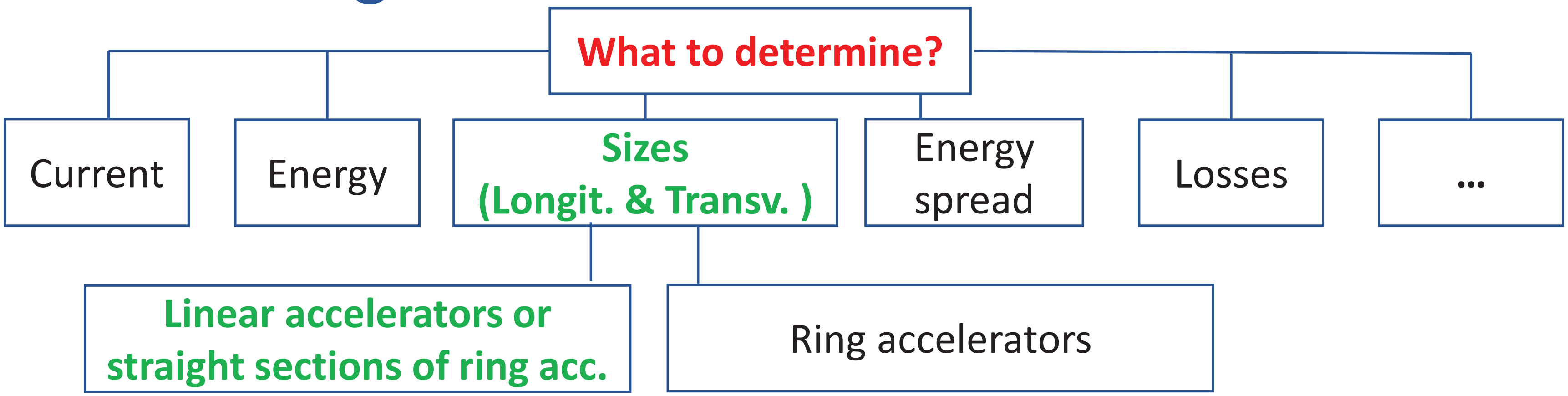
XXVII RUSSIAN PARTICLE ACCELERATOR CONFERENCE RuPAC-2021
26 September – 2 October 2021

# Smith-Purcell radiation in prewave zone for diagnostics of relativistic electron beams


Damir Garaev
Alexey Tishchenko
Daria Sergeeva

# Beam diagnostics




Destructive methods:

- luminescent screens (T)
- profile grid, harp, secondary emission monitor (T)
- laser wire scanner (T)
- Transition radiation (T,L)
- …

Non-destructive methods:

- electro optical sampling (L) – complicated, expensive
- Diffraction radiation (L)
- …

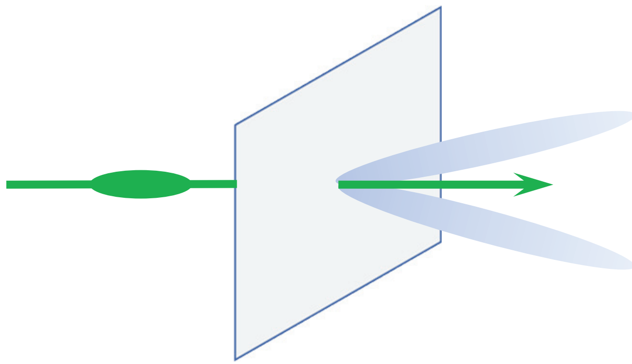
Destructive methods:

- screens (T)
- wire scanners (T)
- …

Non-destructive methods:

- streak camera (L)
- synchrotron radiation (including interferometry) (T, L)
- …

# Beam instrumentation

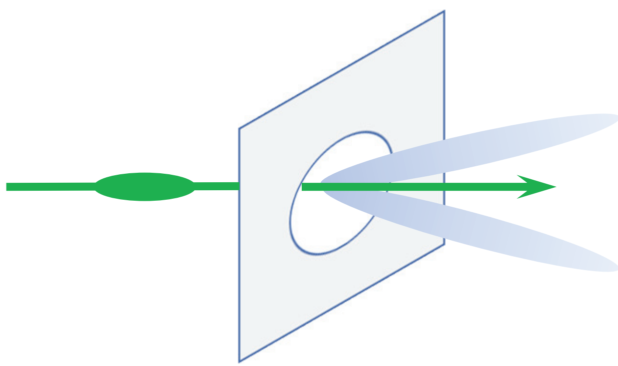


**Coherent Transition Radiation (TR)**

Bunch crosses the foil, excites current, that radiates. Can think of as a reflection of the Coulomb field. Usually generates from angled screen.

Destructive

Widely-used



**Coherent Diffraction Radiation (DR)**

"Advanced TR": similar to TR but with a hole in a screen. Nature and properties similar to TR.

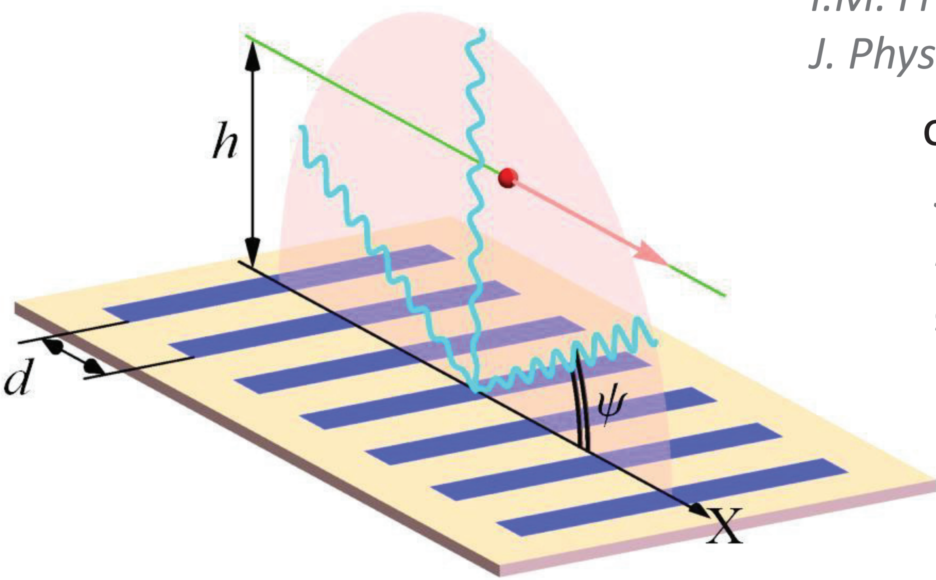
Non-destructive

Also **Smith-Purcell radiation (SPR)** – similar to DR, but from diffraction grating

Monochromatic, directed (large angles to the beam trajectory).

Non-destructive

# Smith-Purcell radiation




predicted

*I.M. Frank, Doppler effect in a refractive medium, J. Phys. U.S.S.R.* ***2*** *(1943).*

observed

*S.J. Purcell and E.M. Smith, Visible Light from localized surface charges moving across a grating, Phys. Rev.* ***92*** *(1953).*

$$d\left(\beta^{-1} - \cos\psi\right) = s\lambda$$

- monochromaticity
- wide spatial distribution

Applications:

- radiation source (orotron)
- beam diagnostics

*F. S. Rusin and G. D. Bogomolov, The orotron, an electronic device with an open resonator and a reflecting grating, Izvestiya VUZ. Radiofizika* ***11*** *(1968)*

# Time profile measurement with Smith-Purcell radiation

$$\left(\frac{dI}{d\Omega}\right)_{N_e} = \left(\frac{dI}{d\Omega}\right)_1 \left(N_e S_{\text{inc}} + N_e^2 S_{\text{coh}}\right)$$

$$\left(\frac{dI}{d\Omega}\right)_{N_e} \cong \left(\frac{dI}{d\Omega}\right)_1 N_e^2 S_{\text{coh}} \approx \left(\frac{dI}{d\Omega}\right)_1 N_e^2 \left|\int_{-\infty}^{+\infty} T e^{-i\omega t} dt\right|^2$$

and setting $\left|\int_{-\infty}^{+\infty} T e^{-i\omega t} dt\right|^2 \equiv \rho^2(\nu),$ where $\nu = \frac{2\pi}{\omega}$

$$\left(\frac{dI}{d\Omega}\right)_{N_e} \cong \left(\frac{dI}{d\Omega}\right)_1 N_e^2 \rho^2(\nu)$$

H. L. Andrews, et al., Reconstruction of the time profile of 20.35 GeV, subpicosecond long electron bunches by means of coherent Smith-Purcell radiation, PR STAB (2014)

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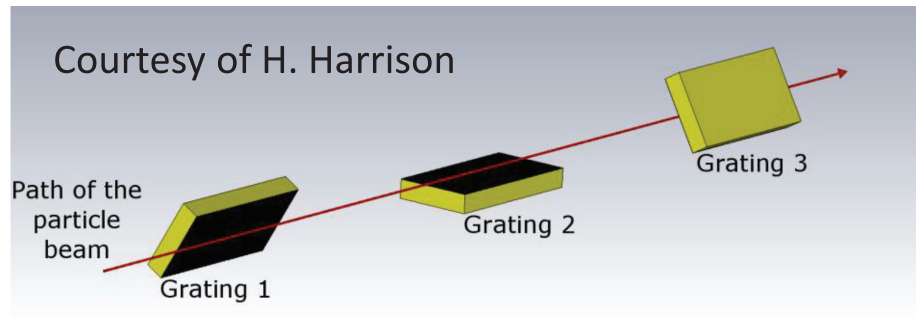


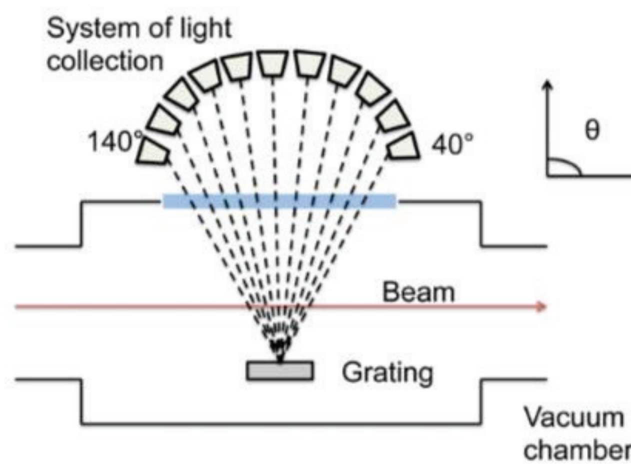

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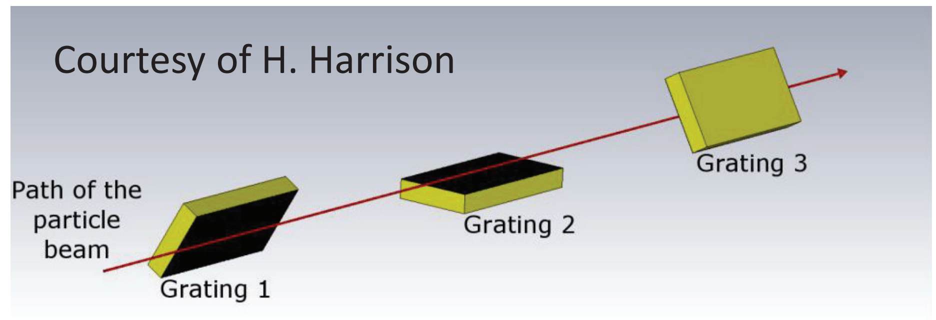
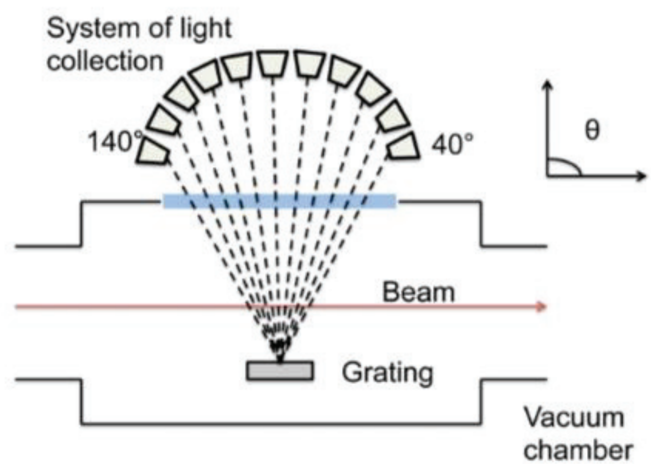


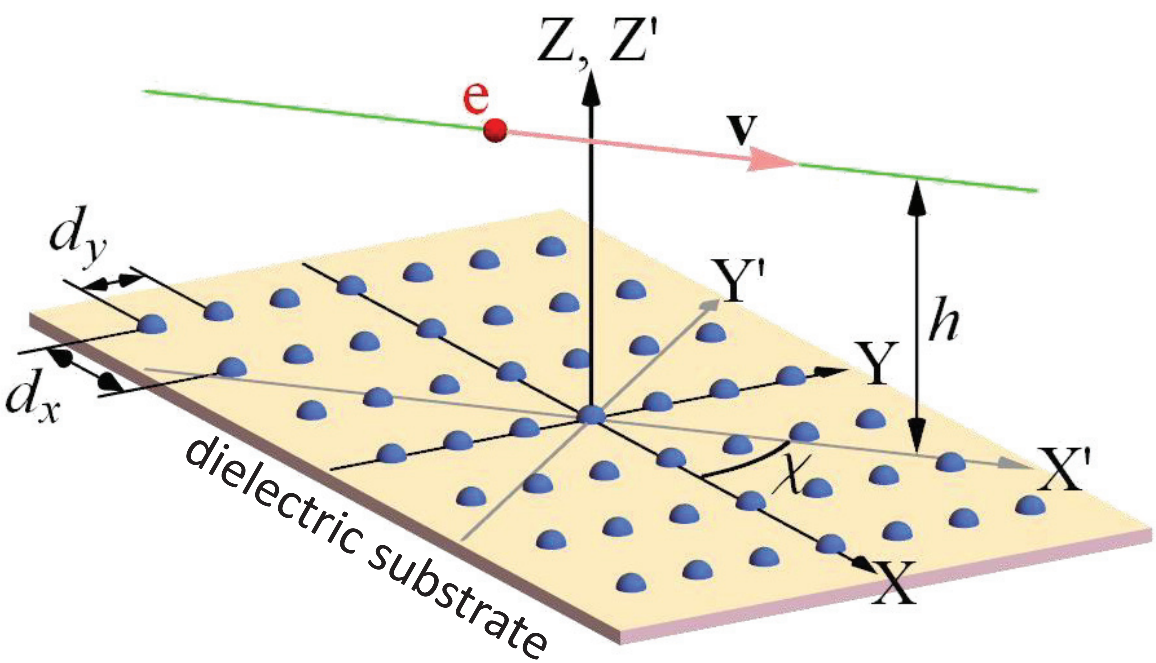



# 2D diffraction lattice or metastructure or photonic crystal

Z. Wang, et al., PRL (2016)

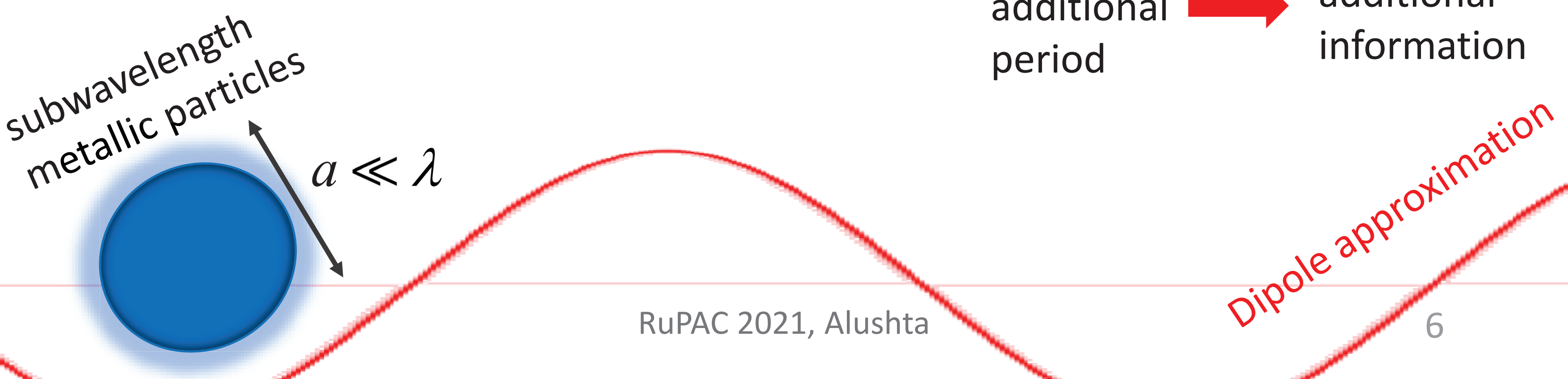
C. Roques-Carmes, et al., Nature Communication (2019)

Z. Su, et al., ACS Photonics (2019)

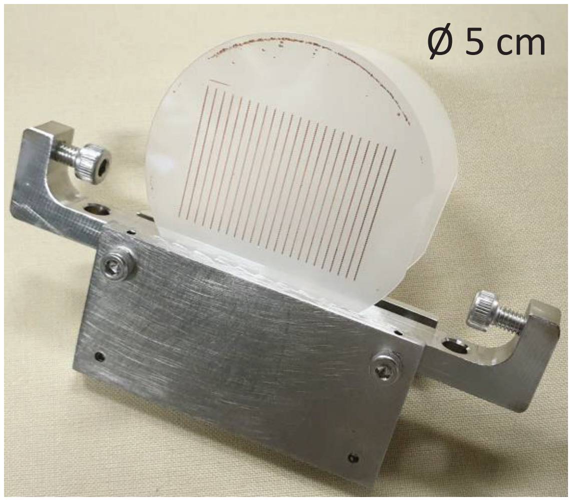


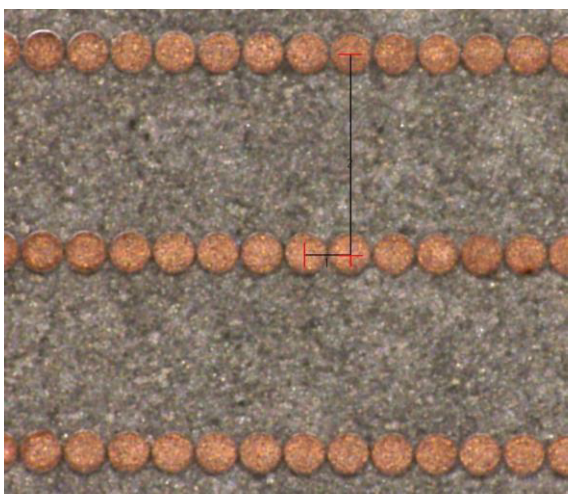

small particles → fully analytical theory

additional period → additional information



# 2D diffraction grating - real



**manufactured at the MEPhI Nanocenter**

- Copper on monocrystalline Sapphire (500 μm)
- Ti/Cu seed layer (400 nm)
- Copper (8 μm) was electroplated from sulfate electrolyte
- Laser Direct Imaging was used
- Truncated cones of
- 300±7 μm in bottom Ø and 150±1 μm in top Ø
- Period: 325.6-330.9 μm
- Second period: 1511.9 μm

D.Yu. Sergeeva, A.S. Aryshev, A.A. Tishchenko, K.E. Popov, N. Terunuma, and J. Urakawa, THz Smith–Purcell and grating transition radiation from metasurface: experiment and theory, Optics Letters (2021)

# Main analytical findings

$$\mathbf{E}^{rad}(\mathbf{r},\omega)\Big|_{wavezone} =$$
$$= \alpha(\omega)\frac{e\omega}{\pi c^2\beta^2\gamma}\frac{e^{ikr}}{r}\sum_m e^{id_x m_x \varphi_x} e^{id_y m_y \varphi_y}\left[\mathbf{k},\left[\mathbf{k},\mathbf{P}_m\right]\right]$$

$$\varphi_x = \frac{\omega v_x}{v^2} - k_x, \qquad \varphi_y = \frac{\omega v_y}{v^2} - k_y,$$
$$\mathbf{P}_m = \frac{i}{\gamma}\frac{\mathbf{v}}{v}K_0\left(L_m\frac{\omega}{v\gamma}\right) + \frac{\mathbf{L}_m}{L_m}K_1\left(L_m\frac{\omega}{v\gamma}\right),$$
$$\mathbf{L}_m = \left[\mathbf{v},\left[\mathbf{v},\mathbf{R}_m - h\mathbf{e}_z\right]\right]/v^2.$$

Dispersion relation X2

$$\lambda s_x = d_x\left(\frac{\beta_x}{\beta^2} - n_x\right)$$
$$\lambda s_y = d_y\left(\frac{\beta_y}{\beta^2} - n_y\right)$$

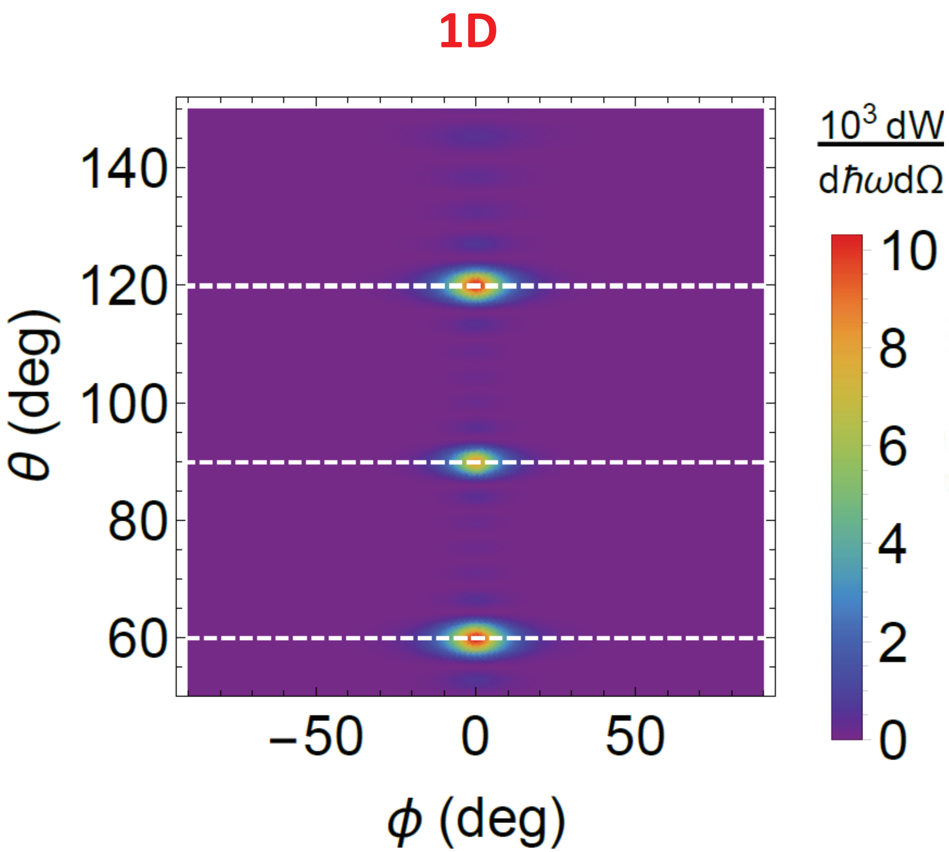
$$d_y = 0$$
$$\beta_y = 0$$

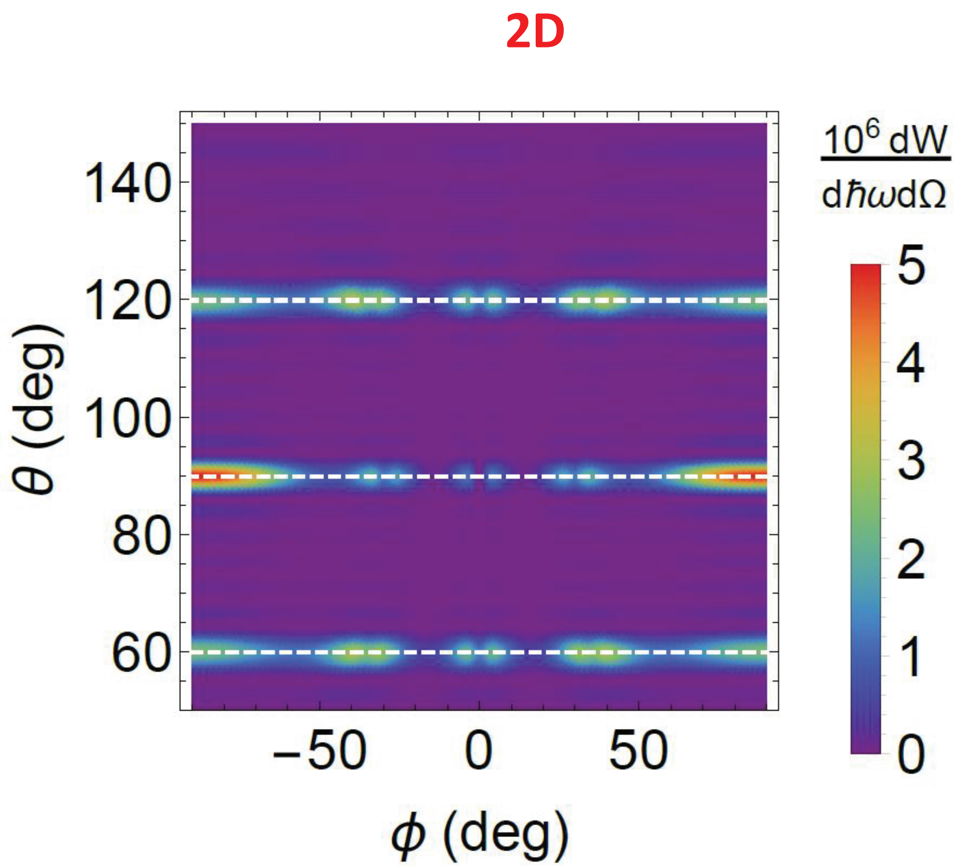
Conventional dispersion relation

$$d\left(\beta^{-1} - \cos\psi\right) = s\lambda$$

*D.I. Garaev, D.Yu. Sergeeva, and A.A. Tishchenko, Theory of Smith-Purcell radiation from a 2D array of small noninteracting particles, Phys. Rev. B* ***103*** *(2021)*

# Numerical calculations

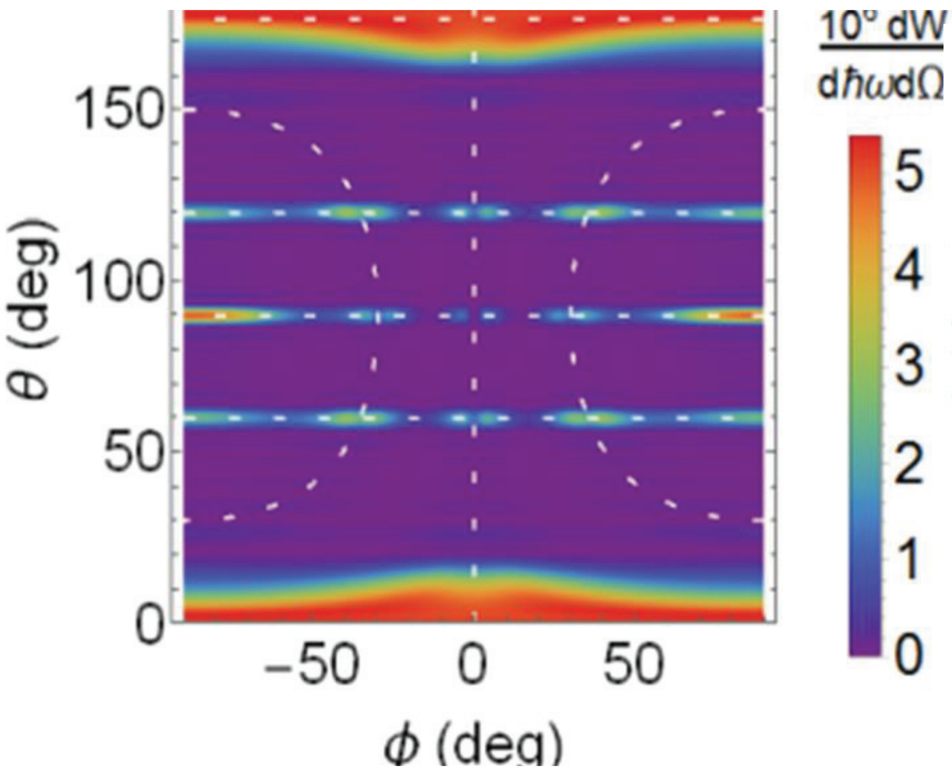


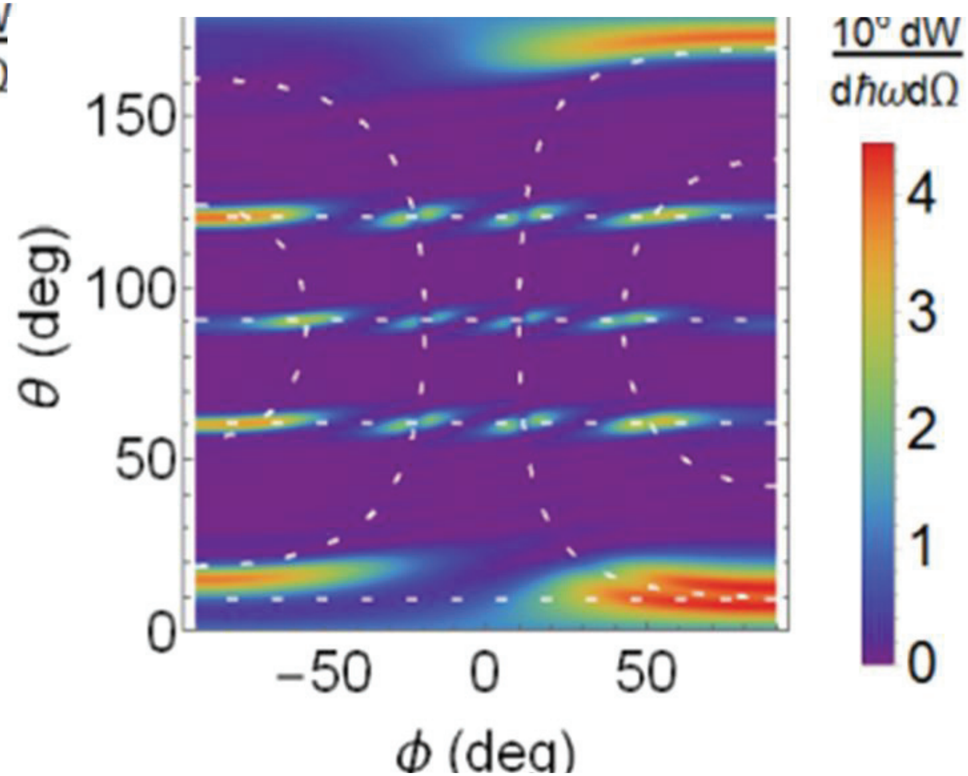

*D.I. Garaev, D.Yu. Sergeeva, and A.A. Tishchenko, Theory of Smith-Purcell radiation from a 2D array of small noninteracting particles, Phys. Rev. B **103** (2021)*

# Numerical calculations

$\chi = 0^\circ$

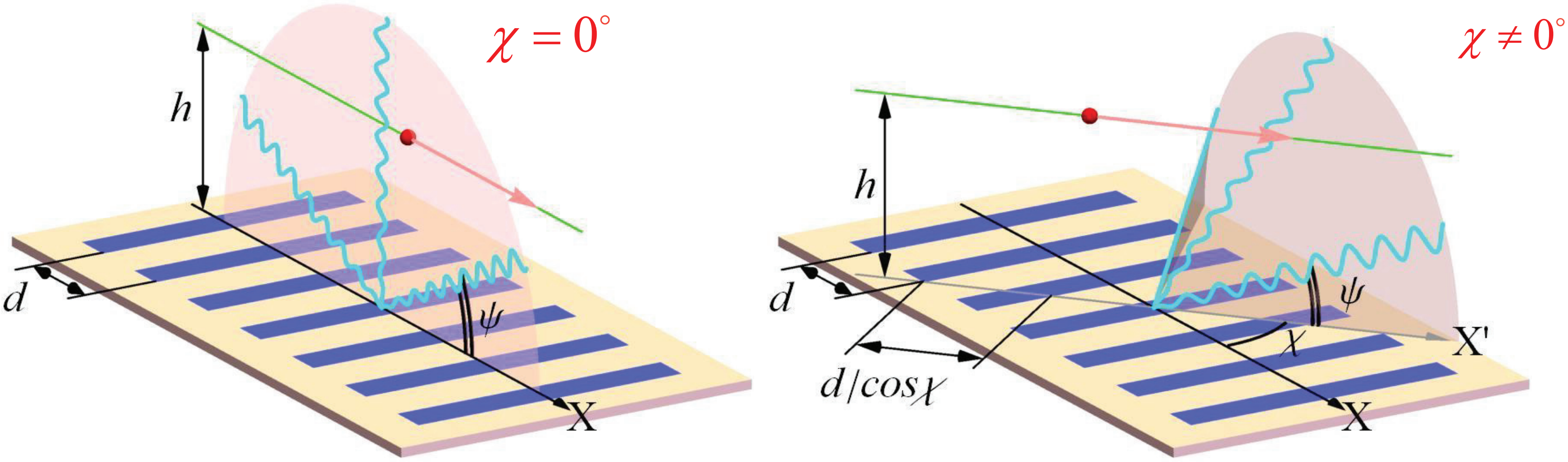



$\chi = 10^\circ$




*D.I. Garaev, D.Yu. Sergeeva, and A.A. Tishchenko, Theory of Smith-Purcell radiation from a 2D array of small noninteracting particles, Phys. Rev. B* ***103*** *(2021)*

# Conical effect




predicted

*D. Y. Sergeeva, A. A. Tishchenko and M. N. Strikhanov, Conical diffraction effect in optical and x-ray Smith-Purcell radiation, Phys. Rev. Accel. Beams **18** (2015)*

and verified experimentally

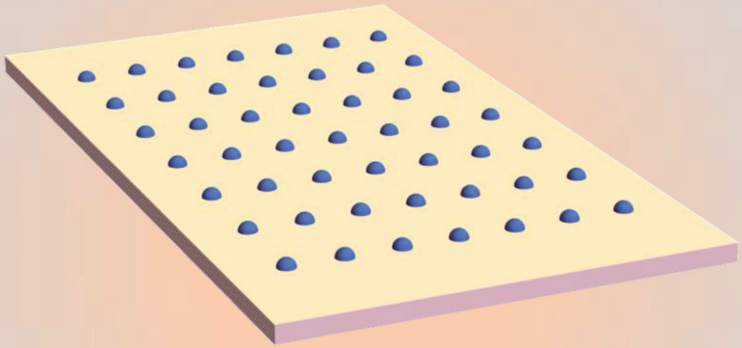
*G.A. Naumenko, A.P. Potylitsyn, D.Yu. Sergeeva, A.A.Tishchenko, M.N. Strikhanov, V.V. Bleko, First experimental observation of conical effect in Smith–Purcell Radiation, JETP Letters (2017)*

# Prewave zone

wave zone
(spherical wave from a point like source)
$$r \gg \Re_{cr} = L^2/\lambda$$

Most theories are not valid here!

near zone $r \sim \lambda$
(non-radiation nature)



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$$r \gg \Re_{cr} = L^2/\lambda$$

for TR $\Re_{cr} = \gamma^2 \lambda$

$$\left.\begin{aligned} E &= 200 MeV \\ \lambda &= 1 mm \end{aligned}\right\} \Rightarrow r \gg 160\, m$$

*V.A. Verzilov, Transition radiation in the pre-wave zone, Physics Letters A **273** (2000)*

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$r \sim L$

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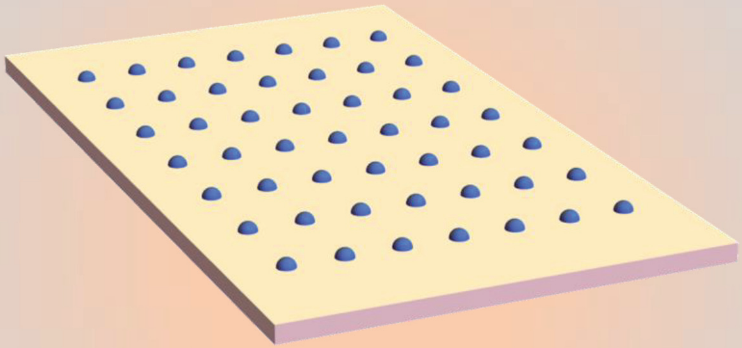

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Most theories are not valid here!

prewave zone

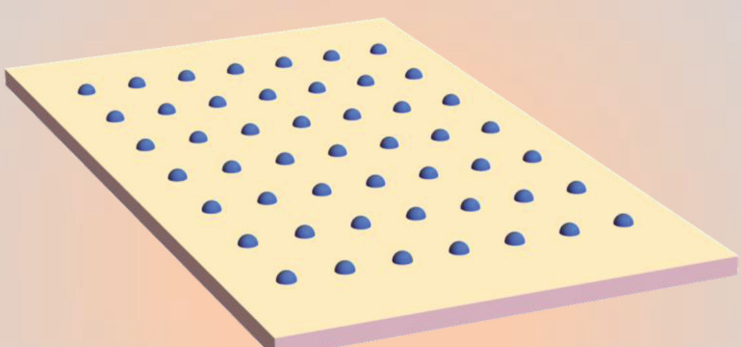
$r \gg \lambda$

$r \sim L$

near zone $r \sim \lambda$
(non-radiation nature)

for SPR $\Re_{cr} = \max\left(L^2/\lambda, \gamma^2 \lambda\right)$

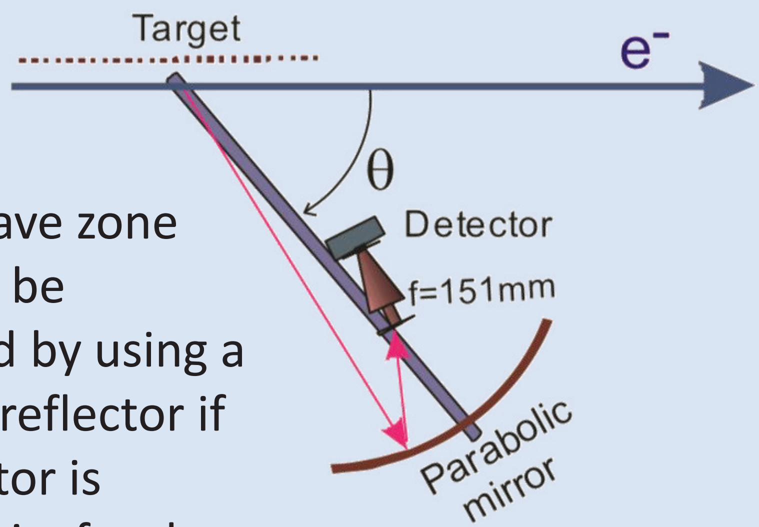
*Д. В. Карловец, А. П. Потылицын, Излучение Смита-Парселла в “предволновой” зоне, Письма в ЖЭТФ (2006)*



# Prewave zone

There are two ways to solve the prewave zone problem

Constructing optic focusing system




The prewave zone effect can be eliminated by using a parabolic reflector if the detector is located in its focal plane

*B.N. Kalinin, et al., JETP Letters (2006)*

*G.A. Naumenko, et al., J. Phys.: Conf. Ser. (2016)*

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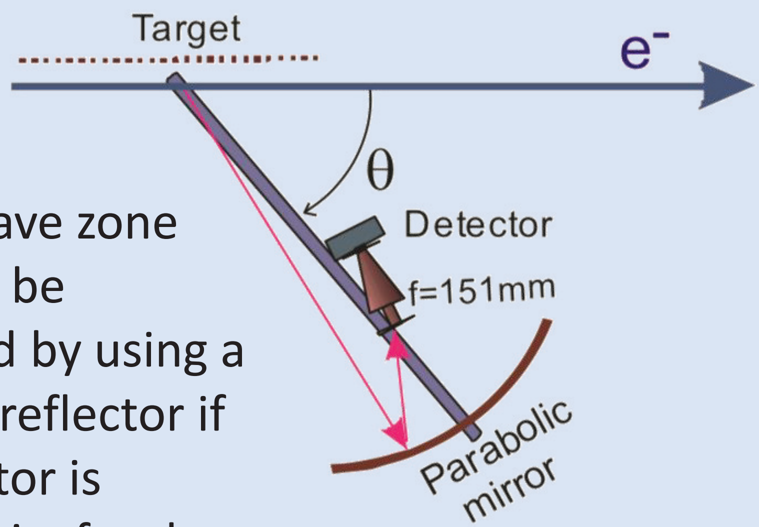


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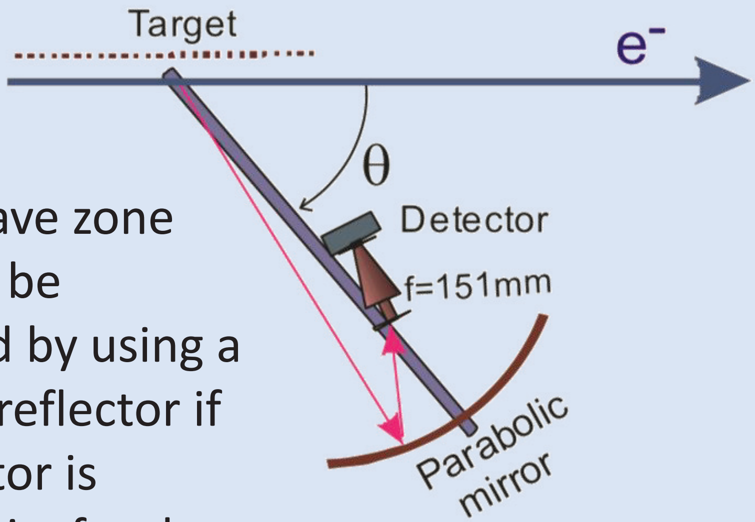
Building prewave zone theory

# Prewave zone

There are two ways to solve the prewave zone problem

Constructing optic focusing system

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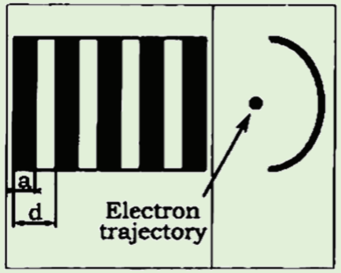
*B.N. Kalinin, et al., JETP Letters (2006)*

*G.A. Naumenko, et al., J. Phys.: Conf. Ser. (2016)*

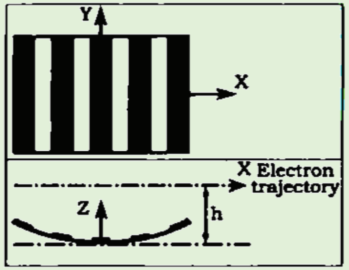
Building prewave zone theory

Focusing effect in a pre-wave zone can be achieved without additional optic devices

azimuthal




polar




*L.G. Sukhikh, et al, PR ST AB (2009)*

*G.A. Naumenko, et al., Proceedings of Channeling (2008)*

# Theory results for the prewave zone

after taking into account values of the second order of smallness:

$$\mathbf{E}^{rad}(\mathbf{r},\omega)\Big|_{prewave\,zone} = \alpha(\omega)\frac{e\omega}{\pi c^2\beta^2\gamma}\times$$

$$\times\sum_m \exp\left(ikr_m - i\omega\frac{\mathbf{v}\mathbf{R}_m}{v^2}\right)\frac{\left[\mathbf{k}_m,\left[\mathbf{k}_m,\mathbf{P}_m\right]\right]}{r_m}$$

$$\mathbf{r}_m = \mathbf{r} - \mathbf{R}_m,$$

$$\mathbf{P}_m = \frac{i}{\gamma}\frac{\mathbf{v}}{v}K_0\left(L_m\frac{\omega}{v\gamma}\right) + \frac{\mathbf{L}_m}{L_m}K_1\left(L_m\frac{\omega}{v\gamma}\right),$$

$$\mathbf{L}_m = \left[\mathbf{v},\left[\mathbf{v},\mathbf{R}_m - h\mathbf{e}_z\right]\right]/v^2.$$

Synphase condition

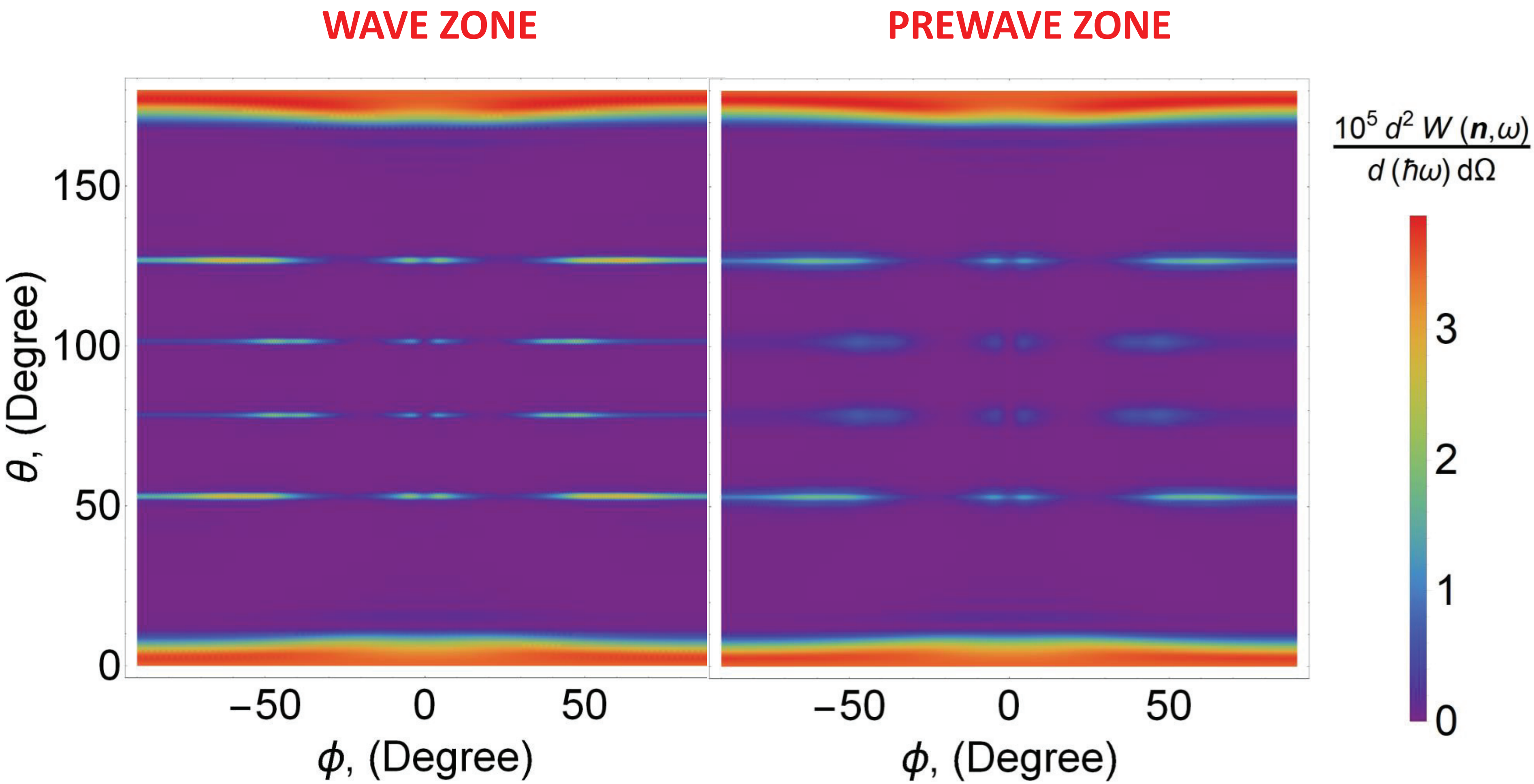
$$\frac{\omega}{v}\left(X_m - X_n\right) + k\left(r_m - r_n\right) = 2\pi s_{mn}$$

$r \to \infty$

Conventional dispersion relation

$$d\left(\beta^{-1} - \cos\psi\right) = s\lambda$$

# Numerical calculations



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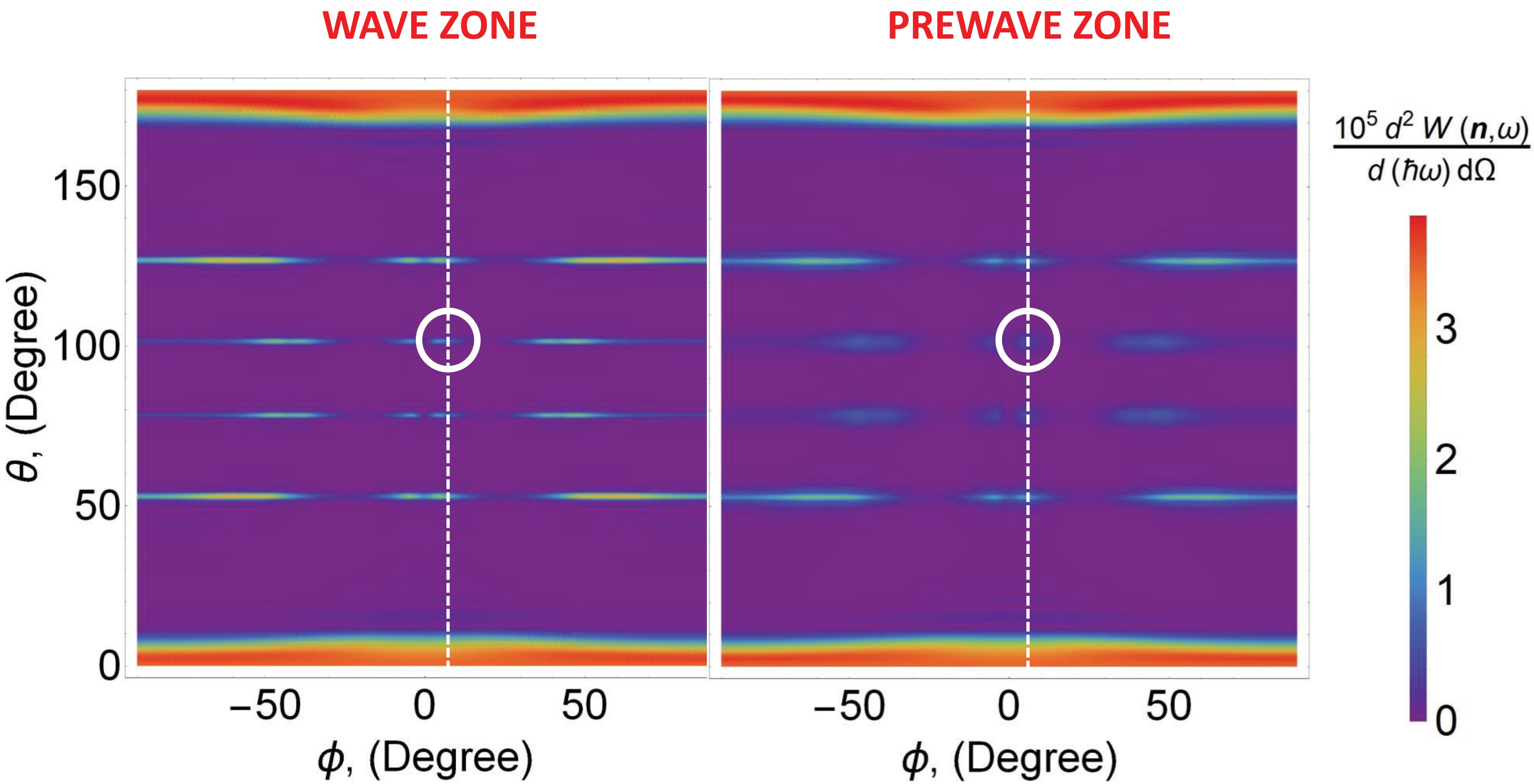
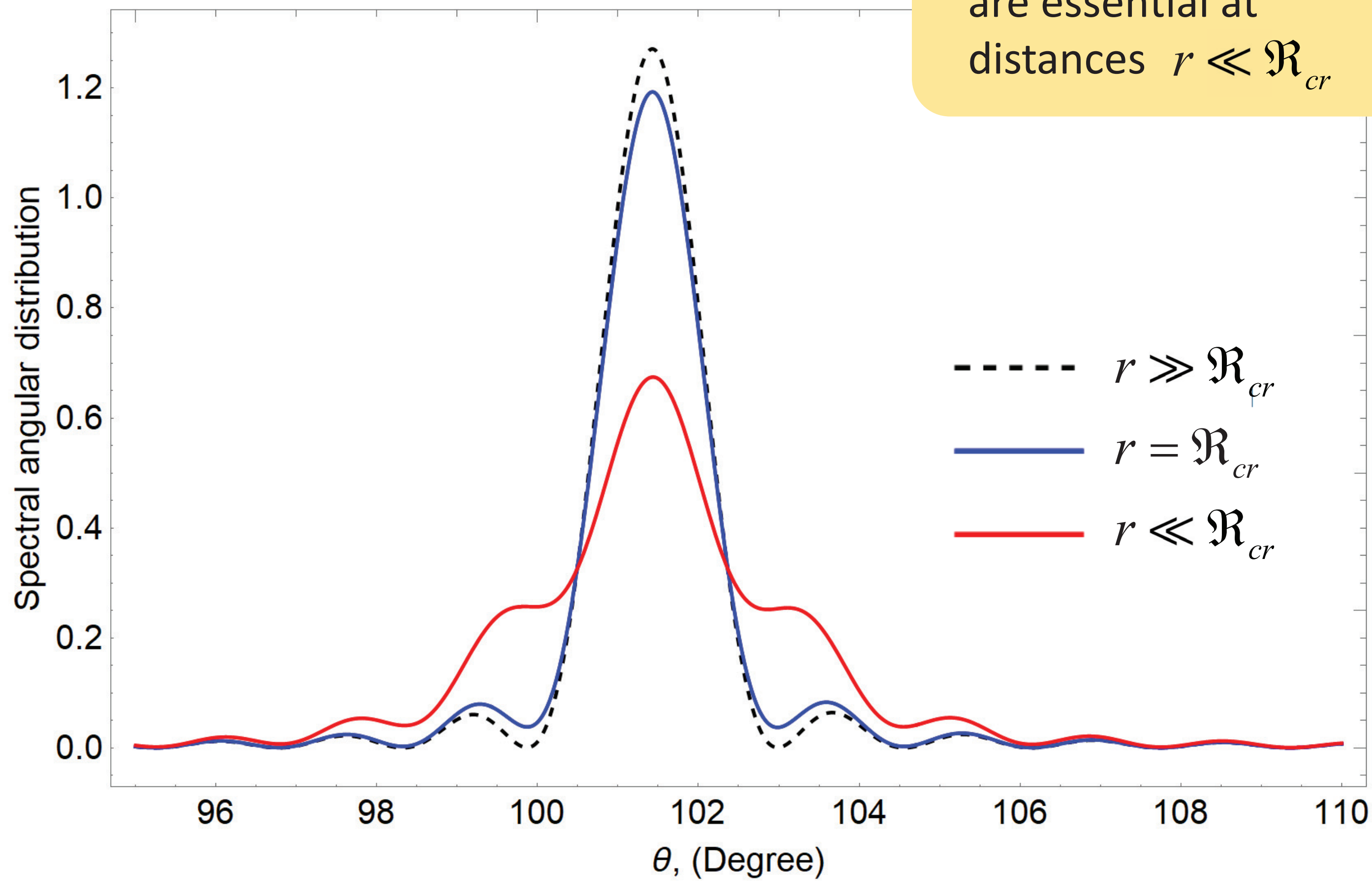


# Numerical calculations

Prewave zone effects are essential at distances $r \ll \Re_{cr}$




coincides with qualitative results from

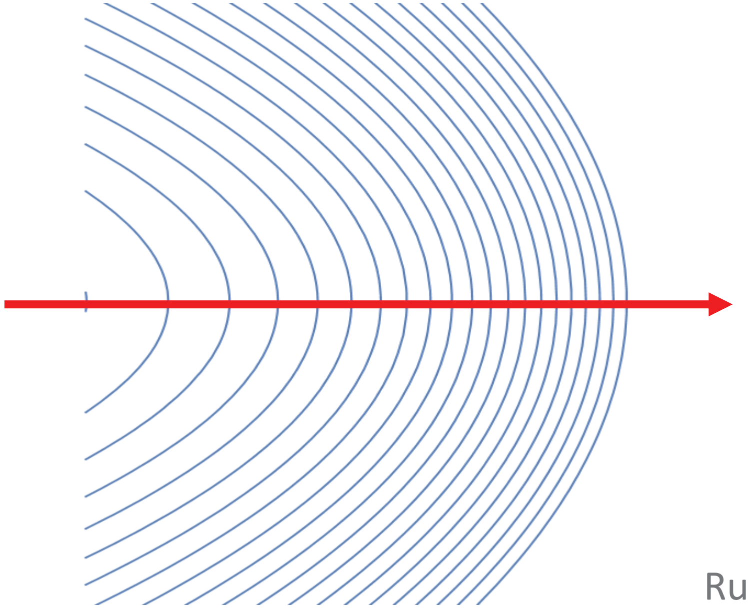
*Д. В. Карловец, А. П. Потылицын, Излучение Смита-Парселла в "предволновой" зоне, Письма в ЖЭТФ (2006)*

# Hyperboloid grating

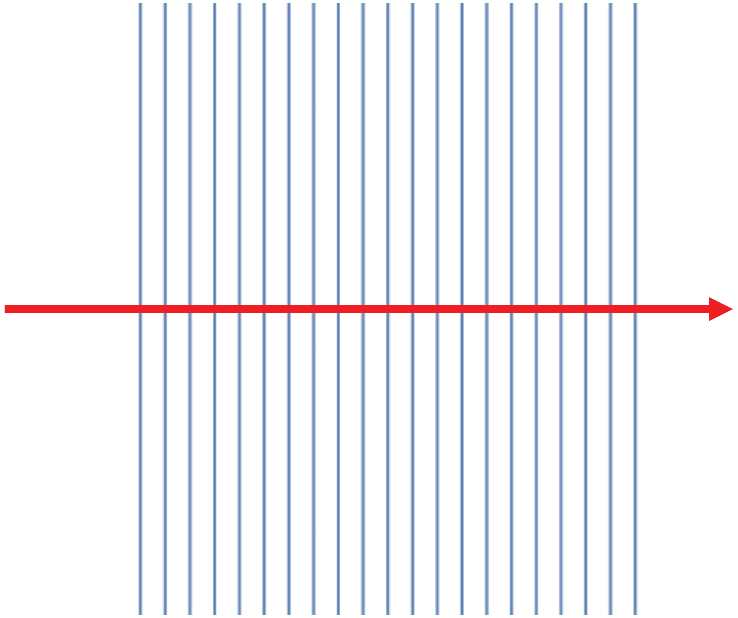
considering synphase condition as an equation for the grating spatial construction

$$\frac{\omega}{v}\left(X_m - X_n\right) + k\left(r_m - r_n\right) = 2\pi s_{mn}$$

brings us to hyperboloid gratings



At $r \to \infty$ the hyperbolic equation degenerates into a linear one



# Coherent diagnostics

Radiation from a beam: $\left(\frac{dI}{d\Omega}\right)_{N_e} = \left(\frac{dI}{d\Omega}\right)_1 F$

form-factor of a beam (contains beam sizes) ← $F$

radiation from a single electron ← $\left(\frac{dI}{d\Omega}\right)_1$

$$F = N_e S_{\text{inc}}\left(r_{tr}, \lambda\right) + N_e^2 S_{\text{coh}}\left(r_{tr}, l, \lambda\right)$$

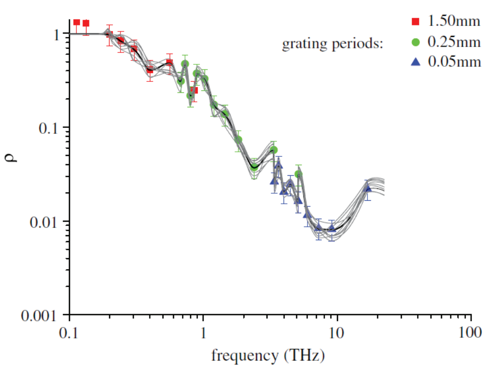
Coherent radiation: $F \approx N_e^2 S_{\text{coh}}\left(l, \lambda\right)$

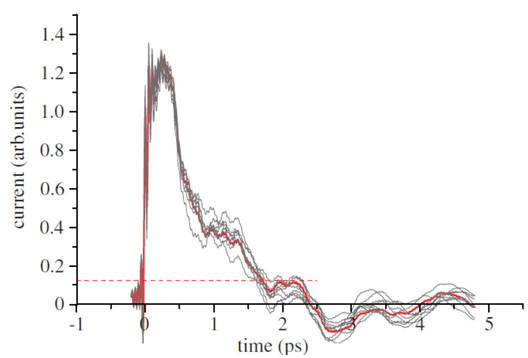
Spectral measurements and Fourier transform

$\left(\frac{dI}{d\Omega}\right)_1$ has to be known!

2D grating,
prewave zone:

$$\left(\frac{dI}{d\Omega}\right)_{N_e} = \frac{dI}{d\Omega}\left(r_{tr}, l\right) \quad ?$$



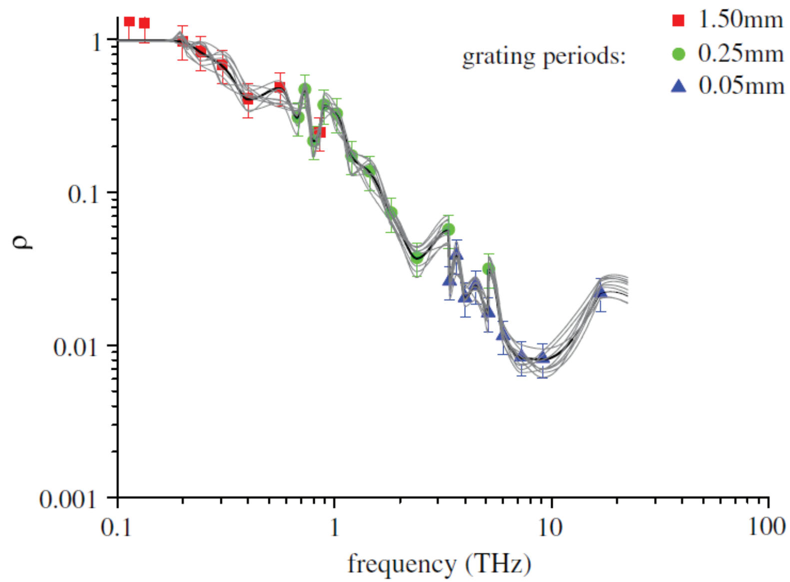
# Conclusions

- In order to improve the diagnostic scheme for the temporal profile of the beam based on SPR, a fully analytical theory was created for a 2D grating in the prewave zone
- Prewave zone effects (broadening of spatial and spectrum distribution) for 2D grating were verified
- It follows from the in-phase condition that for focusing radiation in the pre-wave zone, the grating must have a hyperbolic profile
- To develop a beam diagnostics scheme, it is necessary to obtain the form factors of the radiation of a two-dimensional grating in the prewave zone.

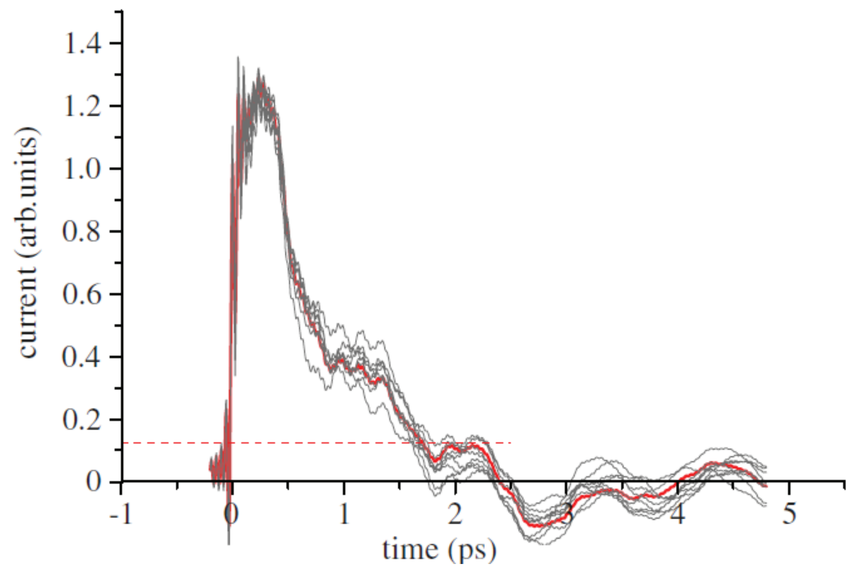
Thanks for your attention!

# Time profile measurement with Smith-Purcell radiation

**Measured spectrum**

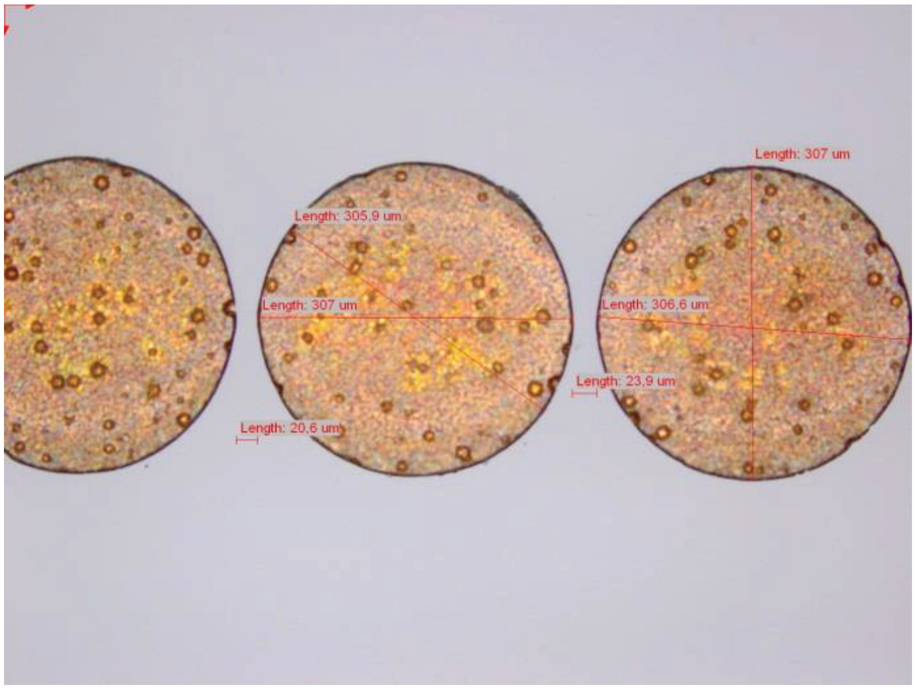



**Reconstructed time profile**




*H. L. Andrews, F. Bakkali Taheri, et al., Reconstruction of the time profile of 20.35 GeV, subpicosecond long electron bunches by means of coherent Smith-Purcell radiation, Phys. Rev. STAB* ***17*** *(2014)*

# Target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Length: 327,7 um
Length: 330,2 um
Length: 1511,9 um
Length: 307 um
Length: 312,2 um
500 um

# Smith-Purcell radiation from metastructure

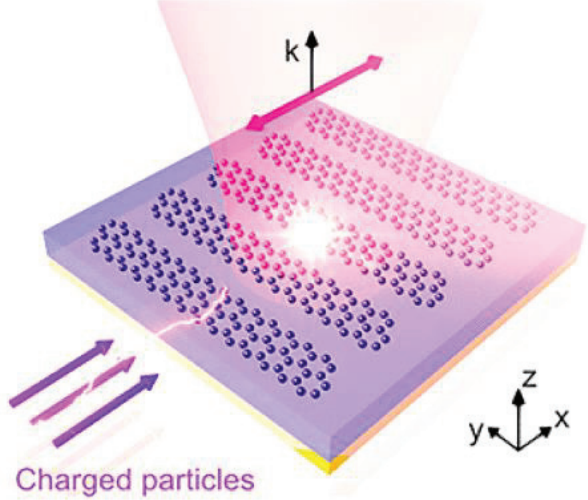
Z. Wang, et al., Manipulating Smith-Purcell Emission with Babinet Metasurfaces, PRL (2016)

L. Liu, et al., Terahertz and infrared Smith-Purcell radiation from Babinet metasurfaces: Loss and efficiency, PRB (2017)

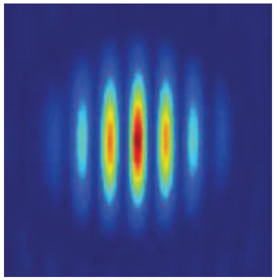
Y. Song, et al., Efficient terahertz and infrared Smith–Purcell radiation from metal-slot metasur- faces, Optics Letters (2018)

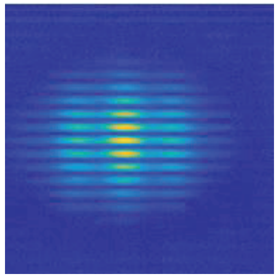
C. Roques-Carmes, et al., Towards integrated tun- able all-silicon free-electron light sources, Nature Communication (2019)

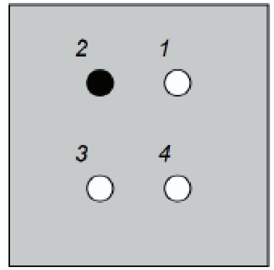
Z. Su, et al., Complete Control of Smith-Purcell Radiation by Graphene Metasurfaces, ACS Photonics (2019)



# Two-dimensional Interferometry








A.I. Novokshonov, A.P. Potylitsyn, G. Kube, Two-Dimensional Synchrotron Radiation Interferometry at PETRA III, Proc. of IPAC (2017)

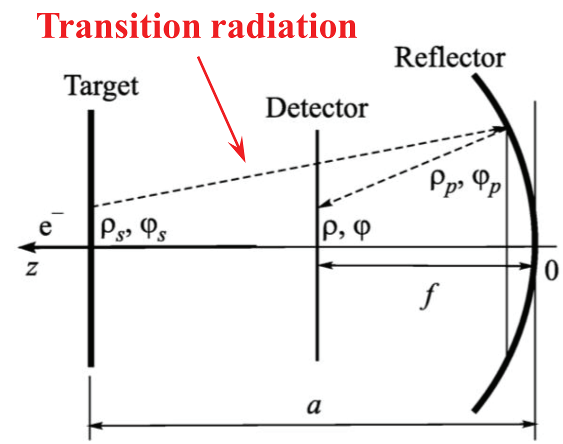
*Implemented idea on SR*

- Two periods of the target → possibility to define two transversal sizes of a beam
- Realization on synchrotron radiation (SR) and polarization radiation (diffraction radiation and Smith-Purcell radiation)
- To define the sizes a theory of radiation from a beam of charged particles is needed

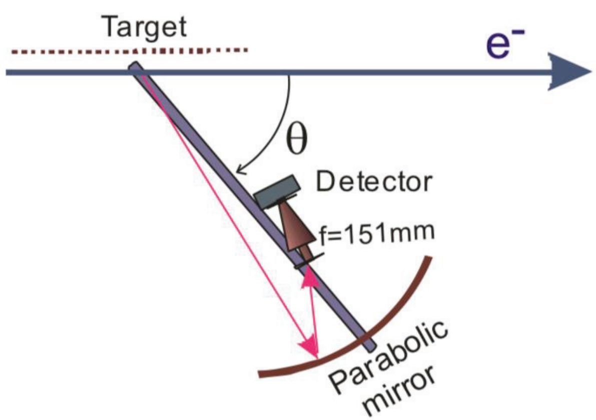
A.I. Novokshonov, A.P. Potylitsyn, G. Kube, 2D Synchrotron Radiation Interferometer for Measuring the Transverse Dimensions of an Electron Beam in a Circular Accelerator, Russian Physics Journal (2017)

# Focusing effect

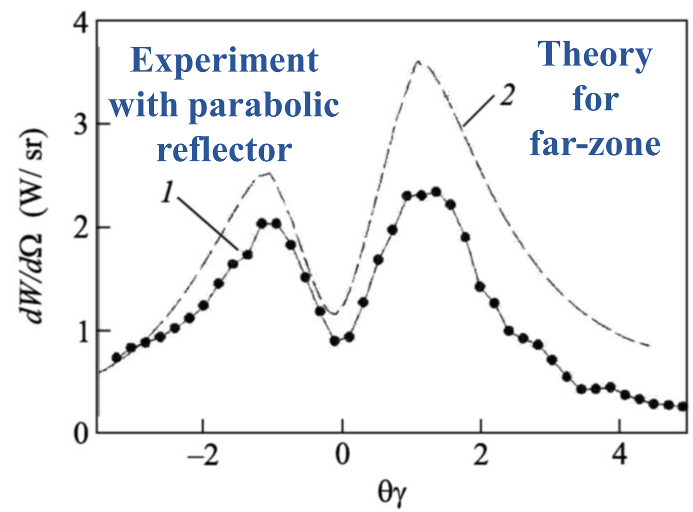
The prewave zone effect can be eliminated by using a parabolic reflector if the detector is located in its focal plane




B.N. Kalinin, G.A. Naumenko, A.P. Potylitsyn, et al. Measurement of the angular characteristics of transition radiation in near and far zones, JETP Letters (2006)




G.A. Naumenko, et al., Characteristics of **Smith-Purcell radiation** in millimeter wavelength region, J. Phys.: Conf. Ser. (2016)



# Focusing effect

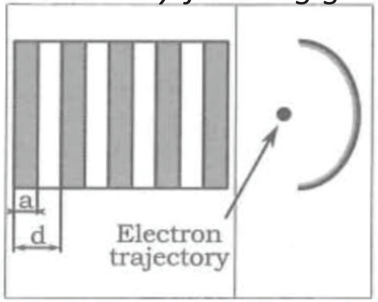
Focusing effect in a pre-wave zone can be achieved without additional optic devicesc

L.G. Sukhikh, et al., Observation of **focusing effect** in optical **transition and diffraction radiation** generated from a spherical target, PR ST AB (2009) (spherical target)

G.A. Naumenko, A.P. Potylitsyn, L.G. Sukhikh, Yu.A. Popov, Experimental Investigation of **Smith–Purcell Radiation** Focusing by Using the **Parabolic Gratings**, Proceedings of Channeling 2008

"SPR intensities from azimuthally- focusing and polar-focusing gratings in pre-wave zone are higher than the ones from the flat grating in corresponding conditions…. One should mention that the radiation intensity maximum is situated at the grating focal distance in the case of polar-focusing grating and at the double focal distance in the case of azimuthally-focusing grating"

*azimuthally-focusing grating*




*polar-focusing grating*