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# STABILITY CONDITIONS FOR A PENNING TRAP WITH ROTATING QUADRUPOLE OR DIPOLE ELECTRIC FIELDS

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*Abstract*

The dynamics of particles in a Penning-Malmberg-Surko trap with Rotating Wall (rotating quadrupole and/or dipole electric field) and a buffer gas is considered. Electromagnetic traps are widely used for the accumulation and confinement of charged particles during various experiments in nuclear and accelerator physics, mass spectroscopy, and other fields. Traps are the main element of sources of charged particles in accelerators. An especially important role is played by traps with efficient accumulation during operation (in a cyclic mode) of ion synchrotrons and colliders with short-lived isotopes. The purpose of this work was to develop algorithms for constructing regions of stability (according to Lyapunov) in the space of parameters describing additional rotating electric fields, and to determine the analytical conditions that must be satisfied by the trap parameters to achieve a given degree of stability. The influence of the space charge of a beam of accumulated particles on the stability of the system is also investigated. The calculation results and the proposed models can be used in the selection and adjustment of the main parameters of the designed traps of the considered type.

## INTRODUCTION

Electromagnetic traps are used to accumulate charged particles for various purposes in accelerator physics, mass spectroscopy, nuclear physics, and some other areas of scientific research. Traps are especially important part of charged particles sources in synchrotrons and colliders with short-lived isotopes. The essence of their action is the localization of charged particles in a limited area of space for a sufficiently long time. For this, special combinations of electromagnetic fields are formed that provide the required behaviour of charged particles inside the trap. The most famous traps of the Paul and Penning type, as well as their various modifications. A detailed review of the main known types of traps and the principles of their action is presented in [1].

In this paper, we consider the dynamics of charged particles in a Penning trap with additional rotating dipole electric field (so called Rotating Wall — RW) and a buffer gas or rotating quadrupole electric field without buffer gas, studied earlier in [2-5]. Note also that the investigated model of motion can be used in the analysis of the Penning-Malmberg-Surko trap - an open cylindrical trap and its modifications [1]. The results of the analysis of the influence of the rotating field, obtained earlier, were either insufficiently rigorous from the mathematical point of view [3, 5], or were insufficiently complete [4]. In [6-9], general approaches were proposed that are applicable to the analysis of stability and the construction of numerical-analytical solutions of the equations of motion of the system under study for arbitrary values of its parameters. Examples of such an analysis were given in the works [10, 11]. In this paper the model of dynamics includes also influence of the space charge of a beam of accumulated particles and combination of dipole and quadrupole rotating electric field.

It should be noted that the work investigates the motion of single particles in ideal (linear) fields. Interest in this formulation of the problem arises from attempts [1, 3] to explain the observed in experiments [2] effect of compression of a bunch of accumulated particles by a rotating field at extremely low concentrations of accumulated particles. In this case, the focusing effect should follow from the analysis of the solution of the equations of motion of single particles in the fields of forces acting in the trap [1].

## EQUATIONS OF MOTION

The dynamics of charged particles in a Penning trap with an additional rotating electric field is considered. Charged particle dynamics is considered in the field of the potentials and homogeneous longitudinal magnetic field:

$$\Phi(r,\theta,z) = \frac{m}{q}\Big(\frac{\omega_z^2}{2}\Big(z^2-\frac{r^2}{2}\Big)-\frac{(q_r r^2+q_z z^2)}{2}+ azr\cos(\theta+t\omega_r)+b\frac{r^2}{2}\cos 2(\theta+t\omega_r+\theta_0)\Big),$$

$$\vec{B}=\vec{e}_z B.$$

Here m and $q$ are the mass and the charge of the particle, $\omega_z$ is the frequency of the particle longitudinal oscillations in the axially symmetric electric field of the trap electrodes; $a$, $b$ and $\omega_r$ are amplitude related parameters and the frequency of the rotating electric dipole and quadrupole fields, $\theta_0$ is initial phase parameter of rotating quadrupole field; $q_r$ and $q_z$ are the parameters associated with the space charge, which determine the linear part of the potential of the axially symmetric accumulated beam; $z$, $r$ and $\theta$ are the axial, radial and angular coordinates with the axis coinciding with symmetry axis of the trap electrodes.

The charged particle motion in these fields is described by the following systems of equations correspondingly:

$$\ddot{x} = \Big(\frac{\omega_z^2}{2}+q_r\Big)x+\Omega_c\dot{y}-k\dot{x}-az\cos(t\omega_r)- b(x\cos 2(t\omega_r+\theta_0)-y\sin 2(t\omega_r+\theta_0)),$$

$$\ddot{y} = \Big(\frac{\omega_z^2}{2}+q_r\Big)y-\Omega_c\dot{x}-k\dot{y}+az\sin(t\omega_r)+ b(y\cos 2(t\omega_r+\theta_0)+x\sin 2(t\omega_r+\theta_0)),$$

$$\ddot{z} = (q_z-\omega_z^2)z-k\dot{z}-a(x\cos(t\omega_r)-y\sin(t\omega_r)).$$

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Here $\Omega_c = qB/m$ is the particle cyclotron frequency, the parameter $k$ presents the friction force related to the particle scattering by the trap buffer gas molecules. In this work, we considered a modification of the Penning trap, a Penning-Malmberg-Surco trap with an rotating electric field along the entire length of the trap.

## REDUCTION TO EQUATIONS WITH CONSTANT COEFFICIENTS

In what follows we confine ourselves to analyzing the solution in the approximation corresponding to the typical experimental values of the parameters:

$$\Omega_c \gg \omega_z \gg \omega_m \approx {\omega_z}^2/2\Omega_c \gg k > 0.$$

Here $\omega_m$ is the magnetron frequency, which describes the oscillations of particles in crossed longitudinal magnetic and radial electric fields.

To study particle motion for arbitrary values of $a$, we introduce new complex variable

$$\psi = (x+iy)e^{i(t\omega_r)}.$$

Then $u = \mathrm{Re}\psi = x\cos(t\omega_r) - y\sin(t\omega_r)$, $v = \mathrm{Im}\psi = y\cos(t\omega_r) + x\sin(t\omega_r)$. It means transition to a moving coordinate system obtained from a stationary by uniform rotation in a transverse plane with a frequency $\omega_r$ around the axis $z$.

As a result, in the new variables we obtain a stationary, with constant coefficients system of equations:

$$\ddot{\psi} + \big(k + i(\Omega_c - 2\omega_r)\big)\dot{\psi} + \Big(\omega_r(\Omega_c - \omega_r - ik) - \frac{\omega_z^2}{2} - q_r\Big)\psi + az + be^{-i(2\theta_0)}\bar{\psi} = 0,$$

$$\ddot{\bar{\psi}} + \big(k - i(\Omega_c - 2\omega_r)\big)\dot{\bar{\psi}} + \Big(\omega_r(\Omega_c - \omega_r + ik) - \frac{\omega_z^2}{2} - q_r\Big)\bar{\psi} + az + be^{i(2\theta_0)}\psi = 0,$$

$$\ddot{z} + k\dot{z} + (\omega_z^2 - q_z)z + \frac{a}{2}(\psi + \bar{\psi}) = 0.$$

The characteristic numbers of the system are the roots of the polynomial:

$$\chi(\lambda) = \det\begin{pmatrix} \mathrm{P}(\lambda) & be^{-i(2\theta_0)} & a \\ be^{i(2\theta_0)} & \bar{\mathrm{P}}(\lambda) & a \\ \frac{a}{2} & \frac{a}{2} & Q(\lambda) \end{pmatrix} = Q(\lambda)(\mathrm{P}(\lambda)\bar{\mathrm{P}}(\lambda) - b^2) - \frac{a^2}{2}(\mathrm{P}(\lambda) + \bar{\mathrm{P}}(\lambda) - 2b\cos 2\theta_0),$$

where

$$Q(\lambda) = \lambda^2 + k\lambda + {\omega_z}^2 - q_z,$$

$$R(\lambda) = \lambda^2 + (i\Omega_c + k)\lambda - \frac{{\omega_z}^2}{2} - q_r,$$

$$\bar{R}(\lambda) = \lambda^2 + (-i\Omega_c + k)\lambda - \frac{{\omega_z}^2}{2} - q_r,$$

$$P(\lambda) = R(\lambda - i\omega_r),\ \ \bar{P}(\lambda) = \bar{R}(\lambda + i\omega_r).$$

The distance on the complex plane from the imaginary axis to the characteristic number with the maximum real part, taken with the corresponding sign, will be called the degree of stability and denoted by $\gamma$. The degree of stability will be taken with a positive sign if all roots of the characteristic polynomial are in the left half-plane (the case of asymptotic stability of the system). If there is at least one root in the right half-plane, the value of the degree of stability will be taken with a negative sign (the case of an unstable system). If the rightmost root (or roots) are located on the imaginary axis, the degree of stability is zero. The degree of stability of the system under study cannot exceed $\gamma_{max} = k/2$.

## CONSTRUCTION OF THE STABILITY REGION

Let us introduce the parameter $\alpha$, with the help of which we will estimate the degree of stability of the system under study. Let $\mu = \lambda + \alpha$. Let us make the corresponding change in the characteristic polynomial:

$$\tilde{\chi}(\mu) = \chi(\mu - \alpha) = d_0\mu^6 + d_1\mu^5 + d_2\mu^4 + d_3\mu^3 + d_4\mu^2 + d_5\mu + d_6.$$

Here $d_0 = 1$, $d_1 = (3k - 6\alpha)$,… The $d_j$ coefficients can also be calculated from the Taylor formula:

$$d_j = \chi^{(6-j)}(-\alpha)/(6-j)!\,,\ j = 0,1,\ldots,6.$$

The asymptotic stability test of the polynomial $\tilde{\chi}(\mu)$ for various values of the parameter $\alpha$ can be organized using the necessary and sufficient Routh – Hurwitz conditions (or its modification — the Lienard-Chipard criterion). The fulfillment of these conditions for the polynomial $\tilde{\chi}(\mu)$ with the selected value of the parameter $\alpha$ will mean that the degree of stability of the polynomial $\chi(\lambda)$ will be greater than the selected value: $\gamma > \alpha$.

## STABILITY CONDITIONS

Let us compose the Hurwitz matrix for the polynomial $\tilde{\chi}(\mu)$ and consider its odd principal minors:

$$\Delta_1 = d_1,\quad \Delta_2 = \begin{vmatrix} d_1 & d_0 & 0 \\ d_3 & d_2 & d_1 \\ d_5 & d_4 & d_3 \end{vmatrix},$$

$$\Delta_5 = \begin{vmatrix} d_1 & d_0 & 0 & 0 & 0 \\ d_3 & d_2 & d_1 & d_0 & 0 \\ d_5 & d_4 & d_3 & d_2 & d_1 \\ 0 & d_6 & d_5 & d_4 & d_3 \\ 0 & 0 & 0 & d_6 & d_5 \end{vmatrix}.$$

As a result, we obtain the system of inequalities

$$d_j > 0,\quad j = 1,\ldots,6;\quad \Delta_3 > 0,\ \Delta_5 > 0,$$

the fulfillment of which guarantees that the degree of stability $\gamma$ of the polynomial $\chi(\lambda)$will lie in the range:

$$\alpha < \gamma \leq \frac{k}{2}.$$

## CALCULATION EXAMPLES

The calculations were carried out for electron with the following values of the main parameters: $\Omega_c = 4.4$ Grad/s, $\omega_z = 59.6$ Mrad/s, $k = 1400\ \mathrm{s}^{-1}$.

In Figures 1 and 2 the found regions of asymptotic stability in the space of parameters $a$ and $\omega_r$ are marked with different colors with an estimate of the range of values for the degree of stability of the system. For exam-

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ple, red means that the degree of stability is more than 10 percent (but not more than 20), green means more than 20 (but not more than 30), etc. (see the legend of the corresponding figure), where $\gamma_{max} = k/2$ is taken as 100 percent. Figure 1 shows stability area with switched off rotating quadrupole field. Figure 2 shows the effect of adding a rotating quadrupole electric field to the system with dipole rotating field.

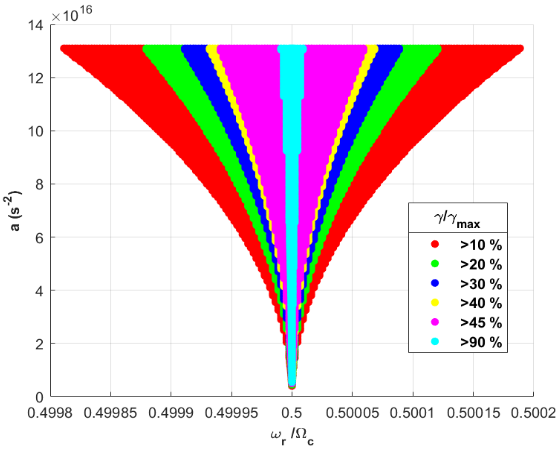


Figure 1: Areas with different ranges of values of the degree of stability (b=0).

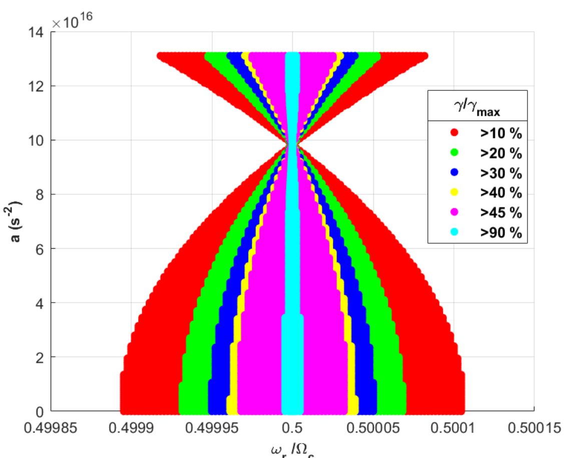


Figure 2: Areas with different ranges of values of the degree of stability ($b = 10^{15}$ s$^{-2}$, $\theta_0 = \pi/2$).

## CONCLUSION

The dynamics of particles in a Penning trap with a rotating quadrupole and/or dipole electric field and a buffer gas is considered. Using the Lienard-Chipart modification of the Routh-Hurwitz stability criterion, an analysis is carried out and regions in the space of parameters of the trap of the asymptotic stability of particle motions (according to Lyapunov) are found. The influence of the space charge of a beam of accumulated particles on the stability of the system is also investigated. Various models of an axially symmetric beam are considered. The stability regions are constructed in the space of parameters characterizing the additional rotating electric field.

Appropriate software has been developed to find and graphically represent areas of stability. In the algorithm for constructing stability regions, stability criteria for families of polynomials linear in the parameter are also used.

The calculation results and the proposed models can be used in the selection and adjustment of the main parameters of the designed traps of the considered type.

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