
Focusing field	1.5~2.7 T	1.56~2.62 T	1.57~2.66 T
Defocusing field	1.0 ~2.4 T	1.77~2.51 T	1.15~2.31 T
Number of lattices	10	10	10
RF fre- quency	44.4 MHz	35.1 MHz	51.6 MHz
Cavity volt- age (single cavity)	1.2 MeV	1.5 MeV	1.5 MeV
Harmonic number	26	16	22
Number of cavities	10	15	15
Turn separa- tion for the extraction	~1.5 cm	~1.5 cm	~1.5 cm

* (From the machining center)

STATIC ORBITS ANALYSIS

Isochronism and Tune

It is well-known that static beam dynamics results are the basis for one to verify the feasibility of the lattice design. Therefore, the results of phase slip, tune diagram, and static region are introduced first.

The magnetic field of 2 GeV/6 MW machine (CYCIAE-2000) has the characteristics of high order gradient (up to 3rd), strong nonlinearity, etc. The physical design and beam dynamics study require higher tracking accuracy of the particle tracking program, especially near the edge field of focusing and defocusing magnets. Besides that, the beam dynamics simulation also needs higher magnetic field interpolation accuracy and more reasonable tracking algorithms which adaptive to wide-range twisty orbits of FFA. Some

improvements of interpolation method are accomplished in closed orbit calculation codes as Cyclops [8].



Figure 1: Integral phase slip of different schemes.



Figure 2: Tune diagram.

In scheme 1 we proposed in 2019, the differential phase slip is controlled below $\pm 1\%$ and the maximum integral phase slip of three schemes is 35°, as shown in Figure 1. It can be concluded that the magnetic field can well meet the isochronous requirement. The harmonic number is set to 26, and the RF frequency is 44.4 MHz, which equal to 100 MeV compact cyclotron CYCIAE-100.

In scheme 2, the harmonic number is set to 16, and the RF frequency corresponds to 35.1 MHz. The phase slip of the beam during the acceleration process is controlled within ± 40 degrees, which ensures a specific energy gain per turn. Due to the introduction of local 3rd magnetic field distribution to avoid integer resonance, a certain amount of isochronism is sacrificed at the injection and extraction position. Therefore, the corresponding frequency error is about 3‰. According to the single-cavity voltage of 1 MV, it takes approximately 91 turns to extract the beam.

In scheme 3, the harmonic number is set to 22, and the RF frequency corresponds to 51.65 MHz. Due to the utilization of an integer resonance suppressor (IRS) [9], it is possible to cross v_r =3 integer resonance while controlling beam envelop growth. With the benefit of IRS, the phase slip is better than Scheme 2 and can be controlled within ±10 degrees, which is beneficial for clean extraction.

Stable Region

The stable region both for radial and vertical phase spaces, should be investigated carefully to find the boundary of the linear region and ensure the beam is accelerated stably. Since strong focusing is realized in FFA machines based on alternative fields, the vertical and radial stable region is much bigger than compact cyclotron. For Scheme 2, near injection and extraction, the radial tune is close to 2, 3 respectively. v_r escapes these integer

resonances and provides a certain width away from resonance lines with some sacrifice of isochronism as shown in Figure 2, which is achieved by 3rd order magnetic field.

In Scheme 2, the working path is closed to $v_r=2$ and $v_r=3$ resonance lines at injection and extraction respectively. Moreover, the working path moves around the coupled resonance over a wide range, which should be evaluated during resonance analysis. In the stable region, the most important resonance is $v_r = 2.5$, and the corresponding energy is 1270 MeV. The stable regions are shown in Figure 3.



Figure 4: Stable Regions of Scheme 3.

As demonstrated in Figure 4, radial stable region is enough for 3 cm off center non-ideal particles. For Scheme 3, ν_r =3, the most important resonance is inhibited using IRS. Besides that, resonances of $3\nu_r$ =10 and $4\nu_r$ =10 are also investigated. Since the highest order of lattice magnetic field is third, the stable region of fourth order resonance is much larger than third order resonance. Moreover, the beam can quickly pass through these two critical inherent resonances with the help of significant energy gain up to 15 MeV per turn based on recent experiments of a scaled RF cavity (Q~42000).

MULTI-PARTICLE BEAM DYNAMICS

Resonance Study

For the analysis of Scheme 1, the most important resonance line is $v_r=3$. To quantify the influence of $v_r=3$ resonance, a B_3 error field is added to the field map of middle plane near the radius of $v_r=3$ (Radial rectangular distribution with radial width 15 cm, phase is fixed at 45°). The energy gain per turn is 10 MeV/turn, and the gradient of v_r is 0.001/MeV (or 0.01/turn). With 0.5 Gs B₃ amplitude, the radial oscillation increases from ~6 mm to ~15 mm. With 1 Gs B3 amplitude, the radial oscillation is about ~26 mm. The details are shown in Figure 5.



Figure 5: Enlargement of beam envelop caused by $v_r = 3$ integer resonance for Scheme 1.

Resonance study is carried out especially for Scheme 2. We have summarized the crossed resonance lines for the three schemes in the Table 2.

Table 2. Summary of Resonance Study

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Reso-	Driving	Scheme 1 Energy	Scheme 2 Energy	Scheme 3
hance	-	Lifergy	Lifergy	Lifeigy
vr=2	B_2	800~850	800~900	/
vr=3	<i>B</i> ₃	1640~1680	1850~2000	1690
2 <i>vr</i> =5	dB_5/dr	1280	1280	1320
vr-vz=0	$\mathrm{d}\bar{B}/\mathrm{d}r$	1560	1120	/
vr+vz=5	dB_5/dr	1030	1640	1870
<i>vr</i> +2 <i>vz</i> =7	d^2B_7/dr^2	/	1390	/
<i>vr-2vz=-</i> 2	d^2B_2/dr^2	/	1250	/
2 <i>vr</i> - <i>v</i> z=2	d^2B_2/dr^2	1230	1030	/
2vr+vz=8	d^2B_8/dr^2	/	1730	1790
3 <i>vr</i> =7	d^2B_7/dr^2	1150	1160	1170
3 <i>vr</i> =8	d^2B_8/dr^2	1400	1410	1420
3 <i>vr</i> =10	$d^2 B_{10}/dr^2$	2000	/	1960

This table shows that low-order resonances, such as integer resonances, are the main barriers for the higher energy acceleration, especially for the CW FFA machine. Part of numerical simulation results can be found in Ref [10]. It can be concluded that with the benefit of a strong focusing of the alternating radial gradient, the axial envelope is effectively controlled for the coupled resonances. In summary, also through the multi-particle simulation, the beam envelope does not increase significantly when considering the non-ideal magnetic field, except for the second harmonic field at some particular phases. These simulations show that the lattice structure design of Scheme 2 can tolerate a certain amount of non-ideal magnetic field components and is feasible in magnet construction.

To maintain better isochronism, $v_r = 3$ is crossed but suppressed by IRS mentioned before in Scheme 3. In addition, Scheme 3 crosses Walkinshaw resonance when the beam is almost extracted. Since energy gain per turn is relatively large, it can be verified by numerical simulation that the beam envelop grows only slightly when the amount of off-center is about 1 cm, and half envelop increases to 10 mm at the case of 3 cm off-center, which is shown in Figure 6.



Figure 6: Vertical beam envelop of off-centered beam, $v_r=2v_z$ (up), $2v_r+2v_z=10$ (down).

Higher order resonance $2v_r+2v_z=10$ is also under consideration. It is an inherent resonance in our machine while the highest of magnetic field is 3rd, only small amount of driving term at extraction. Case of 0.5 Gs/m³ is simulated and the phase of B₁₀ field is random. Figure 6 shows that the beam envelop is growing a little due to this resonance.

Space Charge Effect in the FFA Machine

In FFA, it is unclear whether the high-current beam, coupled with strong nonlinear, large gradient magnetic field and multi-bunch space charge effects, produces the same beneficial dynamical behavior for beam acceleration and extraction in the high-intensity cyclotron. Its feasibility has to be verified numerically by high-performance parallel computation.

OPAL-CYCL (the cyclotron flavor of the beam dynamics simulation framework OPAL) is a good candidate for CW FFA design since it allows parallel calculations of large-scale particles and uses the first 3D PIC algorithm [11]. In which the code considers the effects of space charge among neighboring bunches. OPAL introduced the AMR (adaptive mesh refinement) algorithm based on the massively parallel computing framework AMReX in version 2.4.0, which can significantly reduce the 14th Symp. Accel. Phys. ISBN: 978-3-95450-265-3

computational cost and memory consumption while ensuring computational accuracy. We have modified and verified it, found more in-depth problems related to the mesh accuracy division, and found the direction to optimize the algorithm from the essence. However, there is still room for optimizing the AMR algorithm's domain mapping and resetting the mesh distribution frequency.

After the present analysis, we can conclude that it is reasonable to set a smaller number of grids for directions with smaller beam sizes and more minor variation scales. However, determining the accuracy of such a setting requires larger-scale simulations on a more powerful computer and rigorous theoretical analysis. The isotropic nature of the grid size that the AMR algorithm needs to calculate also makes the ever-changing ratio of the beam size impossible to achieve the best optimization simply by the initial grid size. As shown in Figure 7, for example, when the dimensions or the ratio of the different dimensions of the bundle changes, the dimension that only needs to be divided into two or three grids to maintain the accuracy in the actual region may be mapped to the computational region because the initial setting needs to be divided into four grids, resulting in a waste of computing power.



Figure 7: Schematic diagram of the mapping of a beam of varying size scale onto a fixed computational region.

There are also preliminary results from the multi-particle simulations, specifically for Scheme 3. Figure 8 depicts the outcome of the simulation in the horizontal plane and the vortex effect that is seen. The bunch's initial parameters are: Transverse emittance is 2π mm mrad; phase width is 3°; beam current is 3 mA. In the case of phase width equal to 3°, the vortex motion is incomplete and the initial phase ellipse is not well matched. Moreover, the transverse size of the bunch decreases to around \pm 7 mm at the final turn, making it potentially appropriate for extraction.



Figure 8: Longitudinal and transverse beam shape variation, from left to right, from top to bottom, 1, 16, 31, 46, 61, 80 turns.

The followings are the compared results with and without the space charge effect, with and without flattop cavity in more detail. As compared in the first and second rows of Figure 9, the beam quality is significantly better without a space charge, demonstrating that the space charge effect is primarily responsible for the vortex effect in the CW FFA. It is also indicated in the second row and the third row in Figure 9 that higher beam current results in a more significant space charge effect, which increases the transverse size near ± 1 cm. Nevertheless, with higher energy gain provided by RF cavities and properly arranged extraction elements, turn separation is promisingly increased to 30 mm, which is suitable for extraction.



Figure 9: Longitudinal and transverse beam shape variation, from the 1st column to the 3rd column, 46, 61, 80 turns; from the 1st row to the 3^{rd} row: 0 mA with space charge (green); 3 mA with space charge (blue-green); 6 mA with space charge (yellow).

POSSIBLE SOLUTIONS FOR EXTRACTION

Extraction by Integer Resonance

For high-intensity isochronous accelerators, the critical factor limiting a further increase in current intensity is the beam loss around the extraction region. If the extraction efficiency of the accelerator is not high, crucial components such as the accelerator vacuum pipe and deflector will be bombarded during high-power operation, resulting in activation, which will affect the maintenance of the accelerator. Therefore, improving the extraction efficiency of high-current beams by enlarging the turn separation and reducing the bunch size is a critical issue in the research of extraction simulation for high-energy and high-power accelerators.

The 2 GeV FFA adopts the following two methods to increase turn separation: 1) The layout of long drift sections can add a large number of high-frequency cavities to maximize the energy gain; 2) Off-centered injection can produce precession at the extraction position, increasing the separation of the last turn, but integer resonance needs to be carefully considered.

Scheme 2 utilizes integer resonance for extraction, tunes the working path near the extraction area reasonably, and controls $v_r \approx 3$ resonance to drive radial oscillation. Although the second scheme avoids the problem of crossing integer resonance, there are still some challenges in beam extraction. The reason is that due to the adjustment of the local radial gradient of the magnetic field, the beam is on the longitudinal defocusing phase of the high-frequency field in the last thirty circles, increasing the phase width of the beam, which brings a more significant impact on the extraction of the beam and make it more difficult. After the beam with an initial phase width of 5° is accelerated, the phase width is stretched to about 10°. Increasement of beam phase width will make the extraction effect of precession worse. As shown on the left side of Figure 10, the results of precession extraction in radial phase space indicated large turn separation but the beam size is growing at extraction region.



Figure 10: The precession effect of scheme 2 (left) and the growth of the beam envelope (right).

Extraction by Half Integer Resonance

To avoid growth of beam size and energy dispersion, Scheme 3 is more potential for 3 mA extraction. By adding a B_3 components, radial oscillation of different phases is excited. It realizes a suppression of orbital oscillation and make it possible to cross the integer resonance and utilizing half resonance for extraction. The tolerance of magnetic field error can be improved from 1 Gs to 10 Gs with IRS.

CONCLUSION

Three feasible schemes for 2 GeV/6 MW FFA are found and beam dynamics is introduced and compared in detail. Scheme 2 avoids ν_r =3 resonance using adjustment of 3rd order radial field, which serves as octupole component.

Resonance study is carried out in detail for Scheme 2 and gives tolerance of magnetic field error. Parallel computing included space charge effect is underway and some results supports high-current extraction while the turn separation is potential to \sim 3 cm. Integer and half-integer resonances extraction is studied and provide some possible methods to enlarge the turn separation from 1.5 cm to 3 cm.

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