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METHOD AND SYSTEMATIC ERRORS FOR SEARCHING FOR THE ELECTRIC DIPOLE MOMENT OF CHARGED PARTICLE USING A STORAGE RING

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Abstract

One of possible arguments for CP-invariant violation is the existence of non-vanishing electric dipole moments (EDM) of elementary particles [1]. To search for the EDM the BNL proposed to construct a special ring implementing the frozen spin mode in order to detect the EDM signal. Since systematic errors determine the sensitivity of a method, this article analyses some major methods proposed for searching for the EDM from the point of view of this problem. The frequency domain method (FDM) proposed by the authors does not require a special accelerator for deuterons and requires spin precession frequency measurements only. The method has four features: the total spin precession frequency due both to the electric and the magnetic dipole moments in an imperfect ring in the longitudinal-vertical plane is measured at an absolute statistic error value of ~10^{-7} rad/sec in one ring filling; the ring elements position remain unchanged when changing the beam circulation direction from clockwise (CW) to counter-clockwise (CCW); calibration of the effective Lorentz factor by means of spin precession frequency measurements in the horizontal plane is carried out alternately in each CW and CCW procedure; the approximate relationship between the spin precession frequency components is set to exclude them from mixing to the expected EDM signal at a statistical sensitivity level approaching 10^{-29} e cm. The FDM solves the problem of systematic errors, and can be applied in the NICA facility.

ORIGINAL IDEA

The idea of searching for electric dipole moment of the proton and deuteron using polarized beams in a storage ring is based on "the frozen spin" method and was originally proposed at Brookhaven National Laboratory (BNL) [2]. The concept of the "frozen spin" lattice consists of deflectors with electric and magnetic fields incorporated in one element, in which the spin vector of the reference particle is always orientated along the momentum. This is clearly evident from the Thomas–Bargmann–Michel–Telegdi equation:

$$\frac{d\vec{S}}{dt} = \vec{S} \times \left(\vec{\Omega}_{mdm} + \vec{\Omega}_{edm}\right),$$

$$\vec{\Omega}_{mdm} = \frac{e}{m\gamma}\left\{(\gamma G + 1) - \left(\gamma G + \frac{\gamma}{\gamma+1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right\}, \quad (1)$$

$$\vec{\Omega}_{edm} = \frac{e\eta}{2m}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right),\ G = \frac{g-2}{2} \quad ,$$

where G is the anomalous magnetic moment, g is the gyromagnetic ratio, Ω_{mdm} is the spin precession frequency due to the magnetic dipole moment (hereinafter referred to as MDM precession), Ω_{edm} is the spin precession frequency due to the electrical dipole moment (hereinafter referred to as EDM precession), and η is the dimensionless coefficient defined in (1) by the relation $d = \eta e\hbar/4mc$. The advantages of purely electrostatic machines are especially evident at the "magic" energy, when:

$$G - 1/\left(\gamma_{mag}^2 - 1\right) = 0, \quad (2)$$

and the spin vector initially oriented in the longitudinal direction rotates in the horizontal plane with the same frequency as the momentum Ω_p, i.e., $\Omega_{mdm} - \Omega_p = 0$.

In the case of deuterons with $G = -0.142$ the only possible method is a storage ring with both electric and magnetic fields [3]. This can be done by applying a radial electric field E_r to balance the vertical magnetic field B_v contribution to Ω_{mdm}^p, as shown in Eq. (1):

$$E_r = \frac{GBc\beta\gamma^2}{1-G\beta^2\gamma^2} \approx GB_v c\beta\gamma^2. \quad (3)$$

Thus, for both protons and deuterons there is a general idea of how to construct a ring, but this is realized with the help of different types of deflectors.

METODS OF EDM MEASUREMENT

for searching for the EDM are determined by the success of solving the problem of systematic errors. From this point of view, there are currently three promising methods of searching for the electric dipole moment of protons and deuterons: BNL "frozen spin" method [2], Koop's "spin wheel" method [3] and Frequency Domain method (FDM) [4]. Basically, their difference is delineated by how the problem of systematic errors is solved.

BNL "Frozen Spin" Metod

First, we will consider the "frozen spin" method [2]. In common case the orientation of the spin in 3D space is determined by three frequency projections of spin precession due to magnetic dipole moment $\Omega_r, \Omega_y, \Omega_z$ and electric dipole moment Ω_{edm}:

$$\Omega = \sqrt{(\Omega_{edm} + \Omega_r)^2 + \Omega_y^2 + \Omega_z^2}. \quad (4)$$

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The main idea of the "frozen" spin concept is to create such a configuration of external fields that in an ideal accelerator without imperfection of elements of the storage ring the spin orientation changes only due to the presence of the electric dipole moment Ω_{edm}, that is at $\Omega_r, \Omega_y, \Omega_z \ll \Omega_{edm}$ we have $\Omega \approx \Omega_{edm}$. However, in a non-ideal storage ring with imperfection, when $\Omega_r \neq 0, \Omega_y \neq 0, \Omega_z \neq 0$, the spin changes in accordance with:

$$\tilde{S}_y = \sqrt{\left(\frac{\Omega_y \Omega_z}{\Omega^2}\right)^2 + \left(\frac{\Omega_r + \Omega_{edm}}{\Omega}\right)^2} sin(\alpha + \phi), \alpha = \Omega \cdot t \quad (5)$$

In BNL method the deviation of the spin vector in the vertical plane is measured, that is, the amplitude of the changing part of the signal $\tilde{S}_y$. Expecting it at the level of $\tilde{S}_y \approx 10^{-6} rad$ after $t \approx 1000$ sec and assuming that it is necessary to correct all misalignments to such a magnitude $\Omega_y, \Omega_z, \Omega_r << \Omega_{edm}$, that is the contribution will be determined only by the EDM signal. However, each of the residual frequencies $\Omega_y, \Omega_z, \Omega_r$ plays its own negative role as a systematic error. The most important factor determining systematic errors is the presence of errors in the installation of the elements (imperfections) of the ring, which leads to the appearance of vertical E_v and radial B_r components of the electric and magnetic fields, respectively. They both change the spin components in the vertical plane, in which the EDM signal is expected, and create the systematic errors that imitate the EDM signal. Even if we assume that the vertical component of the Lorentz force averaged over the ring F_v is equal to exactly zero due to the ideal adjustment of fields in elements of the ring to provide the stable motion [5],

$$\overline{F_v} = e\left(\overline{vB_r} - \overline{E_v}\right) = 0,. \quad (6)$$

we would still observe a non-zero rotation of the spin in the vertical plane, that is to say the "fake EDM" signal. Assuming that n number of arbitrary elements of length L are installed on the ring with the rms vertical error $\langle \delta h \rangle$ and that the condition (6) for them is fulfilled, one computes the standard deviation of the MDM spin precession frequency in the vertical plane defined by the radial axis:

$$\langle \Omega_{r,mdm} \rangle = \frac{e}{m\gamma} \cdot \frac{G+1}{\gamma} \cdot \frac{\langle B_r \rangle}{\sqrt{n}}. \quad (7)$$

where $\langle B_r \rangle$ is the rms value of the radial magnetic field. In the given case $\Omega_{r,mdm}$ and Ω_r are the same. The value of the radial component of the field $\langle B_r \rangle = B_v \cdot \langle \delta h \rangle / L$ is thus determined by (a) the slope of the magnet in the transverse plane defined by the longitudinal axis and (b) the vertical component of the magnetic field B_v. If we assume a realistic rms value of the installation error of an arbitrary magnet $\langle \delta h \rangle$=100µ, the spin precession frequency in the vertical plane will be on the order of $\langle \Omega_{r,mdm} \rangle$ ≈100 rad/sec when the size of the magnets is L ≈1 m and the total number of elements on the ring is $n \approx 100$. To solve this problem in the BNL method it was suggested the procedure of injecting two beams in the ring in two opposite directions, clockwise (CW) and counter-clockwise (CCW) [5]. If in the CW direction the deviations of the spin-vector from the horizontal plane due to the MDM and the EDM add up, then they subtract in the case of the CCW circulating beam. Adding the CW and CCW results together, the EDM can be separated from a systematic error arising due to the MDM. However, in the case of a deuteron ring, the magnetic component of the Lorentz force depends on the direction of motion, which therefore means that the polarity of the magnetic field needs to be changed when the direction of injection is different. This is a fundamental problem for the implementation of the CW-CCW procedure in the deuteron case.

Another unresolved problem from our point of view is the so-called geometric phase effect. If the frequencies $\Omega_y, \Omega_z, \Omega_r$ (see 4,5) in all three planes are of equal order of magnitude and close in value, but not equal to zero, then the invariant spin axis is completely undefined, that is, in each element of ring the spin rotates around the most pronounced axis with an indefinite amplitude. The effect of mixing the frequencies with the frequency of the EDM occurs, which, despite the use of two beams moving in opposite directions clockwise CW and counter clockwise CCW, eliminates the certainty of the measurements. This effect is called the "geometric phase" and it remains unresolved in the BNL method. Eq. (1)].

Koop's "Spin Wheel" Metod

Now we need to discuss the idea of Koop's "spin wheel" method. This method uses a transverse magnetic field instead of the CW-CCW procedure, causing the spin-vector to rotate in the vertical plane perpendicular to it first in the clockwise and then the counter-clockwise direction. Quoting I. Koop, we can formulate the basic concept of the "spin wheel" method. The idea of the method is to apply a relatively strong radial magnetic field B_y to provide for rapid spin rotation in the vertical plane, say about 0.1÷1 Hz instead of 10^{-9} Hz, as in the frozen spin scenario. If one controls the accompanied beam orbit splitting with a required accuracy, then it is possible to extract the EDM contribution to a measured spin precession rate just comparing runs with a positive $\langle z1 \rangle - \langle z2 \rangle = +\Delta$ and negative $\langle z1 \rangle - \langle z2 \rangle = -\Delta$ orbit separation. Measuring the spin precession frequency $\Omega_x(\pm\Delta)$in the vertical plane one obtains

$$\Omega_{edm} = \frac{\Omega_x(+\Delta) + \Omega_x(-\Delta)}{2} \text{ at } \Omega_r = \Omega_{r,mdm} \pm \Omega_{B_x} \quad (8)$$

$$\Omega_x(\pm\Delta) = (\Omega_{edm} + \Omega_r)\sqrt{1 + (\Omega_y^2 + \Omega_z^2)/(\Omega_{edm} + \Omega_r)}$$

The author makes an estimate of the contribution of the average radial magnetic field at a level of 10^{-13} Gauss, which produces a mimic effect comparable with the EDM at the level $d = 10^{-29}$ e·cm. According to D. Kawall, the accompanying beam orbit splitting is on the order of 10^{-12}m. Here the author supposes two things: (1) that they

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can measure the average value of the orbit with an accuracy of 10^{-12} m using SQUIDs and (2) that the MDM spin precession frequency is completely determined by the average orbit, hence $\Omega_x(+\Delta) = \Omega_x(-\Delta)$. We disagree with assumption (1) on the grounds that such an orbit displacement measurement accuracy has never been shown experimentally, and we believe assumption (2) to be wrong because the spin precession frequency of a bunched beam in the presence of an RF field depends on the beam orbit length, but not the average orbit shift Δ. Besides, it is not clear how one can eliminate the contribution from the MDM frequency Ω_x^{mdm}arising due to imperfections. Assuming $\Omega_y, \Omega_z << \Omega_{B_x}$ we get

$$\frac{\Omega_x(+\Delta)+\Omega_x(-\Delta)}{2} = \Omega_{edm} + \Omega_r^{mdm}, \quad (9)$$

but not Ω_{edm}. This is only possible if $\Omega_r^{mdm} = 0$. This conclusion is obvious, since two terms Ω_x^{mdm} and Ω_{B_x} are aligned with the EDM.

FREQUENCY DOMAIN METHOD

In FDM [4] only spin precession frequency measurements are involved and at an accuracy that already has been experimentally verified [6]. The method is based on four fundamental features: the total spin precession frequency due to the electric and magnetic dipole moments in an imperfect ring in a vertical plane is measured at an absolute statistic error value of ~10^{-7} rad/sec for one ring filling; a position of the ring elements is unchanged from clockwise (CW) to counter-clockwise (CCW) procedures; the calibration of the effective Lorentz factor using the spin precession frequency measurement in the horizontal plane is carried out alternately in each CW and CCW procedure; the approximate relationship between the frequencies of the spin in different planes is set to exclude them from mixing to the vertical frequency of the expected EDM signal at a statistical sensitivity level approaching 10^{-29} e cm. The total spin precession frequency in the vertical plane is measured with a clockwise direction of the beam $\Omega_{CW} = \Omega_{r,mdm}^{CW} + \Omega_{edm}$ and compared with counterclockwise measurements $\Omega_{CCW} = -\Omega_{r,mdm}^{CCW} + \Omega_{edm}$. The sum of the frequencies of these two signals $\Omega_{edm} = (\Omega_{CW} + \Omega_{CCW})/2 + (\Omega_{r,mdm}^{CCW} - \Omega_{r,mdm}^{CW})/2$ allows one to identify the frequency of the EDM signal, which in turn converts into the EDM value. However, given an accuracy of the EDM measurement it is completely determined how exactly the condition $\Omega_{r,mdm}^{CCW} = \Omega_{r,mdm}^{CW}$ must be fulfilled after changing the polarity of the magnetic field. We must therefore reformulate the global problem regarding how to restore the conditions for the equal contribution of the MDM spin frequency. Studying the spin-orbital dynamics of the beam, we introduced a fundamental parameter, the effective Lorentz factor $\gamma_{eff} = \gamma_s + \beta_s^2\gamma_s \cdot \Delta\delta_{eq}$,which determines the spin precession in 3D space [4,7]:

$$\Delta\delta_{eq} = \frac{\gamma_s^2}{\gamma_s^2\alpha_0 - 1}\left[\frac{\delta_m^2}{2}\left(\alpha_1 - \frac{\alpha_0}{\gamma_s^2} + \frac{1}{\gamma_s^4}\right) + \left(\frac{\Delta L}{L}\right)_\beta\right], \quad (10)$$

where $\Delta\delta_{eq}$is the deviation of the equilibrium level (average value) of momentum due to the orbit increasing in length in the transverse plane $(\Delta L/L)_\beta$ and due to synchrotron oscillation with amplitude δ_m, $\alpha_0, \alpha_1$are the zero and first order momentum compaction factors, while γ_s is the Lorentz factor of the synchronous particle. Using γ_{eff}, we can assert: two particles are assumed to be the same, or, equivalently, the beams are identical in terms of spin behaviour if they have the same effective Lorentz factor averaged over all particles in the beam. This ensures it is no longer necessary to obtain a coincidence of trajectories, but instead only requires the condition of equality γ_{eff} for the CW and CCW beams. In this regard, before changing the polarity, we must calibrate the effective Lorentz factor. Calibration of the effective Lorentz factor is done via measuring spin precession in the horizontal plane where we have no contribution from Ω_{edm}. For that purpose, a special transverse spin rotator Wien filter is used in order to suppress the spin precession in the vertical plane without beam trajectory perturbation together with a small detuning of the beam energy from the magic value. This procedure allows one to change the direction of the invariant spin axis from horizontal to vertical. Using the fact that ν_s is an injective function of γ_{eff}, it follows that there exists a unique value, γ_{eff} at which the polarization vector is frozen with respect to the beam's momentum vector in the horizontal plane, i.e., $\nu_s = 0$ in the rest frame. Since the tilt of the spin precession axis is the same for the CW and the CCW beams, $\lim_{\nu_s^{CCW} - \nu_s^{CW} \to 0} \Omega_{r,mdm}^{CCW} - \Omega_{r,mdm}^{CW} \to 0$. After calibrating the effective Lorentz factor, we turn off the Wien filter transferring the invariant spin precession axis from horizontal to vertical position and measure $\Omega_{CW} = \Omega_{r,mdm}^{CW} + \Omega_{edm}$.

Another important problem, the "geometric phase" (GP) error, is the accumulation of spin rotation in the vertical y−z plane caused by non-commuting rotations in the horizontal x−z and transverse vertical x−y planes. Formulated in the frequency domain language, it is a result of a lack of a definite direction of the spin precession axis. Our goal in minimizing the GP effect is to make the Ω_{edm} contribution to $\Omega = \left((\Omega_{edm} + \Omega_r)^2 + \Omega_y^2 + \Omega_z^2\right)^{1/2}$ much larger than that of Ω_y and Ω_z. That is, we have to fulfil the requirement $(\Omega_{edm} + \Omega_r)^2 > \frac{1}{2}\frac{\Omega_y^2+\Omega_z^2}{\Omega_{edm}}$. According to this equation, the restriction occurs at the values of Ω_y and Ω_z, which should have less of an effect on the total frequency Ω than the EDM: $\frac{\Omega_y^2+\Omega_z^2}{2\Omega_r} < \Omega_{edm}$. Since we expect the Ω_r in the range of 50 to 100 rad/sec, it follows that making Ω_y and Ω_z. < 10−3 rad/sec is sufficient to minimize the GP error to below the Ω_{edm} value. Note that the solution of the GP problem does not require knowledge of the precise values of Ω_y and Ω_z, they just have to be small.

Thus, FDM has significant advantages over the two methods discussed above: the method is based on measuring the spin precession frequency and the problems of the geometric phase and the transition from CW to CCW are solved.

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