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X-RAY THOMSON INVERSE SCATTERING FROM PERIODICALLY MODULATED LASER PULSES*

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Abstract

Being a compact source of x-rays based on the Thomson backscattering, Thomson source has potential to be used in medicine and biology and in other areas where narrow band x-ray beams are essential. We propose and investigate theoretically the idea to use laser pulses modulated with a short period in Thomson backscattering. The coherent radiation is obtained with intensity proportional to the squared number of micro-pulses in the whole laser pulse.

INTRODUCTION

Thomson (or Compton) backscattering happens when a laser pulse scatters off with a counter propagating relativistic free electron. Thomson backscattering underlies a bright and compact X-ray source. The sizes of such source are much smaller, than synchrotron's sizes, while the brightness is comparable with that. The general layout of this process is shown in Fig. 1.

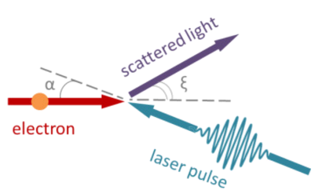

Figure 1: Layout of Thomson backscattering: laser pulse scattering on a moving relativistic electron.

The frequency of the scattered light is defined as without recoil effect:

$$\omega = \omega_0 \frac{1+\beta\cos\alpha}{1-\beta\cos\xi}, \quad (1)$$

where ω_0 is the frequency of incident laser pulse, $\beta = v/c$ with c being the speed of light in vacuum and v being the initial speed of the electron, α is an angle of interaction, ξ is the angle between the electron's trajectory and the propagation direction of the initial laser pulse.

The number of X-ray photons depends on the number of oscillations of electrons in the external field, i.e. the laser pulse duration T:

$$N_{\text{ph}} = f\left(N_{\text{oscillations}}\right) \propto T. \quad (2)$$

Increasing T leads to incoherent enhancement of radiation. It is similar to incoherent radiation from a long electron beam [1]:

$$I = I_0\left(N + N^2 F\right). \quad (3)$$

Here the first summand describes incoherent radiation, while the second one describes the coherent radiation. In order to switch on coherence effects we propose to use the laser with periodical longitudinal profile, the layout see in Fig. 2.

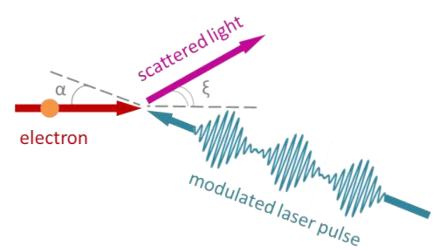

Figure 2: Periodically modulated laser pulse is scattered on a relativistic electron beams, giving rise to the radiation with frequency higher than initial laser.

As the first approximation the laser wave can be considered as a plane wave:

$$\mathbf{E}^{\text{ext}}\left(\mathbf{r},t\right) = \mathbf{E}_0 \cos\left(\omega t - \mathbf{kr} + \varphi_0\right), \quad (4)$$

where ω is the laser frequency, $\mathbf{k}$ is its wave-vector, $\mathbf{E}_0$ is the laser amplitude, φ_0 is the initial phase. The modulated laser pulse can be described by [2]:

$$\mathbf{E}^{\text{ext}}\left(\mathbf{r},t\right) = \mathbf{E}_0\left(t\right) \cos\left(\omega t - \mathbf{kr} + \varphi_0\right). \quad (5)$$

For periodically modulated beam $\mathbf{E}_0\left(t+t_0\right) = \mathbf{E}_0\left(t\right)$. It means that instead of wave with a modulated profile we can use the set of N short laser pulses each of which is described by the field of a plane wave, see Fig. 3.

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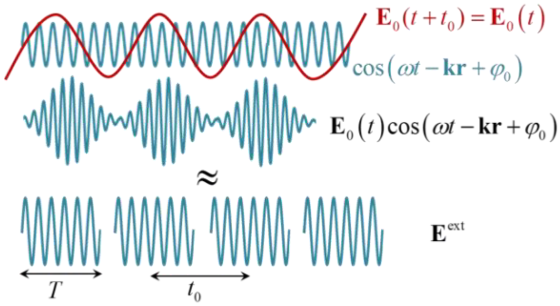

Figure 3: Different ways to perform the modulated laser pulse.

In this paper, we propose the new way to increase the intensity of X-ray inverse Compton source significantly, using the modulated laser pulses.

GENERAL THEORY

The energy of light scattered on the relativistic electron per the unit frequency and per unit solid angle is

$$\frac{dW_1(\mathbf{n},\omega)}{d\Omega d\omega}=\frac{e^2\omega^2}{4\pi^2c^3}\left|\int_0^T dte^{i\omega t}e^{-i\mathbf{kR}(t)}\left[\mathbf{n},\mathbf{v}(t)\right]\right|^2, \quad (6)$$

where e is the electron's charge, $\mathbf{k}$ is the wave-vector of scattered light, $\mathbf{k}=\mathbf{n}\omega/c$ with $\mathbf{n}$ being the unit wave-vector, T is the duration of a single laser pulse, $\mathbf{R}(t)$ and $\mathbf{v}(t)$ are the trajectory and the velocity of the electron in the external field (laser). As the electron having the initial speed starts to oscillate in the external field, $\mathbf{R}(t)$ describes this periodical motion along with drift motion in a single laser pulse [3].

If an external field is periodical (modulated laser pulse), the electron's trajectory repeats in every period of the external field t_0. So, for the electron moving in the modulated laser pulse we can write $\mathbf{R}(t)=\mathbf{R}(t+t_0)$ and $\mathbf{v}(t)=\mathbf{v}(t+t_0)$. It means that in the case of scattering of the modulated laser pulse on the electron the integral in Eq.(6) reads

$$\frac{dW(\mathbf{n},\omega)}{d\Omega d\omega}=\frac{e^2\omega^2}{4\pi^2c^3}\left|\sum_{n=0}^{N-1}\int_{nt_0}^{T+nt_0} dte^{i\omega t}e^{-i\mathbf{kR}(t)}\left[\mathbf{n},\mathbf{v}(t)\right]\right|^2, \quad (7)$$

where N is the number of periods of the external field.

ELECTRON MOTION

In order to find the coordinates and the velocity of the electron in the electromagnetic field of laser beam one need to solve the system of motion equations:

$$\frac{d\mathbf{p}}{dt}=e\mathbf{E}+\frac{e}{c}[\mathbf{v},\mathbf{H}],\quad \frac{d\varepsilon}{dt}=e\mathbf{vE}, \quad (8)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields of laser wave, which are related as $\mathbf{H}=[\mathbf{n},\mathbf{E}]$, $\mathbf{p}$ is the momentum of the electron moving under action of the laser pulse, ε is its energy. Wave function describing the motion of the electron in the field of the plane wave was calculated strictly by Volkov proceeding from Dirac equation [4]. Later, the solution for the momentum was obtained by Ritus in form of four-vectors [5]:

$$p_\mu=p_{0\mu}-\frac{e}{c}A_\mu+k_\mu\left(\frac{e}{c}\frac{pA}{kp}-\frac{e^2}{c^2}\frac{A^2}{2kp}\right), \quad (9)$$

where $p_\mu=(\varepsilon/c,\mathbf{p})$, $p_{0\mu}=(\varepsilon_0/c,\mathbf{p}_0)$, $\mathbf{p}_0$ coincides with the electron's momentum when there is no the external field, $\mathbf{A}_\mu=(0,\mathbf{A})$ with $\mathbf{A}$ being the vector-potential: $\mathbf{H}=rot\mathbf{A}$, $\mathbf{E}=-(1/c)\partial A/\partial t$, $k_\mu=(\omega_0/c,k_0)$, $pA=-\mathbf{p}_0\mathbf{A}$, $kp=-\mathbf{k}_0\mathbf{p}_0+\varepsilon_0\,\omega_0/c^2$. The similar solution can be obtained solving Eqs.(8)-(9) by classical methods like it was done in [3, 2].

Let the external wave propagate along z-axis. Then the wave-vector is $\mathbf{k}_0=\mathbf{e}_z\,\omega_0/c$, the vector-potential can be written in form [6] $\mathbf{A}=\mathbf{e}_xA_0\cos(\omega_0t-k_{0z}z)$, where $\mathbf{A}_0$ is the amplitude, and, consequently, the electrical and magnetic fields are

$$\begin{aligned}\mathbf{E}^{ext}&=\mathbf{e}_xA_0k_0\sin(\omega_0t-k_0z),\\ \mathbf{H}^{ext}&=\mathbf{e}_yA_0k_0\sin(\omega_0t-k_0z).\end{aligned} \quad (10)$$

Let us introduce the laser strength parameter $a_0=eA_0/(mc^2)$, where e and m are the electron charge and mass correspondingly. In order to avoid nonlinear effects below we will consider the case $a_0<<1$, that means that it means that the terms proportional to a_0^2 will be further excluded from our consideration. Second approximation is $\Delta p_z<<p_z$, where Δp_z is the additional momentum induced by the electromagnetic field: $\Delta p_z=p_z-p_{0z}$. Taking into account these two approximations we can write the expressions for the electron's trajectory:

$$\mathbf{R}(t)=\mathbf{R}_0+\mathbf{v}_xt-\frac{a_0c}{\gamma_0\omega_0}\frac{\sin(t\eta)-t\eta}{1-\beta_{0z}}\left\{1,0,\frac{\beta_{x0}}{1-\beta_{0z}}\right\}, \quad (11)$$

where $\eta\approx\omega_0(1-\beta_{0z})$. In order to calculate the energy of scattered radiation per unit frequency and per unit solid angle one needs to find the velocity of the electron in external field:

$$\mathbf{v}=\mathbf{v}_{0x}-\frac{a_0c}{\gamma_0}(\cos(t\eta)-1)\left\{1,0,\frac{\beta_{x0}}{1-\beta_{0z}}\right\}. \quad (12)$$

INTENSITY OF RADIATION

Substituting the obtain expressions for the velocity and trajectory of the electron in the external field (i.e. Eqs. (11)-(12)) in Eq. (7) we obtain:

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$$\frac{dW(\mathbf{n},\omega)}{d\Omega d\omega}=\frac{e^2\omega^2}{4\pi^2c^3}\left|\sum_{n=0}^{N-1}e^{iD}\times\right.$$
$$\left.\times\int_{t_0 n}^{T+t_0 n}dte^{iB\sin\eta t}e^{iAt}\left(\mathbf{H}+\mathbf{K}\cos\eta t\right)\right|^2, \quad (13)$$

where we designate

$$A=\omega-\omega\left(\mathbf{nv}_0c^{-1}+a_0\gamma_0^{-1}\left[n_x+n_z\,\beta_{x0}/(1-\beta_{0z})\right]\right), (14)$$

$$B=\omega\left[n_x+n_z\,\beta_{x0}/(1-\beta_{0z})\right]a_0\gamma_0^{-1}\omega_0^{-1}\left(1-\beta_{0z}\right)^{-1}, (15)$$

$$\mathbf{K}=\frac{a_0c}{\gamma_0}\left[\frac{\beta_{x0}}{(1-\beta_{0z})}\left(\mathbf{e}_yn_x-\mathbf{e}_xn_y\right)-\left(\mathbf{e}_yn_z-n_y\mathbf{e}_z\right)\right], (16)$$

$$\mathbf{H}=\left[\mathbf{n},\mathbf{v}_0\right]-\mathbf{K},\qquad D=-\mathbf{nR}_0\,\omega/c. \quad (17)$$

Integration can be performed with help of the following formula:

$$e^{ix\sin y}=\sum_{s=-\infty}^{\infty}e^{isy}J_s(x), \quad (18)$$

where $J_s(x)$ is the Bessel function of the s-th order. As a result we find:

$$\frac{dW(\mathbf{n},\omega)}{d\Omega d\omega}=\frac{e^2\omega^2}{4\pi^2c^3}\left|\sum_{s=-\infty}^{\infty}F(s,t_0)\frac{\sin\left(T(A+s\eta)/2\right)}{(A+s\eta)/2}\times\right.$$
$$\left.\times e^{i\frac{Nt_0-t_0+T}{2}(A+s\eta)}\left[\mathbf{H}J_s(B)+\frac{\mathbf{K}}{2}\left(J_{s-1}(B)+J_{s+1}(B)\right)\right]\right|^2, \quad (19)$$

where the factor $F(s,t_0)$ defines the coherent effect due to the modulation of the laser pulse

$$F(s,t_0)=\frac{\sin\left(N(A+s\eta)t_0/2\right)}{\sin\left((A+s\eta)t_0/2\right)}. \quad (20)$$

Actually, $F(s,t_0)$ is proportional to N and being squared is proportional to N^2, which is maximal coefficient enhancing the radiation. The condition of maximal enhancement is

$$\omega=\frac{2m\pi-s\omega_0t_0(1-\beta_{0z})}{t_0\left[1-\frac{\mathbf{nv}_0}{c}-\frac{a_0}{\gamma_0}\left(n_x+n_z\frac{\beta_{x0}}{1-\beta_{0z}}\right)\right]}, l=0,\pm1,.... \quad (21)$$

For $l=0$ the radiation is maximal, what follows from maximum of relation of the sine to its argument in Eq. (19):

$$\omega=\frac{-s\omega_0(1-\beta_{0z})}{1-\frac{\mathbf{nv}_0}{c}-\frac{a_0}{\gamma_0}\left(n_x+n_z\frac{\beta_{x0}}{1-\beta_{0z}}\right)}. \quad (22)$$

In this case these two factors are maximal simultaneously and, consequently, the maximal enhancement of radiation can be observed.

CONCLUSION

In this paper, we propose the new way to increase the intensity of X-ray inverse Compton source significantly, using the modulated laser pulses. We construct the theory describing the radiation in all its spectral and angular details. Also, we obtain the conditions of the coherent enhancement of radiation.

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