

Precise Analysis of Beam Optics at the VEPP-4M by Turn-by-Turn Betatron Phase Advance Measurement

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Motivation

- Improve current workflow for frequency and optics measurement
- Add TbT data processing, anomaly detection and noise estimation
- Accurate frequency measurement
- Amplitude and phase measurement with error propagation
- Optics measurement from amplitude and phase

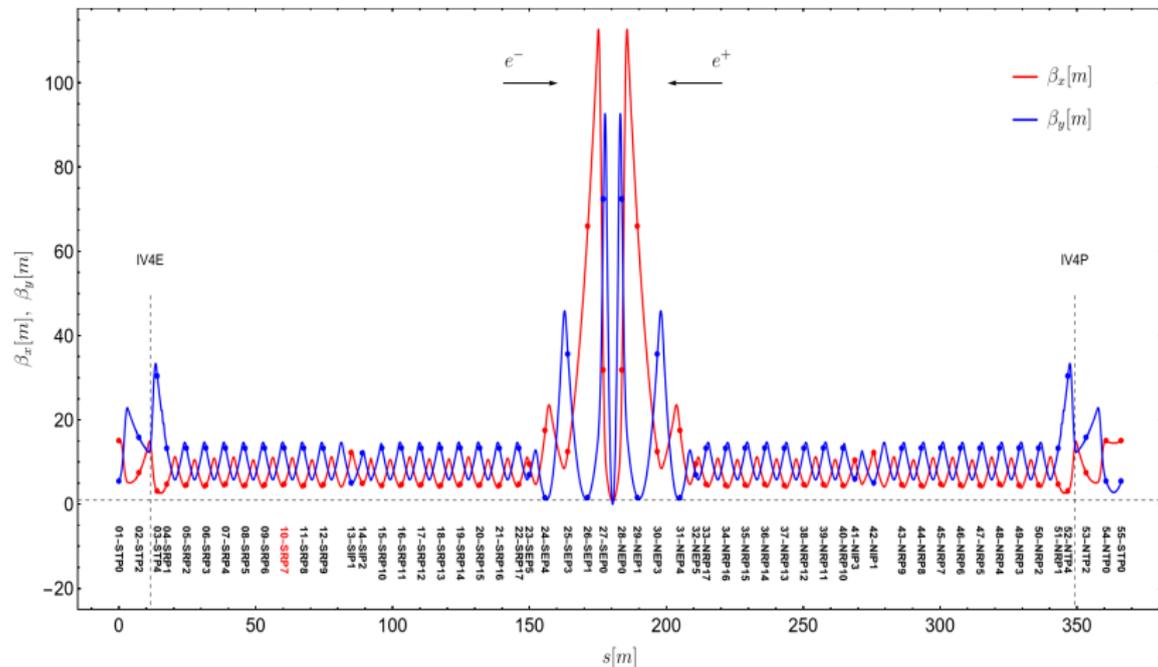
Contents

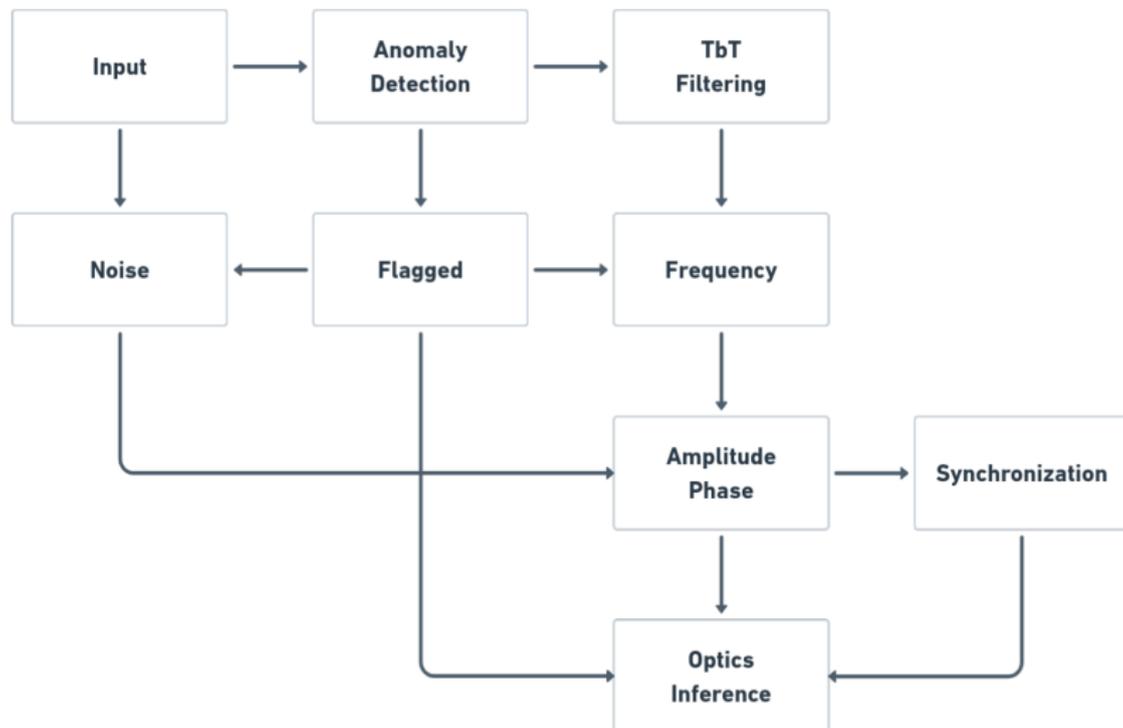
- VEPP-4M BPM system
- TbT analysis workflow
- TbT processing and BPM noise estimation
- Anomaly detection
- Frequency measurement
- Amplitude and phase measurement
- Optics measurement from amplitude and phase
- Conclusion

VEPP-4M BPM System

Bekhtenev, E. A., Karpov, G. V. & Piminov, P. A.

A beam-position monitor system at the VEPP-4M electron-positron collider. Instrum Exp Tech 60, 679–685 (2017)





TbT Processing

TbT cleaning with SVD

$M = U\Sigma V^T \rightarrow$ keep selected number of singular values

Signal representation in the matrix form (Hankel–Prony)

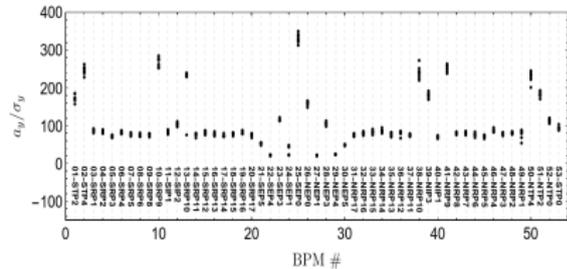
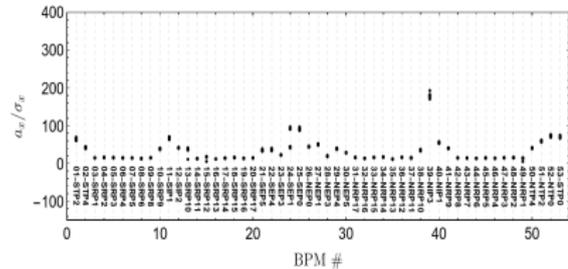
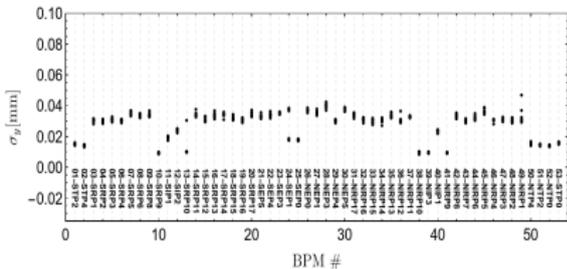
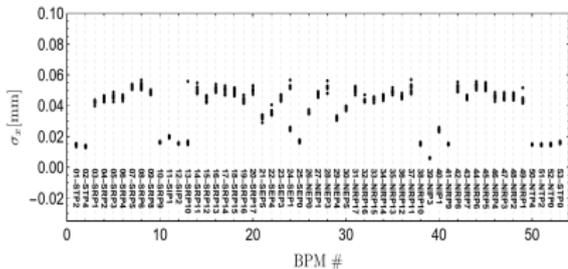
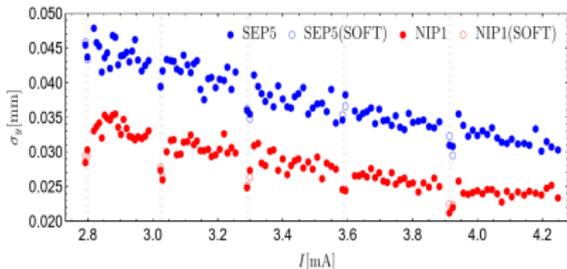
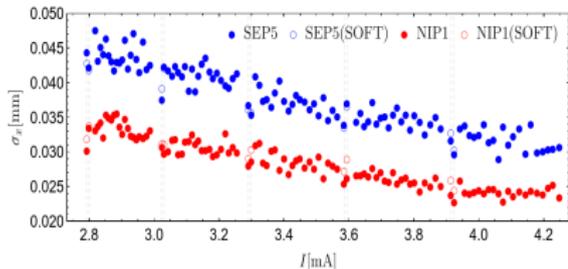
$$S = [s_1 \ s_2 \ s_3 \ s_4 \ s_5 \ s_6 \ s_7 \ s_8] \rightarrow M_S = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2 & s_3 & s_4 & s_5 \\ s_3 & s_4 & s_5 & s_6 \\ s_4 & s_5 & s_6 & s_7 \\ s_5 & s_6 & s_7 & s_8 \end{bmatrix}$$

Individual signals filtering

$$S \rightarrow M_S = U\Sigma V^T \rightarrow \bar{M}_S \rightarrow \bar{S}$$

Optimal SVD truncation

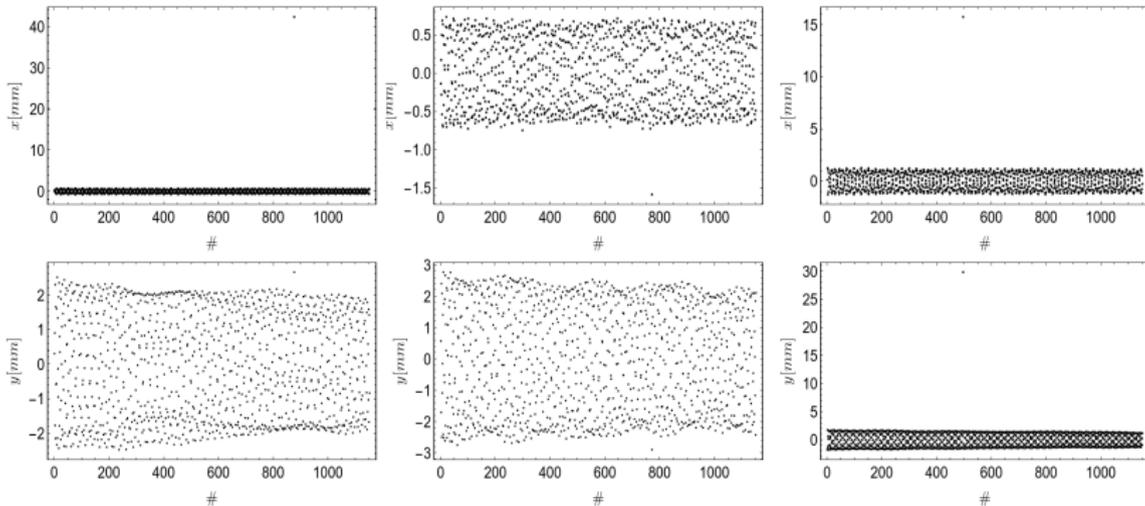
Matan Gavish and David L. Donoho, The Optimal Hard Threshold for Singular Values is $4/\sqrt{3}$, 2014



Anomaly Example

What is an anomaly?

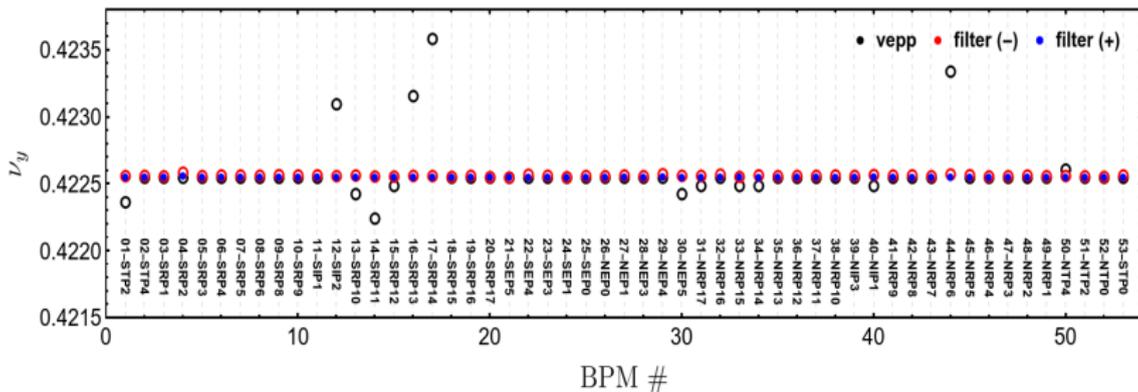
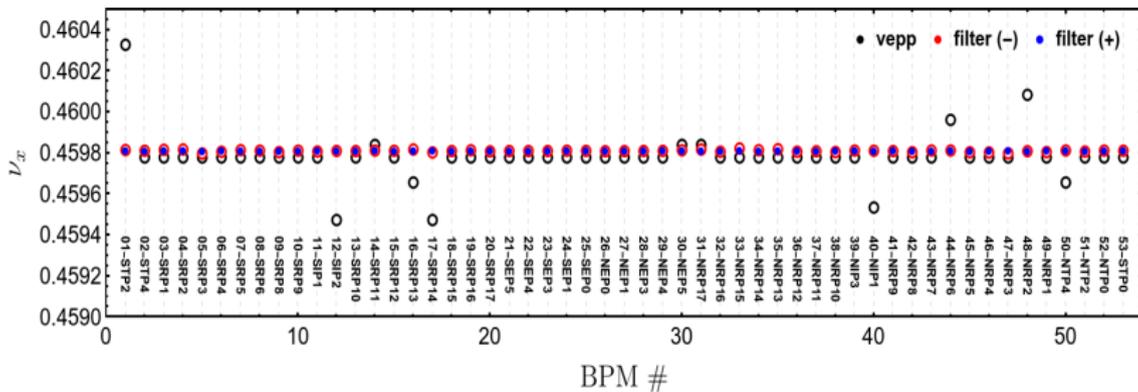
Something that deviates from what is standard, normal, or expected.



How to take anomalies into account?

Remeasure, flag and exclude, use robust estimators.

Frequency Measurement Example



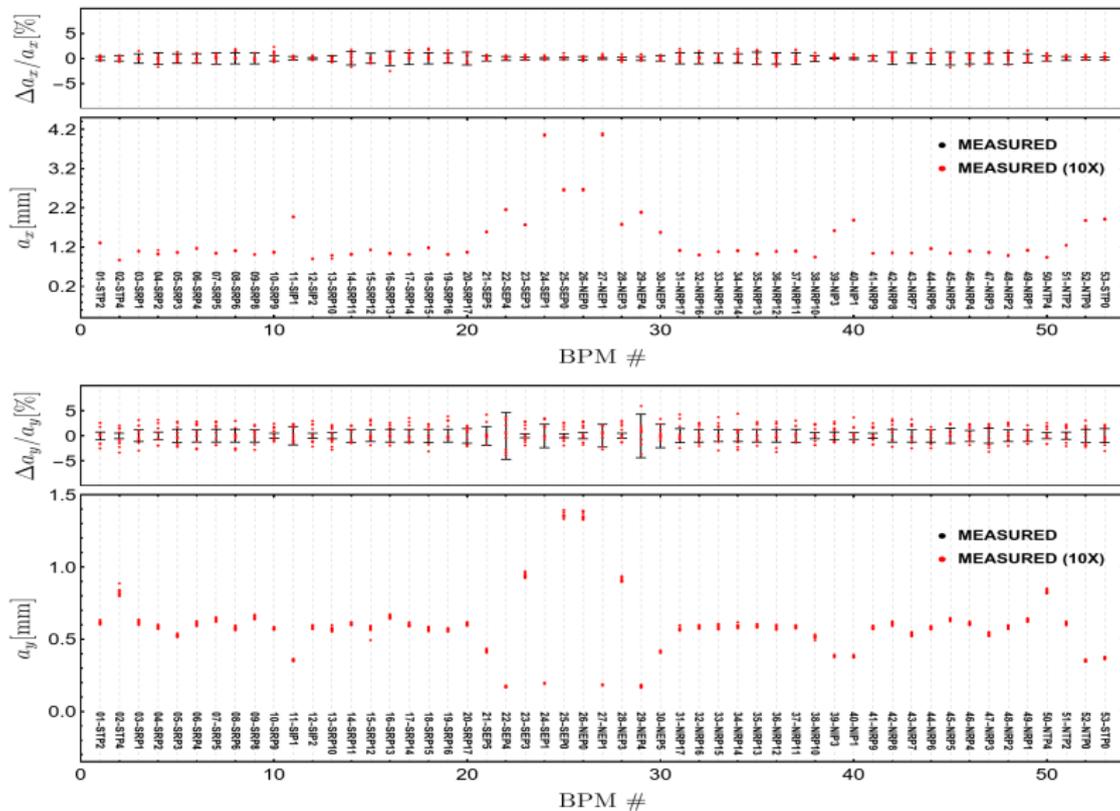
Quasiperiodic decomposition

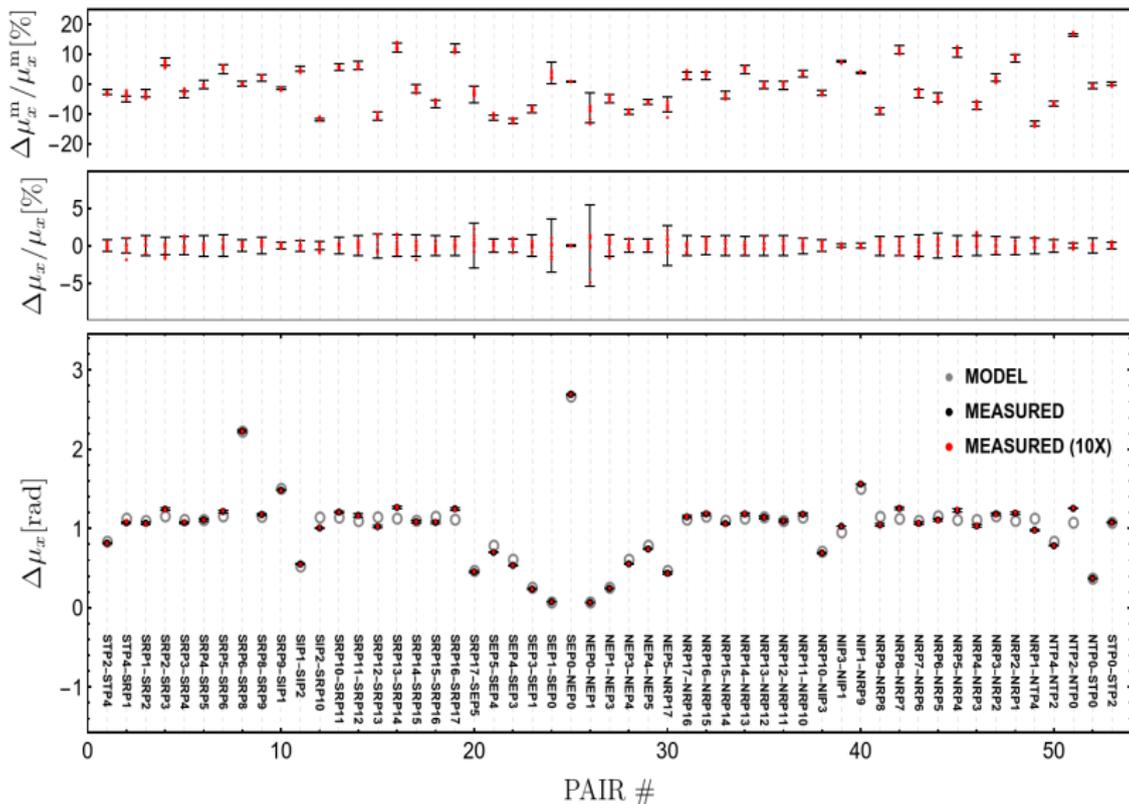
$$x(n) = \sum_{m=1}^M (c_m \cos(2\pi\nu_m n) + s_m \sin(2\pi\nu_m n)) = \sum_{m=1}^M a_m \cos(2\pi\nu_m n + \phi_m)$$

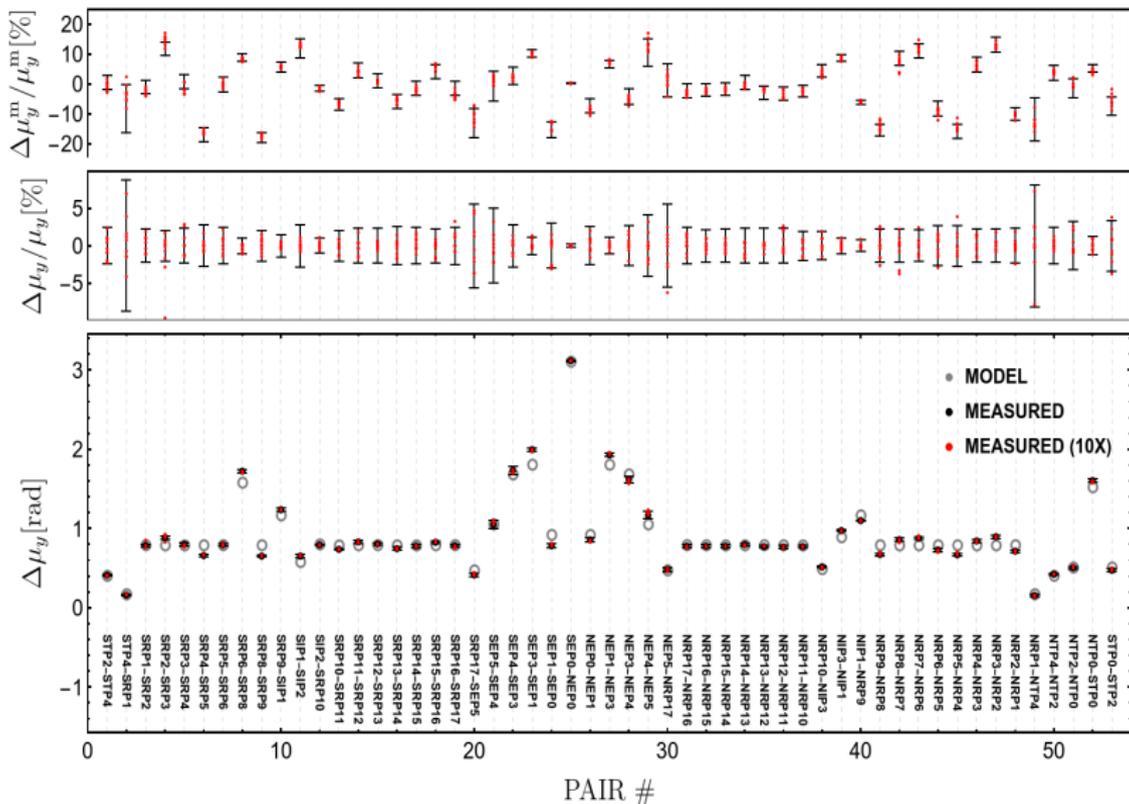
Estimation of parameters

$$c_m = \frac{2}{N} \sum_{n=1}^N x(n) \cos(2\pi\nu_m n), \quad s_m = \frac{2}{N} \sum_{n=1}^N x(n) \sin(2\pi\nu_m n)$$

Amplitude Measurement







Optics Measurement

β^a

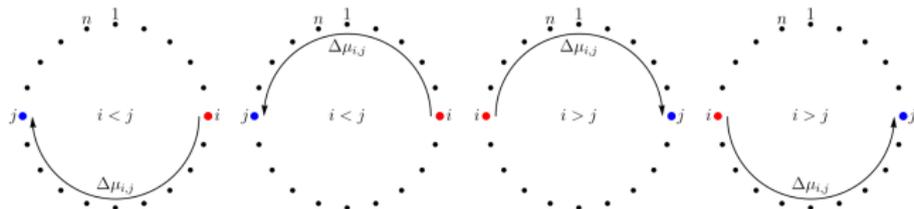
$$x^m(n) = a_x^m \cos(2\pi\nu_x n + \varphi_x^m) = \sqrt{2I_x\beta_x^m} \cos(2\pi\nu_x n + \varphi_x^m)$$

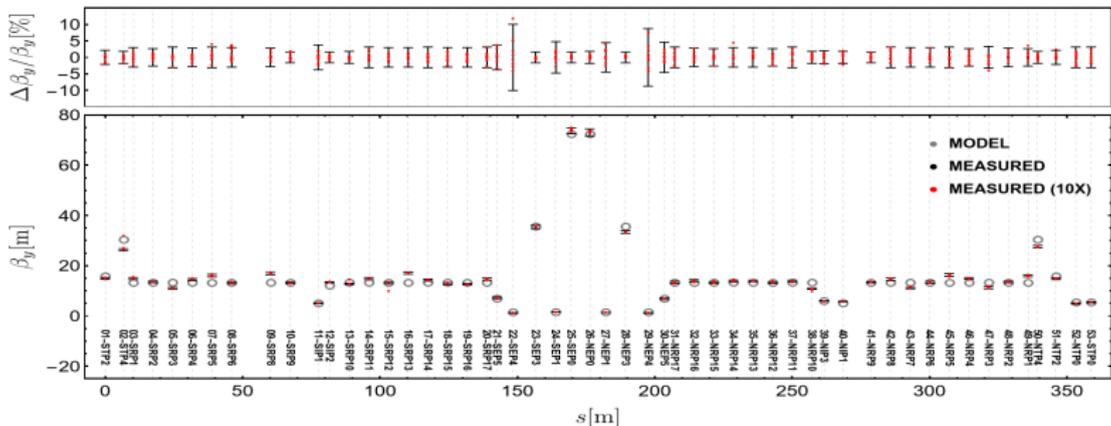
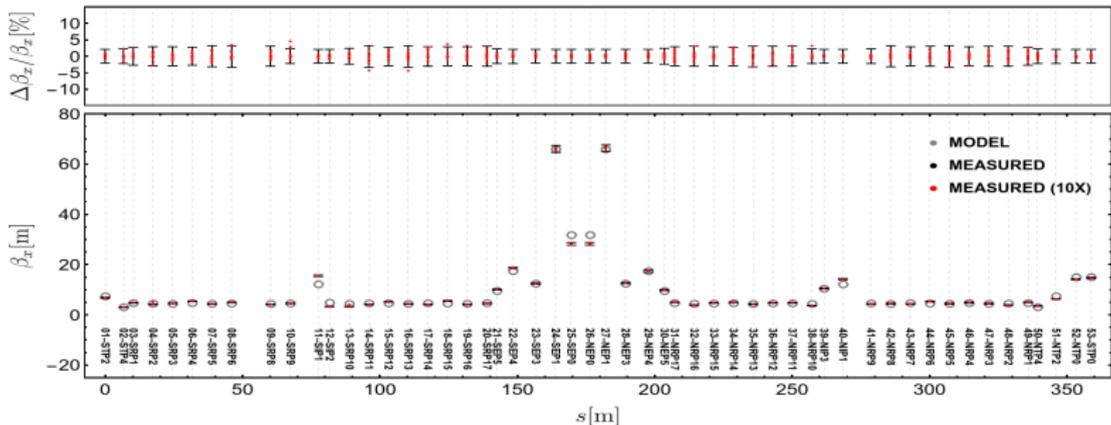
$$\hat{I}_x = \frac{1}{2} \left\langle \frac{a_x^m \cdot a_x^m}{\beta_{x, \text{model}}^m} \right\rangle_m$$

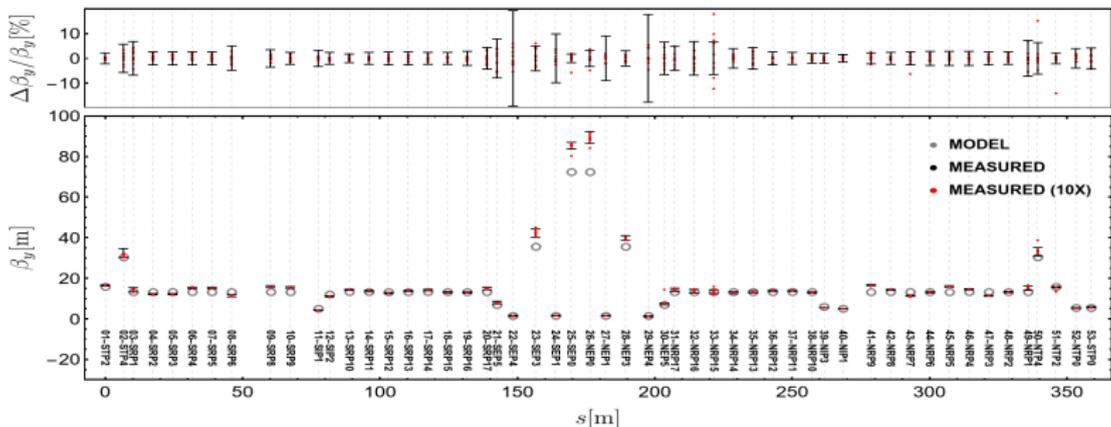
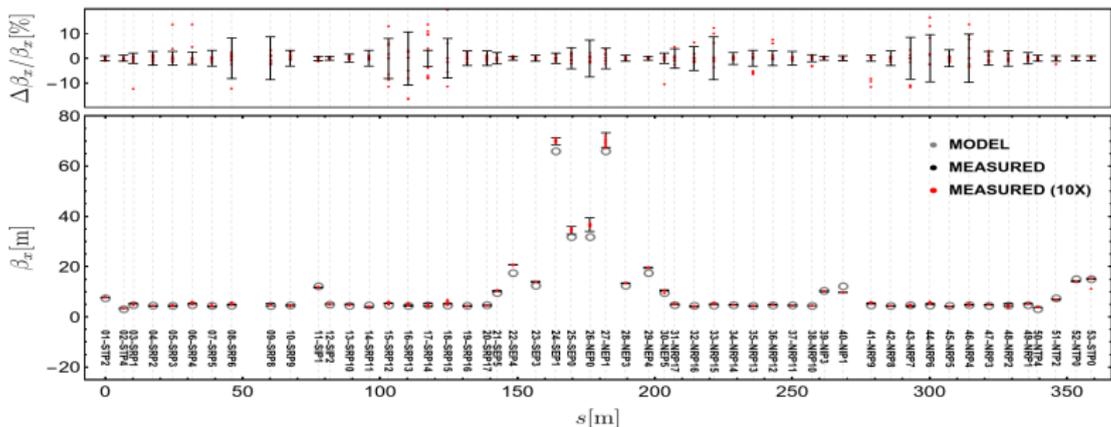
β^φ

$$M_{i,j} = N_j R_{i,j} N_i^{-1}$$

$$\begin{bmatrix} \sqrt{\beta_j} & 0 \\ -\frac{\alpha_j}{\sqrt{\beta_j}} & \frac{1}{\sqrt{\beta_j}} \end{bmatrix} = \begin{bmatrix} m_{1,1}^{(i,j)} & m_{1,2}^{(i,j)} \\ m_{2,1}^{(i,j)} & m_{2,2}^{(i,j)} \end{bmatrix} \begin{bmatrix} \sqrt{\beta_i} & 0 \\ -\frac{\alpha_i}{\sqrt{\beta_i}} & \frac{1}{\sqrt{\beta_i}} \end{bmatrix} R_{i,j}^{-1}(\Delta\mu_{i,j})$$

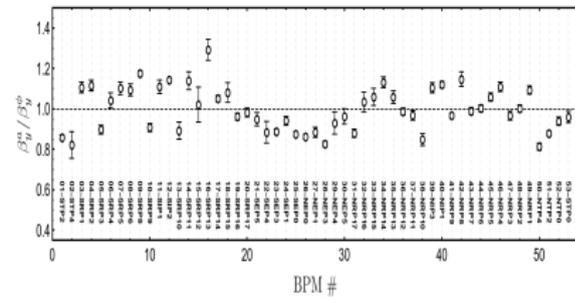
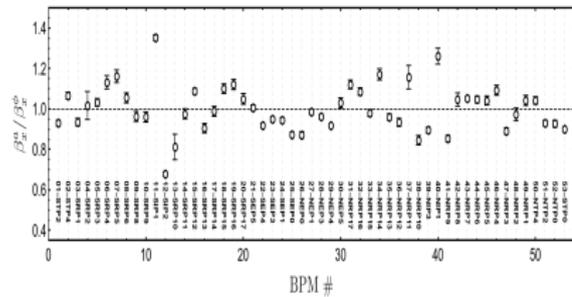
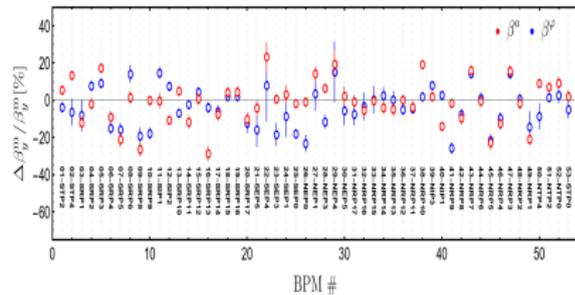
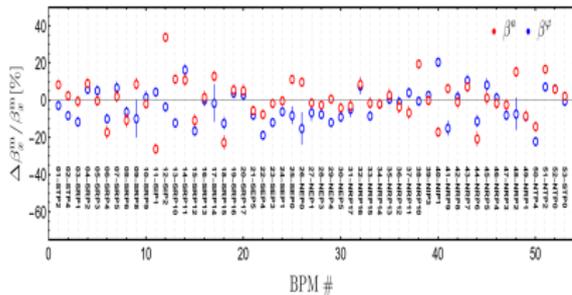


β^a 

β_4 

β^a vs β^φ

- β^a and $\beta^\varphi \approx 20\%$ deviation from model
- β^a and β^φ ratio $\approx 15\%$



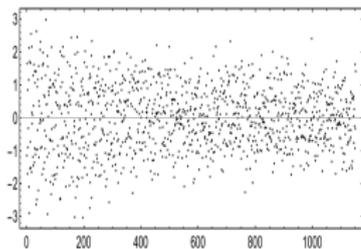
Conclusion

- Data processing and noise estimation
- Accurate and robust frequency measurement $\sigma_\nu \approx 10^{-6}$
- Amplitude ($< 5\%$) and phase ($\approx 5\%$) measurement with error propagation
- Robust action estimation
- β^a accuracy $\approx 5\%$ @ $\approx 20\%$ deviation from model
- β^φ accuracy $\approx 10\%$ @ $\approx 20\%$ deviation from model
- β^a vs β^φ ratio $\approx 15\%$

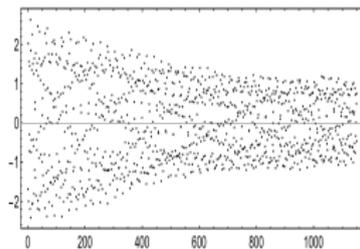
Thank you for your attention

Signal Processing Example

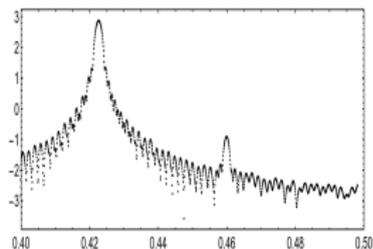
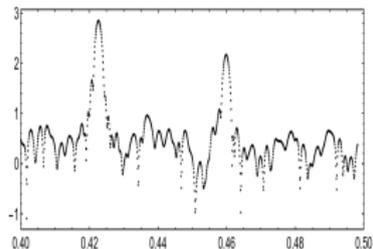
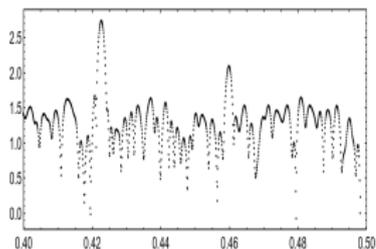
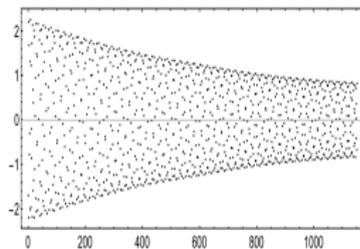
INPUT



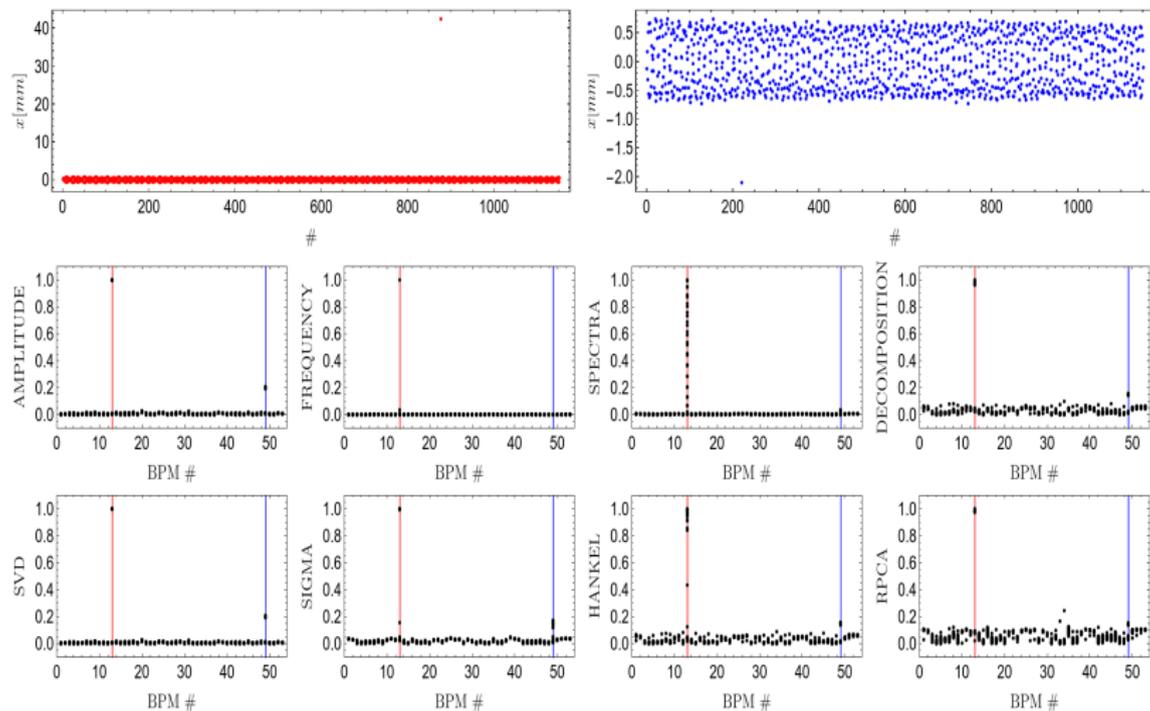
SVD



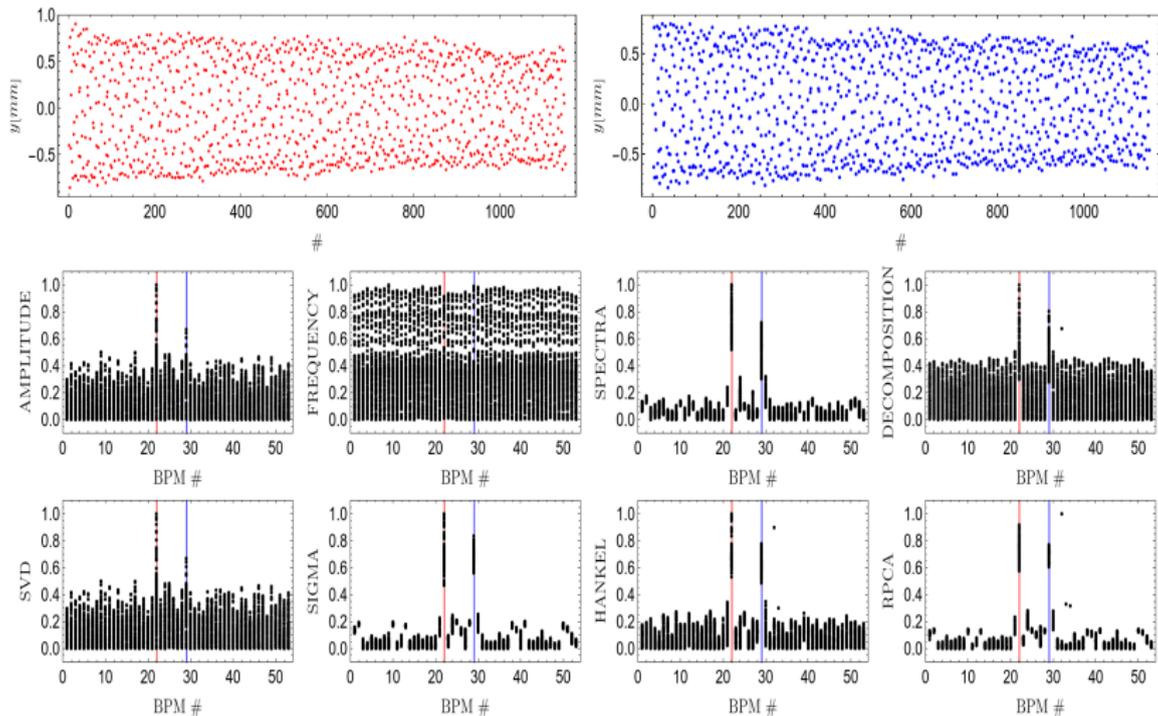
HANKEL



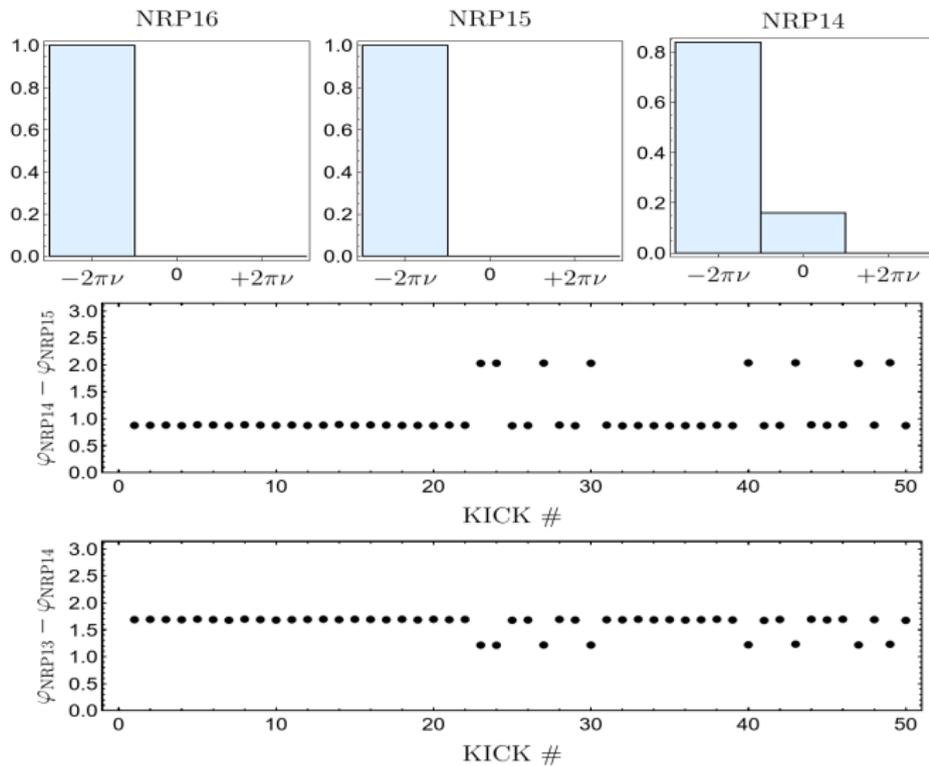
Spike Anomalies



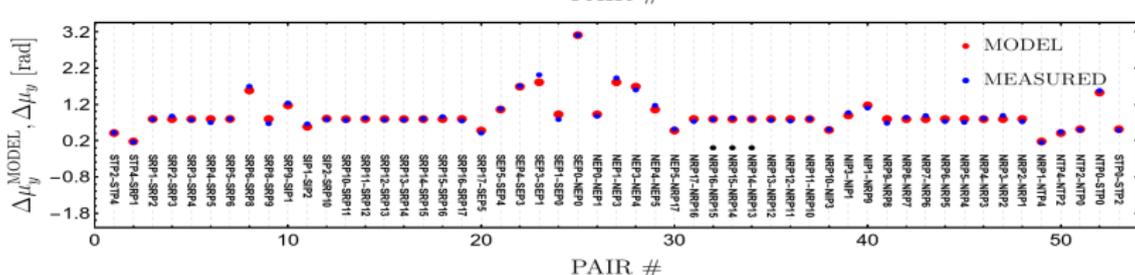
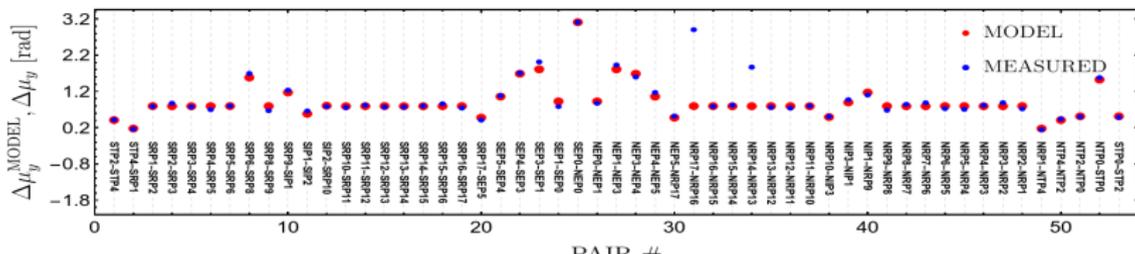
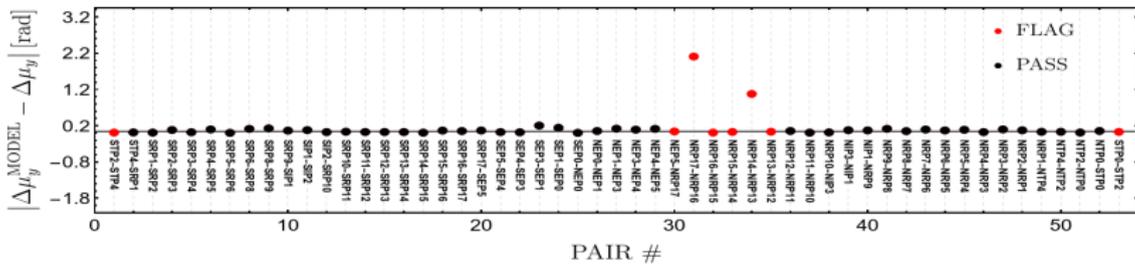
Systematic Anomalies



BPM Synchronization

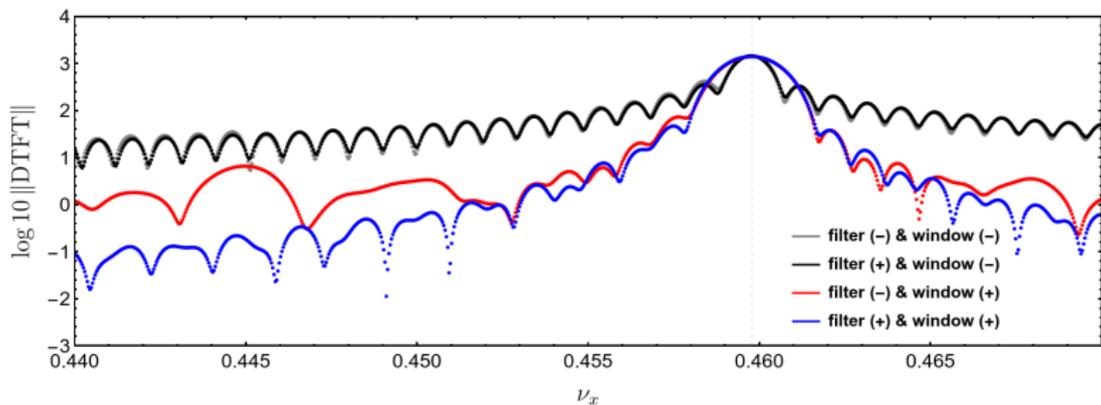
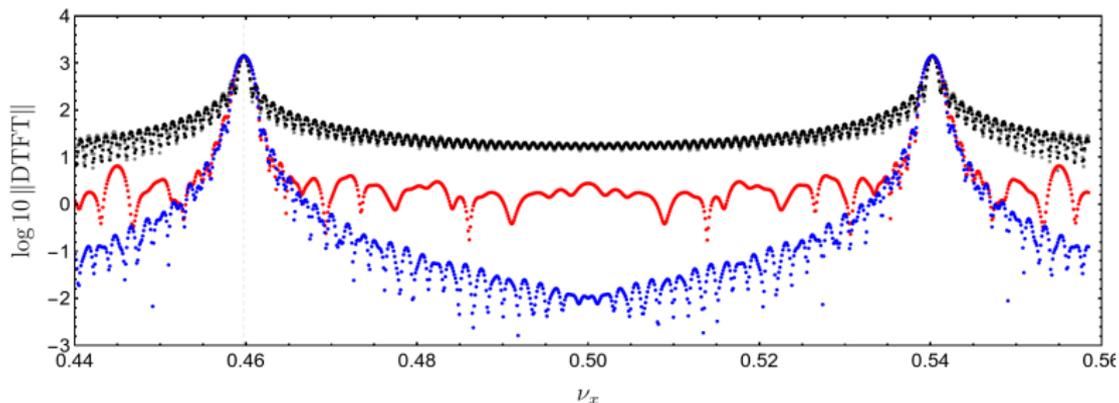


Synchronization Example

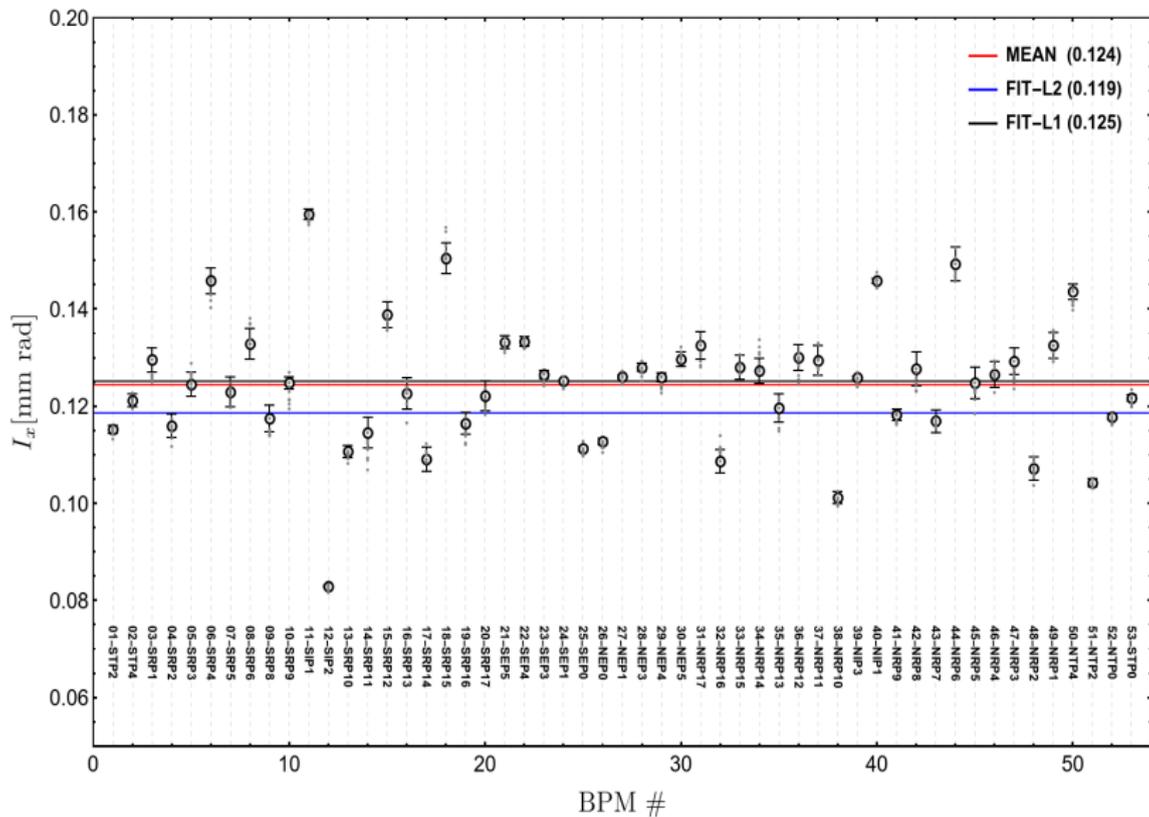


Frequency Estimation

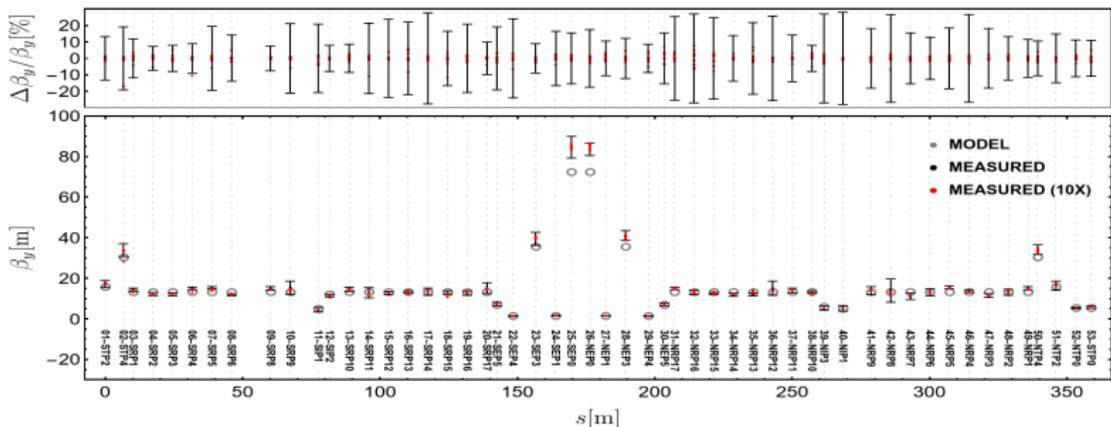
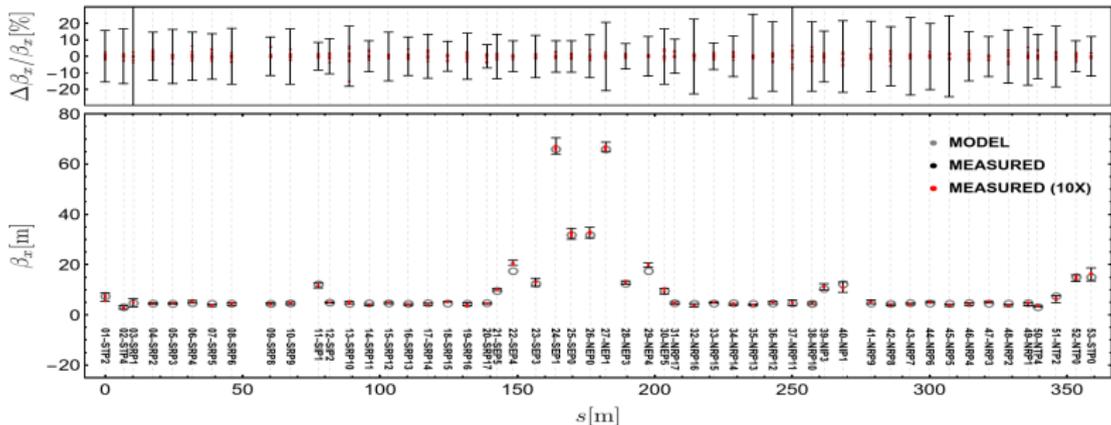
Tunes (8.54, 7.58) \rightarrow (0.46, 0.42)



Action Estimation



$\beta\phi$ (combinations)



α 