ISBN: 978-3-95450-240-0 ISSN: 2673-5539 doi:10.18429/JACoW-RuPAC2021-TUPSB17

INVESTIGATIONS OF CHARGE PARTICLE DYNAMICS IN SPACE CHARGE FIELDS

A.S. Chikhachev[1], All-Russian Electrotechnical Institute, 111250, Moscow, Russia

Abstract

The paper studies the nonstationary dynamics of single-component systems. The problem of the dynamics of a plane layer is considered. The classical collisionless system described by the "Meshchersky integral" and the "conjugate" integral of motion is considered. States characterized by a constant charge in nonstationary coordinates are obtained.

INTRODUCTION

The study of non-stationary systems that intensively interact with their own field is of great interest from both experimental and theoretical points of view. In this paper, we will use the nonstationary Hamiltonian, which follows from Meshchersky's work [1]. A similar Hamiltonian was used in [2,3,4], in which, apparently, the problem of exact accounting for the eigenfield was solved for the first time. It should be noted that the model Hamiltonian of a nonstationary system can be used both for a quantum mechanical system and for a classical one. In this paper, the classical system is considered.

CLASSIC COLLISIONLESS SYSTEM

Let us first consider a one-dimensional system of charged particles described by a collisionless kinetic equation. Let x be the coordinate and t be the time. A one-dimensional system can be described by the following nonstationary Hamiltonian:

$$H=\frac{p^2}{2m}+\frac{1}{\xi^2(t)}U\left(\frac{x}{\xi(t)}\right) \tag{1}$$

Where $p=m\dot{x}$, $\xi(t)$ - the characteristic function that satisfies the equation $\ddot{\xi}=\frac{\lambda}{\xi^3(t)}$, Where λ - constant.

If, next, enter new variables $x_*=\frac{x}{\xi(t)}$, $\tau=\int\frac{dt'}{\xi^2(t')}$, then from (1.1) we can obtain an expression for the integra of motion:

$$I=\frac{m}{2}\left(\frac{dx_*}{d\tau}\right)^2+\frac{m\lambda}{2}x_*^2+U(x_*). \tag{2}$$

Potential function $U(x_*)=q\xi^2\Phi\left(\frac{x}{\xi}\right)$, where is the elementary charge , is the electrostatic potential. In variables, the expression (2) is essentially the Hamiltonian of a stationary (i.e., independent of) systems, and the role of the potential is played by the expression $\frac{m\lambda x_*^2}{2}+U(x_*)$.

In addition to (2), there is a conjugate integral:

$$J_I^{\pm}=\pm\int_0^{x_*}\frac{dx_*'\sigma\left(\frac{2}{m}\left(I-U(x_*')\right)-\lambda x_*'^2\right)}{\sqrt{\left(\frac{2}{m}\left(I-U(x_*')\right)-\lambda x_*'^2\right)}}-\tau,$$

here σ is the Heaviside function. The velocity of a particle can be represented as the sum of the portable (or nasal) velocity - this is the magnitude $x\frac{\dot{\xi}}{\xi}$ and relative-

$$\upsilon_x'=\pm\sqrt{\frac{2}{m}\left(I-U(x_*)\right)+\frac{x_*^2}{4\tau_0^2}}.$$

In this case, the relative movement in the positive and negative directions of the axis is possible x. Consider the case of relative forward motion: $\upsilon_x'>0$, then

$$J_I^{=}=\int_0^{x_*}\frac{dx_*'\sigma\left(\frac{2}{m}\left(I-U(x_*')\right)-\lambda x_*'^2\right)}{\sqrt{\left(\frac{2}{m}\left(I-U(x_*')\right)-\lambda x_*'^2\right)}}-\tau.$$

Performing equality $\frac{dJ_I^+}{d\tau}\equiv 0$ is trivial.

Previously, conjugate integrals of motion, apparently for the first time, were considered in [2].

There are, further, two possibilities: to study the dynamics of a clot with an increasing and decreasing function $\xi(t)$. Let us first consider the case of a decreasing $\xi(t)=\sqrt{\xi_0^2-\frac{t}{\tau_0}}$. In this case,

[1]churchev@mail.ru

ISBN: 978-3-95450-240-0 ISSN: 2673-5539 doi:10.18429/JACoW-RuPAC2021-TUPSB17

$\xi(\tau)=\xi_0 \exp\left(-\frac{\tau}{2\tau_0}\right)$. Let, further, the density $n=\int d\dot{x} f\left(I, J_I^+\right)$, potential - $U\left(\frac{x}{\xi}\right)=U(x_*)$.

Poisson equation

$$\frac{d^2U(x_*)}{dx_*^2}=-4\pi q^2\xi^3\int \frac{dIf\left(I,J_I^+\right)}{\sqrt{\frac{2}{m}(I-U)-\lambda x_*'^2}}.$$

Complete self-consistency is achieved when the integral on the right side is proportional to ξ^{-3}. It is enough to take the distribution function in the form :

$$f=\kappa_*\delta(I-I_0)\exp\left(-\frac{3}{2\tau_0}J_I^+\right) \quad (3)$$

Note that $\exp\left(\frac{\xi_0^3}{\xi}\right)=\frac{\xi_0^3}{\xi^3}$ In this case, the density is as follows:

$$n=\frac{\kappa}{\xi^4\sqrt{\frac{2}{m}(I_0-U)-\lambda x_*^2}}\exp\left\{-\frac{3}{2\tau_0}\int_0^{x_*}\frac{dx_*'\sigma\left(\frac{2}{m}\left(I_0-U\left(x_*'\right)\right)-\lambda x_*'^2\right)}{\sqrt{\frac{2}{m}\left(I_0-U\left(x_*'\right)\right)-\lambda x_*'^2}}\right\}$$

The Poisson equation is reduced to the form:

$$U''(x_*)=-4\pi q^2\xi_0^3\frac{\kappa}{\sqrt{\frac{2}{m}\left(I_0-U(x_*)\right)-\lambda x_*^2}}\times$$
$$\exp\left\{-\frac{3}{2\tau_0}\int_0^{x_*}\frac{dx_*'\sigma\left(\frac{2}{m}\left(I_0-U\left(x_*'\right)\right)-\lambda x_*'^2\right)}{\sqrt{\frac{2}{m}\left(I_0-U\left(x_*'\right)\right)-\lambda x_*'^2}}\right\} \quad (4)$$

Denote, next,

$$\upsilon_0^2=\frac{2I_0}{m}, s=\frac{x_*}{2\tau_0\upsilon_0}, s_0=\frac{x_{*0}}{2\tau_0\upsilon_0}, y=\frac{2U}{m\upsilon_0^2},$$
$$u(s)=\int_{s_0}^{s}\frac{ds'\sigma\left(1-y(s')+s'^2\right)}{\sqrt{1-y(s')+s'^2}}.$$

Then the Poisson equation follows:

$$y''(s)=-\kappa_1 u'(s)\exp\{-3u(s)\},$$
$$u'(s)=\frac{\sigma\left(1-y(s)+s^2\right)}{\sqrt{1-y(s)+s^2}}. \quad (5)$$

Note that the current density consists of two terms - the first of which is the "nose" type, independent of the field. The second term-defined by the relative velocity, $\upsilon_*=\sqrt{\frac{2}{m}(I_0-U)-\lambda s^2}$, depends on the field.

The density and current density are: $n=\frac{n_0}{\xi^4}u'\exp\{-3u\}, j_x=\frac{n_0 v_0}{\xi^5}(1-su')\exp\{-3u\}$.

Let's rewrite the system (5) in the following form:

$$y'=\frac{\kappa_1}{3}\exp\{-3u(s)\}-C_0,$$
$$u'(s)=\frac{\sigma\left(1-y(s)+s^2\right)}{\sqrt{1-y(s)+s^2}}. \quad (6)$$

in this case, the value κ_1 determines the boundary condition: $y_0'=\frac{\kappa_1}{3}+C_0$.

Figure 1 shows the solution of this system when $\kappa_1=3, y(0)=0, u(0)=0, C_0=-1,$ and Fig. 2 shows the dependence of the total charge of the clot (layer) and the current from s.

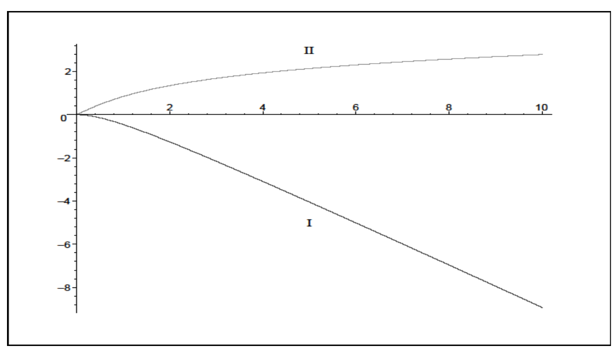

Figure 1: Dependencies y(s) (I) and u (s) (II).

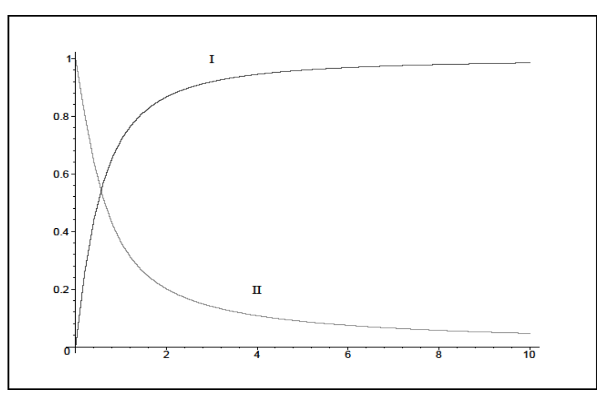

Figure 2: Dependence of the total charge.

ISBN: 978-3-95450-240-0 ISSN: 2673-5539 doi:10.18429/JACoW-RuPAC2021-TUPSB17

layer from s (I) and current from s (II).

The curve I in Fig. 2 shows the output to the state with the full charge of the layer in the variables s, τ. In the variables x, t this corresponds to the decrease in the charge of the layer and its size proportionally $\xi(t) = \sqrt{\xi_0^2 - \frac{t}{\tau_0}}$. In the case of a growing function $\xi(t)$ when the charges move in the negative direction of the axis x, then the total charge of the layer, constant in the variables in the variables x, t will grow. Note that the constant charge of the layer in non-stationary variables can be obtained only in the case when the transport and relative velocities have opposite directions.

CONCLUSION

In this paper, we study nonstationary self-consistent solutions for the potential of a flat layer and a spherical cluster of charges interacting with an eigenfield. The collisionless kinetic problem was solved.
The results of numerical solutions for the density and potential as functions of a self-similar variable are presented.

It is shown that it is possible for a collisionless system to reach a state with a constant charge in nonstationary coordinates x_*, τ. The rate of change of the total charge in the variables is determined by the time parameter τ_0 by $\tau_0 \to \infty$ the charge can remain almost constant. It is possible that the considered problem is interesting when solving the problem of accumulation of large charges.

Previously, solutions of problems in systems interacting with an eigenfield were considered in [2,3,4].

REFERENCES

[1] J. Mestschersky, "Uber die Integration der Bewegungs-gleichungen im Probleme zweier Korper von verandlicher Masse", *Astron.Nachr.*, v.159, p.229, 1902.

[2] A.S. Chikhachev, "Nonstationary self-consistent model of an ensemble in its own field", *Technical Physics*, vol. 59, No. 4, p. 474, 2014.

[3] A.S. Chikhachev "Dynamics of electric charges in a self-consistent field in a spherical-symmetric system", *PEPAN*, vol. 17, No. 3, pp. 325-327, 2020.

[4] A.S. Chikhachev, "System of charges in an eigenfield" *IISJ*, vol. 20, pp.8-17, 2020.