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WAKEFIELD UNDULATOR BASED ON A SINUSOIDAL DIELECTRIC WAVEGUIDE*

I. L. Sheinman†, O. S. Alekseeva, Saint-Petersburg Electrotechnical University "LETI", Saint-Petersburg, Russia

Abstract

The idea of creating an undulator based on the wake principle by passing a beam through a sinusoidal dielectric waveguide is proposed. A numerical analysis of the dynamics of a short electron beam in a wake undulator on a bending wave of a waveguide with a dielectric filling is carried out. The possibility of reducing the instability of the beam by choosing the initial phase of the flexural wave and the initial transverse positioning of the beam is considered.

INTRODUCTION

Undulators are key elements of free electron lasers [1]. They are devices in which the electron beam in the process of movement experiences the action of transverse periodic force. The oscillations of particles arising under the force in the transverse direction are accompanied by accelerated motion of the particles, which, in turn, generates electromagnetic radiation in the direction of the beam movement. To create a transverse force, electromagnetic fields created by periodically located dipole magnets are used. Undulator radiation also arises when particles move through a periodic lattice of crystals, where the local fields of atoms play the role of deflecting fields.

Undulators have a number of features that make them attractive for creating free electron lasers: a large beam aperture; short undulator period, the ability to generate waves of both circular and plane polarization, dynamic control of the wavelength and undulator coefficient.

Linear charged particle accelerators are used as sources of electron bunch sequences for free electron lasers. An intensively developing direction in recent years is the use of linear wakefield accelerating structures with dielectric filling. They are based on the principle of excitation of Cherenkov radiation in dielectric waveguides by a high-current relativistic electron beam.

The leading bunch (driver) with a large charge, moving along the axis of the vacuum channel of the dielectric waveguide, generates behind itself a wake electromagnetic wave of Cherenkov radiation, the phase velocity of which is equal to the speed of the driver. This wave has a longitudinal field component, which is used to accelerate the witness bunch.

A significant drawback of this approach is the exponentially increasing displacement of the beam relative to the waveguide axis, associated with the interaction of the beam with the wakefield generated by it itself, as a result of which the particles are attracted to the waveguide wall, and at high energy they can destroy its integrity.

In [2-5], it was proposed to use the parasitic effect of beam deflection from the waveguide axis in a wakefield accelerating structure to create a wakefield undulator. The use of the bunch-generated intrinsic transverse fields in a microwave cavity to create an undulator effect was proposed in [2]. In [3], it was proposed to create an alternating transverse electromagnetic field for the generation of undulator radiation with the oncoming motion of the generator and undulator bunches. However, due to significant deflecting fields acting on the generator bunch, the range of its flight turned out to be limited [4], which reduced the effectiveness of the method.

An alternative idea is to use a sinusoidal bent waveguide to create transverse beam oscillations [5]. In such a waveguide, the tail of the main beam or the secondary beam of charged particles, being attracted either to one wall or to the other, will vibrate in the transverse direction (Fig. 1). Due to such oscillations, the beam electrons will move with transverse acceleration and generate electromagnetic waves, which is a necessary condition for creating a free electron laser.

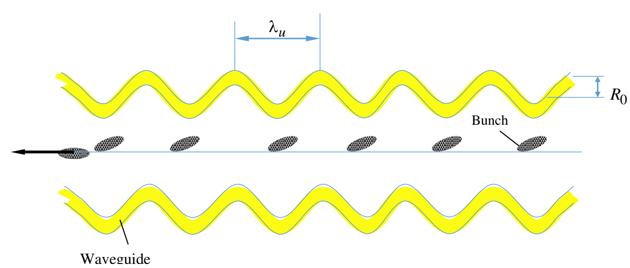

Figure 1: Wakefield undulator.

Modern undulators based on an alternating magnetic field are very expensive and require precise tuning. The proposed approach assumes the use of a bent waveguide with correctly selected parameters as a terminal device for an accelerator for a free electron laser, which will significantly reduce the cost of creating undulators.

BEAM DYNAMICS IN A WAKEFIELD WAVEGUIDE

To describe the beam dynamics with deviations from the waveguide axis not exceeding $R_c/2$, it is possible with a good degree of accuracy to take into account only the first transverse mode of the wake waveguide with a linear dependence of the transverse force on the distance from the particle generating the force to the waveguide axis.

Let us first consider a filamentary electron bunch (beam) with a longitudinal charge profile $f(\zeta)$ moving

* Work supported by Baltic Science Network powerhouse.
† ishejnman@yandex.ru

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parallel to the waveguide axis with a displacement $r(\zeta, t)$. We will assume that the charge in the bunch is uniformly distributed in the longitudinal direction $f(l)=1/l$, where l is the length of the bunch. The change in the magnitude of the relativistic factor with time can be neglected.

In the case when the amplitude of the first mode dominates over the others, the radial dynamics of the beam in a regular waveguide is described by the equation [4-6]

$$\frac{\partial^2 r(\zeta,t)}{\partial t^2} - A\int_0^{\zeta} \sin\left(k_z(\zeta-\zeta_0)\right) r(\zeta_0,t)d\zeta_0 = 0,$$

where $A = -\frac{eq}{4m_e\gamma_0}k_\perp\psi_{F\perp} > 0$, noting that electron beam charge is negative, q is the charge of the electron beam ($q<0$), k_z and $k_\perp$ are the longitudinal and transverse wavenumbers of the first mode of the waveguide, $r(\zeta,t)$ is the displacement of the bunch from the axis of the waveguide, $\psi_{F\perp}$ is the series coefficient in the expansion of the radial force for the first mode, depending on the geometry and dielectric constant waveguide.

In the case of a point charge, which flies in without an initial radial velocity into the field of action of a transverse harmonic force, only the cosine dependence of the force ensures the finite motion of the particle in the transverse direction. The dependence of the force according to the sine law leads to the average displacement of the particle, which increases linearly with time. However, in the case of an extended bunch, as will be shown below, there is an accumulation of action from the head of the bunch, and the purely cosine dependence of the waveguide bend gives rise to instability of the beam, leading to its exit from the waveguide channel.

In this regard, the bending of the axis of the waveguide relative to the longitudinal axis z will be taken into account by adding to the expression for the force an additional periodic displacement of the beam relative to the axis of the waveguide: $R_{0c}\cos(\kappa z)$ or $R_{0s}\sin(\kappa z)$, where $\kappa = 2\pi/\lambda_u$, λ_u is the flexural wavelength. By introducing a parameter $\chi = \mathrm{arctg}(R_{0s}/R_{0c})$ characterizing the initial phase of the flexural wave and varying χ, using the numerical simulation, it is possible to select conditions that correspond to the optimal ratio between the flight distance and the amplitude of the bunch oscillations.

For electron bunches with a length less than a quarter of the wavelength of the Cherenkov wake radiation of the waveguide $\Lambda = 2\pi/k_z$, the sine under the integral can be replaced by its argument. Then the equation of radial dynamics with single-mode excitation of a bent waveguide takes the form:

$$\frac{\partial^2 r_c(\zeta,t)}{\partial t^2} - A\int_0^{\zeta} f(\zeta_0)\left(k_z(\zeta-\zeta_0)\right)\left(r(\zeta_0,t)+R_{0c}\cos(\kappa z)\right)d\zeta_0 = 0,$$

$$\frac{\partial^2 r_s(\zeta,t)}{\partial t^2} - A\int_0^{\zeta} f(\zeta_0)\left(k_z(\zeta-\zeta_0)\right)\left(r(\zeta_0,t)+R_{0s}\sin(\kappa z)\right)d\zeta_0 = 0$$

with initial conditions

$$r(\zeta,0) = r_0, \quad \left.\frac{dr(\zeta,t)}{dt}\right|_{t=0} = v_{r0} = 0.$$

The combined analytical solution of the obtained equations has the form:

$$r(\zeta,t) \approx r_{free}(\zeta,t) + r_{\sin}(\zeta,t) + r_{\cos}(\zeta,t),$$

where the term

$$r_{free}(\zeta,t) \approx \frac{r_0}{2}\left(I_0\left(2\sqrt{\sqrt{Ak_z}\zeta t}\right) + J_0\left(2\sqrt{\sqrt{Ak_z}\zeta t}\right)\right) + \\ +\frac{v_{r0}}{2\sqrt{\sqrt{Ak_z}\zeta}}\left(I_1\left(2\sqrt{\sqrt{Ak_z}\zeta t}\right) + J_1\left(2\sqrt{\sqrt{Ak_z}\zeta t}\right)\right)$$

represents the free movement of a bunch under the influence of forces created by it itself,

$$r_{\cos}(\zeta,t) \approx \frac{AR_{0c}k_z}{\left(Ak_z - \kappa^4 v^2\right)} \times \\ \left(\cos\left(\frac{\sqrt{Ak_z}\zeta}{\kappa v}\right)\cos\kappa vt - \frac{\kappa^2 v}{\sqrt{Ak_z}}\sin\left(\frac{\sqrt{Ak_z}\zeta}{\kappa v}\right)\sin\kappa vt - \cos\left(\kappa\zeta + \kappa vt\right)\right) + \\ +R_{0c}\frac{Ak_z\zeta^2}{\kappa^2 c^2}\sum_{n=0}^{\infty}\left(\frac{\left(\frac{Ak_z(\zeta L)^2}{c^2}\right)^n}{(2n+1)!(2n+3)!}\left((2n+3)(2n+1) - \kappa^2 L\zeta\right)\right),$$

$$r_{\sin}(\zeta,t) \approx \frac{AR_{0s}k_z}{\left(Ak_z - \kappa^4 v^2\right)} \times \\ \left(\cos\left(\frac{\sqrt{Ak_z}\zeta}{\kappa v}\right)\sin\kappa vt + \frac{\kappa^2 v}{\sqrt{Ak_z}}\sin\left(\frac{\sqrt{Ak_z}\zeta}{\kappa v}\right)\cos\kappa vt - \sin\left(\kappa\zeta + \kappa vt\right)\right) + \\ +R_{0s}\frac{Ak_z\zeta^2}{\kappa^2 c^2}\sum_{n=0}^{\infty}\left(\frac{\left(\frac{Ak_z(\zeta L)^2}{c^2}\right)^n}{(2n+1)!(2n+3)!}\left(\kappa L(2n+3) + \kappa\zeta(2n+1)\right)\right).$$

describe the forced motion of a bunch under the influence of a force periodically changing in space.

RESULTS OF NUMERICAL SIMULATION OF BEAM DYNAMICS

Let us present the results of modelling the flight dynamics for a beam energy of W = 10 GeV, a charge q = 100 nC, a length of l = 60 µm, and a base frequency of the waveguide f = 300 GHz.

The calculation results for a single uniformly distributed bunch along the length of the bunch for the cosine dependence of the waveguide bend are shown in Fig. 2. The initial displacement of the beam particles from the waveguide axis was taken equal to $r_0 = 10$ µm, the initial radial velocity was assumed to be zero, and the waveguide bend amplitude $R_0 = 0.5R_c$.

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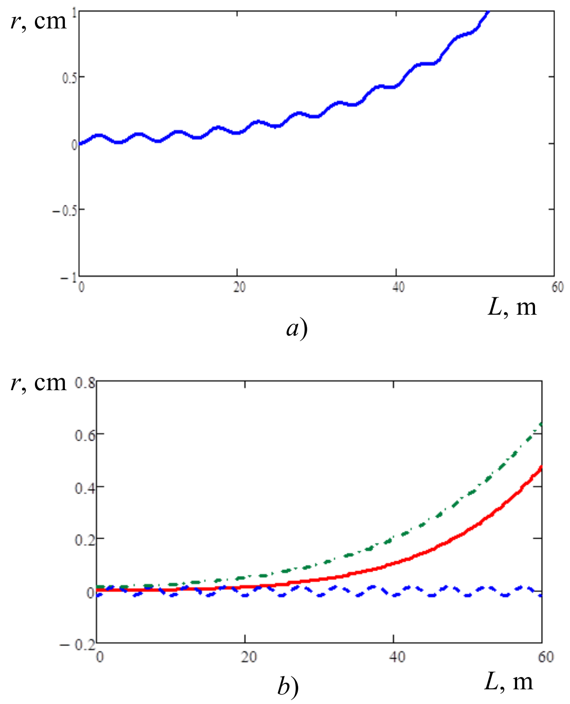

Figure 2: Results of modelling the beam dynamics for the cosine dependence of the waveguide bending: *a*) the beam trajectory, *b*) the effect of individual terms (red solid line is r_{free} , blue dashed line – the first term of $r_{\cos}$, green dash-dotted line – the second term of $r_{\cos}$).

As can be seen from Fig. 2, the solution without taking into account the initial phase is unstable, and the bunch quickly "settles" on the wall. Fig. 2b shows that the second term in $r_{\cos}$ leads to the deflection of the beam from the waveguide axis.

The introduction into the equation of dynamics of an additional term with a displacement that changes according to the sine law, and the selection of the initial phase of the flexural wave make it possible to achieve a much greater flight range.

The simulation results taking into account the correction by selecting the initial phase of the waveguide bending ($\chi = -0.085$) are shown in Fig. 3. The effects of the initial displacement of the bunch from the axis r_{free} and the second term $r_{\cos}$ cancel each other out, which makes it possible to increase the flight range by more than 2 times.

The inverse formulation of the problem is also possible: selection of the transverse positioning and radial velocity of the bunch at the entrance to the curved wake waveguide to maximize the flight range of the beam with a constant waveguide profile.

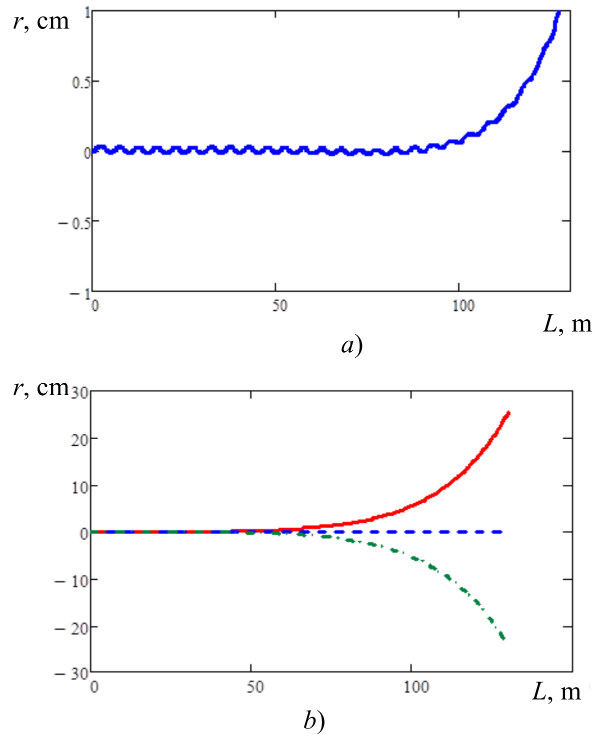

Figure 3: Beam dynamics simulation results with correction by introducing bias with sine ($\chi = -0.085$).

CONCLUSION

The choice of the initial phase of the bending of the waveguide and its matching with the axial positioning of the bunch at the entrance to the undulator make it possible to achieve a significant increase in the range of the beam flight and, thereby, to increase the emitted power of X-ray radiation.

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