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DEVELOPMENT OF A PROGRAM CODE FOR CALCULATION OF CHARGED PARTICLE DYNAMICS IN RFQ

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Annotation

The code for the beam dynamics simulation in radio-frequency quadrupole (RFQ) accelerating structure was written in the C++ programming language. Charged particle beam dynamics was simulated in RFQ structure aimed to accelerate ion beams up to energies of 1.25 MeV / nucleon.

The characteristics of the beam at the exit of the structure (velocity, particle capture coefficient, beam profile, transverse emittances, longitudinal phase portrait) were determined.

INTRODUCTION

The considered accelerating structure is intended for the initial acceleration of ions with an A/Z ratio from 1 to 3.2 (A is the mass number, Z is the charge of the ion). Particles from H to O fall within the specified range.

In order for the capture coefficient in the acceleration mode to be high, it is necessary to use systems that are effective at a low initial particle velocity. These requirements are met by accelerating structures in which acceleration, grouping and focusing of charged particles by a radio-frequency (RF) electromagnetic field occur simultaneously.

RFQ ACCELERATOR

Focusing systems use conventional electrostatic or magnetic quadrupole lenses. To focus the beam in two transverse directions, it is necessary to change the polarity of the poles. When using an alternating RF field, it is possible to use a quadrupole system of electrodes, which is uniform along the axis of the accelerator [1]. Such a system is shown in Fig. 1.

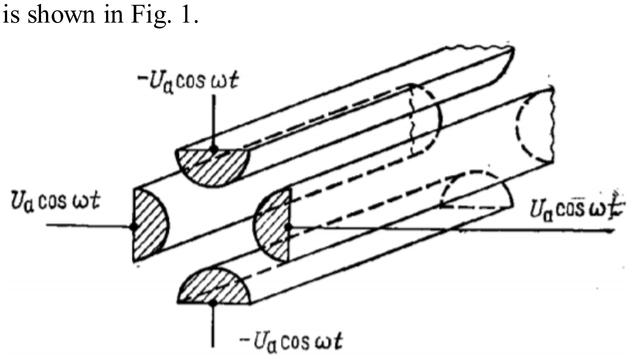

Figure 1: Quadrupole symmetric four-wire line.

The value $2U_a$ is the amplitude value of the voltage between adjacent electrodes. Since an RF voltage is applied to the electrodes, the particles, when moving along the axis, successively experience the action of fields with an alternating sign of the gradient.

The longitudinal accelerating component of the electric field in a four-wire line is created if the distance between opposite electrodes of the same polarity changes periodically along the axis [2]. The spatial period of the change in the distance between the electrodes should be equal to the path that the equilibrium particle travels during the RF period, and the phases of the change in the distances in the perpendicular planes are shifted by half a period.

PROTON AND LIGHT ION BEAM DYNAMICS SIMULATION

The selection of the parameters of any accelerating structure for calculating the particle dynamics begins with the selection of the operating frequency. It is selected based on the condition of the minimum length of an accelerator with a RFQ.

The closest to the optimal value is the widely used frequency of 81.25 MHz from the range of frequencies from 40.625 to 972 MHz.

Taking into account the literature data on the recommended field strength on the surface of the electrodes [3], the value of the Kilpatrick criterion was chosen 1.85.

A pre-buncher is usually placed between the ion source and the accelerating structure. Taking into account the buncher, the phase length of the beam at the entrance to the structure with the RFQ is set equal to 1.4 π.

EQUATION OF MOTION IN THE APPROXIMATION OF ANALYTICALLY GIVEN FIELDS

To simulate the action of an electromagnetic field on accelerated particles in the equation of motion, it is sufficient to take into account only the electric component of the RF field:

$$\frac{d\vec{p}}{dt} = e\vec{E}(r,t)\,. \tag{1}$$

In the Cartesian coordinate system, the expression for the accelerating potential in the RFQ structure on the axis of beam motion in the one-wave approximation has the form [3]:

$$U(x,y,z,t) = \frac{U}{2}\Big[A_{01}(x^2-y^2) + A_{10}I_0\left(k\sqrt{x^2+y^2}\right) \cdot \sin(kz)\Big]\cos(\omega t), \tag{2}$$

where U – voltage between electrodes;
k – wave number;
ω – RF field frequency;
I_0 – modified Bessel function;
A_{01} и A_{10} – focusing and accelerating parameters respectively.

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Using expression (2), we find the electric field strength. Next, we substitute in (1) and pass into Cartesian coordinates and obtain the equations of motion:

$$\begin{cases} \frac{dp_x}{dt} = \frac{eU}{2}\left[2A_{01}x + A_{10}I_1\left(k\sqrt{x^2+y^2}\right)\cdot\frac{kx}{\sqrt{x^2+y^2}}\cdot\sin(kz)\right]\cos(\omega t) \\ \frac{dp_y}{dt} = \frac{eU}{2}\left[2A_{01}y - A_{10}I_1\left(k\sqrt{x^2+y^2}\right)\cdot\frac{ky}{\sqrt{x^2+y^2}}\cdot\sin(kz)\right]\cos(\omega t) \\ \frac{dp_z}{dt} = \frac{eU}{2}\left[A_{10}I_0\left(k\sqrt{x^2+y^2}\right)\cdot k\cos(kz)\right]\cos(\omega t) \end{cases} \quad (3)$$

NUMERICAL SIMULATION OF ION DYNAMICS

At the first stage, the simulation of ion dynamics was carried out in the BEAMDULAC-RFQ program [4], [5].

Own program code for particle dynamics simulation has been written. When writing, a wide analysis of methods for calculating the dynamics of particles taking into account the space charge effect was carried out. The most interesting are the CIC method (Cloud in Cell) and the "large particle method". The latter method is easier to implement and allows direct calculation of the action of Coulomb forces without preliminary calculation of the potentials and intensities at the grid nodes. This calculation, in turn, imposes a certain error on the calculation results.

In the BEAMDULAC-RFQ program, the Runge-Kutta method of the 4th order is used to solve equations (3), and the CIC method is used to take into account the effect of space charge.

DEVELOPING OUR OWN PROGRAM CODE

Using the C ++ programming language, a physico-mathematical model of the ion beam motion in an accelerating structure with a RFQ has been created. To simulate the action of the RF field on the dynamics of particles, the equations of motion are used (3).

The developed program uses a direct calculation of the space charge "by the method of large particles" [6]. The particle beam is represented as a set of bunches located on the axis of the accelerating structure. Each bunch is represented by N uniformly charged balls of radius R (large particles). The force acting on the i-th particle from the j-th is calculated by the formula [6]:

$$F_{ij} = \begin{cases} \frac{q^2}{4\pi\varepsilon_0(2R)^2}\cdot\left(\frac{2R}{r_{ij}}\right)^3\cdot\frac{r_{ij}}{2R}, \quad r_{ij} \geq 2R \\ \frac{q^2}{4\pi\varepsilon_0(2R)^2}\cdot\left[\begin{array}{c} 2\left(\frac{2R}{r_{ij}}\right)^3 \\ -9\left(\frac{2R}{r_{ij}}\right)+8 \end{array}\right]\cdot\frac{r_{ij}}{2R}, \quad 0 \leq r_{ij} < 2R \end{cases} \quad (4)$$

Thus, summing up the forces for each particle, we obtain the influence of the remaining particles of the bunch on it:

$$F_i = \sum_j^N F_{ij}\,. \quad (5)$$

Taking the superposition of the forces acting on the particle from the RF field (3) and the forces of the own field of the particles (4), equation (1) is solved.

The counting time starts to grow noticeably when a larger number of particles are specified. However, the use of a hybrid computational model allows the use of a parallel (GPU) computation mode, for which the problem of calculating particle dynamics is ideally suited.

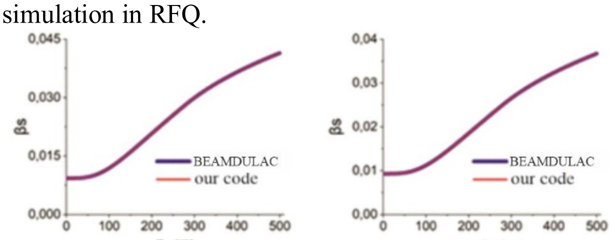
ION BEAM DYNAMICS SIMULATION RESULTS

The calculation of the dynamics of ions was carried out using the initial data presented in Table 1.

Table 1: Initial Data for Modeling Dynamics

Parameter	Value	
Ion type	H^+	O^{5+}
Frequency, MHz	81.25	
Structure length, mm	5043	
Emittance (x, y), π × cm × mrad	0.03	
Longitudinal pulse spread,%	± 0.1	
Beam size (x, y), mm	1	2
Voltage between adjacent electrodes, kV	40	140
Beam initial phase	-0.2 π	-0.2 π
Final phase of the beam	1.2 π	1.2 π
Minimum aperture, mm	7	
Particle current, mA	2	0.2

Figure 2 shows the velocities of an equilibrium particle during it's propagation along accelerating structure, calculated in different programs. It can be seen that the curves repeat each other perfectly. The discrepancy between the calculation results in the two programs is less than 0.1%. This fact makes it possible to use the developed code for solving problems of charged particle dynamics simulation in RFQ.



(a) (b)

Figure 2: Equilibrium particle velocity for protons (a) and ions O^{5+} (b).

As a result of calculations, phase portraits were obtained in all planes for protons (Fig. 3) and O^{5+} ions (Fig. 4).

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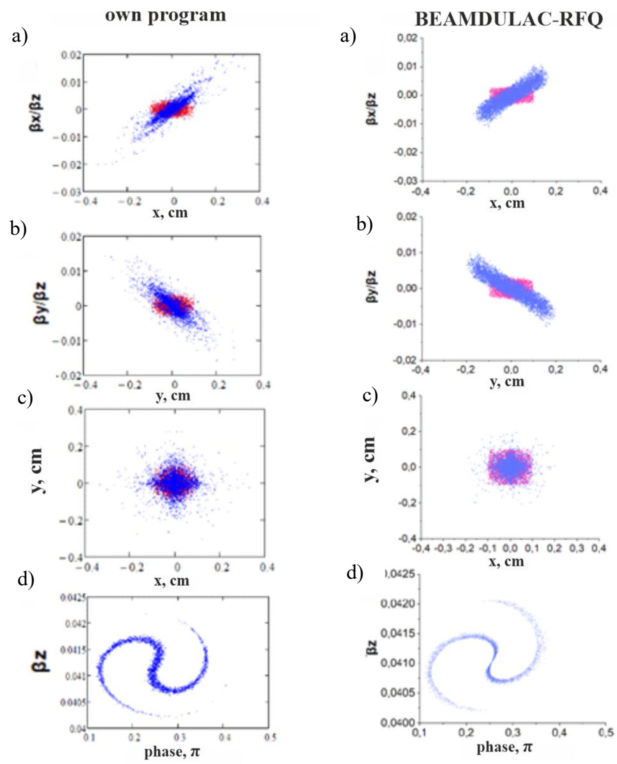

Figure 3: Results of proton dynamics simulation: on the left - in your own program; on the right - BEAMDULAC-RFQ (red dots - particles at the beginning of acceleration, blue dots - particles at the end of acceleration). a, b – transverse phase portraits; c – bunch cross-section; d – longitudinal phase portrait.

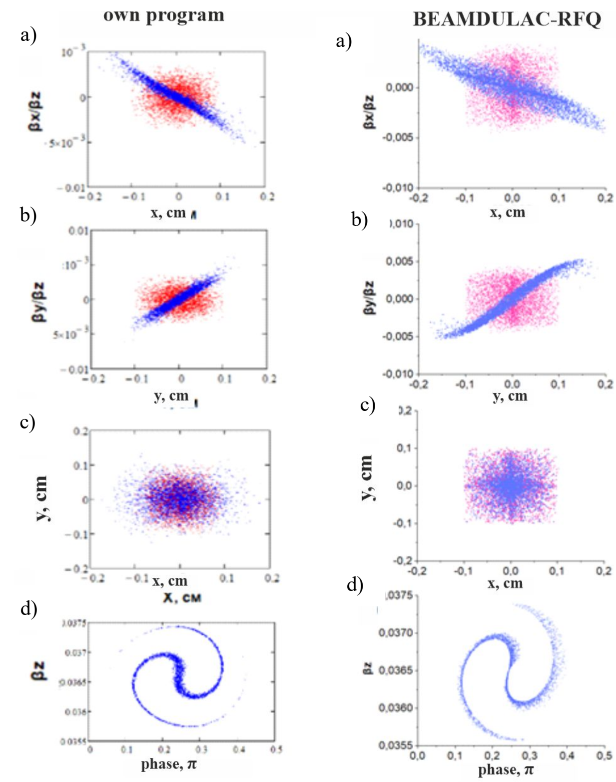

Figure 4: Results of O^{5+}dynamics simulation: on the left - in your own program; on the right - BEAMDULAC-RFQ (red dots - particles at the beginning of acceleration, blue dots - particles at the end of acceleration. a, b – transverse phase portraits; c – bunch cross-section; d – longitudinal phase portrait.

CONCLUSION

A program code has been developed for the ion beam dynamics simulation taking into account the space charge effect using a hybrid computation model, based on the "method of large particles" to estimate the intrinsic field of the beam. To verify the obtained results, control calculations were performed in the BEAMDULAC-RFQ program, which has proven itself in the accelerating community. The dynamic parameters of ions (Figs. 3 and 4), obtained as a result of simulation in our own program, are in full agreement with similar results obtained in the BEAMDULAC-RFQ program, despite different methods for calculating the space charge of the beam. At the exit from the accelerating structure, the phase length of the beam is 0.3 π at the initial 1.4 π, which indicates a successful longitudinal bunching. Thus, it was shown that the developed program code is in no way inferior to the analogue in terms of the counting time and allows to do charged particle dynamics simulation in the RFQ accelerating structures.

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