

# SIMULATION AND DESIGN OF THE PERMANENT MAGNET MULTIPOLE FOR DC140 CYCLOTRON

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## Abstract

Permanent magnet (PM) multipoles are very attractive for beam transportation and focusing in accelerators. The primary advantages over electromagnets are no power supply and no cooling systems, better maintainability.

A PM quadrupole is supposed to be utilized in the DC140 cyclotron destined for acceleration of heavy ions which is under construction in JINR, Dubna. The extracted ion beam passes through a region where the stray field reduces sharply. Horizontal focusing of the beam line will be provided with a passive magnetic channel (MC1) and a PM quad (PMQ) in the strong and low field regions, respectively.

## REQUIRED PMQ PARAMETERS

Table 1 lists design parameters of the quad.

Table 1: PMQ Design Parameters

Field gradient ( $G_0$ )	8.1 T/m
Working region (hor. $\times$ vert.)	64 mm $\times$ 25 mm
Aperture (hor. $\times$ vert.)	80 mm $\times$ 32 mm
Overall sizes (hor. $\times$ vert.)	170 mm $\times$ 106 mm
Effective length* ( $L_{\text{eff0}}$ )	299.26 mm
Error in working region** ( $\Delta_{x,y}$ )	$\pm 1\%$

\*The effective length of the quad is calculated as:

$$L_{\text{eff0}} = \frac{1}{G_0} \int_{-L/2}^{L/2} \frac{\partial B_y(0,0,z(s))}{\partial x} dz(s)$$

\*\*A linear approximation error in the horizontal and vertical directions is evaluated as:

$$L_{\text{eff0}} \Delta_x = \frac{1}{G_0 x} \int_{-L/2}^{L/2} \left[ B_y(x,y,z) - \frac{\partial B_y(0,0,z)}{\partial x} x \right] dz = L_{\text{effx}} - L_{\text{eff0}}$$

$$L_{\text{eff0}} \Delta_y = \frac{1}{G_0 y} \int_{-L/2}^{L/2} \left[ B_x(x,y,z) - \frac{\partial B_x(0,0,z)}{\partial y} y \right] dz = L_{\text{effy}} - L_{\text{eff0}}$$

Also, additional criteria should be considered together with the above magnetic specification, see Table 2.

Table 2: PMQ Operating Conditions

External field	0.35 T
Vacuum	$10^{-7}$ Torr
Operating temperature	30-40 °C
Temperature range	20-70 °C
Life time	10-15 years

The DC140 quad will be located near the dees and designed as a set of identical PMs rigidly fixed in a non-magnetic housing encircling the aperture.

Additional aspects of the quad specification include:

- simple PM shape, preferably cuboidal bricks,
- minimized number of PM in assembly,
- minimized nomenclature,
- commercial availability of PM

## STAGES OF PMQ DESIGN

The initial quad design is selected from an analytical study with the use of a simplified 2D model [1]. At this stage the number and positions of PMs are determined. Then the chosen configuration is optimized in iterative 2D and 3D parametric simulations with realistic PM shape and magnetic characteristics in mind. Simulated data are used to select candidate magnet materials, number, dimensions, and tolerated mechanical and magnetic errors of PM blocks. As a result, the quad design is finalized, and an assembly procedure is proposed. Additional adjustment may be required on the basis of measurements of supplied PM. The assembled quad is magnetically inspected to make sure the desired field requirements are reached.

## SIMPLIFIED 2D MODEL OF PMQ

An initial configuration has been generated with the use of 2D analytical models [1] based on the mathematically strict reasoning:

- In the absence of degaussing (the magnetization  $\mathbf{M} = \text{const}$ ), a field integral of PM with a finite length  $L_{\text{PM}}$  is equal to a two-dimensional field  $\mathbf{B}_{2D}$  generated by an infinite PM multiplied by  $L_{\text{PM}}$ :

$$\int_{-\infty}^{+\infty} \mathbf{B}_{3D}(x,y,z) dz = \mathbf{B}_{2D}(x,y) \cdot L_{\text{PM}}$$

So, the 2D approximation is quite suitable to determine the initial PM quad configuration which then will be corrected with respect to the demagnetization.

- With distance, the field generated by an infinite PM of an arbitrary shape is approaching the field of an infinite PM cylinder with the same dipole magnetic moment. Thus, the cylindrical representation is applicable for the initial study of magnetic performance. At the next stage, modifications are introduced reasoning from practical implementation.

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Particularly, the PM shape has been adjusted for cuboidal bricks.

- The field of a radially magnetized PM cylinder is described with a simple analytical expression:

$$H = \frac{(\mathbf{n}, \mathbf{m})\mathbf{n} - \mathbf{m}/2}{\pi r^2}$$

where  $\mathbf{m} = \mathbf{M} \cdot \pi R^2$  – dipole magnetic moment per unit length;  $\mathbf{M}$  – magnetization vector,  $R$  – magnet radius;  $r = |\mathbf{r}|$  – distance between the magnet center and an observation point,  $r > R$ ;  $\mathbf{r}$  – position vector to the observation point;  $\mathbf{n} = \mathbf{r}/r$  – unit vector from the magnet center to the observation point. The cylindrical representation is convenient to use as the field generated by the magnet cylinder depends on the product of its area and magnetization, not on these parameters individually.

Finally, the simplified 2D model of the PM quad is composed as a set of infinite rods magnetized in the radial direction and placed around the aperture. At the next stage orientations and positions of the magnets are optimized in order to ensure desired field quality.

### Field Quality Criteria

Field quality criteria is deduced from two assertions:

- Linear approximation errors in the horizontal and vertical directions,  $\Delta_x$  and  $\Delta_y$ , (see Table 1) are below the field gradient error  $\varepsilon_G$ :

$$|\Delta_{x,y}| \leq \varepsilon_G = \frac{|\mathbf{G} - \mathbf{G}_0|}{|\mathbf{G}_0|}, \quad \mathbf{G} = \nabla B_y$$

- The beam in the magnet aperture is localized within a region of the elliptical shape.

Therefore, the field quality of the quad is assessed through the maximal deviation of the field gradient from the required level of 8.1 T/m within the ellipse inscribed in the 64 mm × 25 working region.

### PMQ Optimization Using 2D Model

– PM rods with unknown dipole magnetic moments are positioned over the XY plane around the aperture.

– Unknown components ( $m_x$ ,  $m_y$ ) of the dipole magnetic moments form the vector of unknowns  $\mathbf{X}$ .

– Inside the working region a set of reference points is taken with a step  $\Delta\varphi = 1^\circ$  over the line  $x = a_x \cos \varphi$ ,  $y = a_y \sin \varphi$ ,  $0 \leq \varphi \leq \pi/2$ ,  $a_x = 32$  mm,  $a_y = 12.5$  mm.

– Target values of the field gradient  $\mathbf{G}_0 = \nabla B_{0y} = (8.1, 0)$  T/m at the reference points create the right-hand vector  $\mathbf{Y}_0$ .

– Values of field gradient at the reference points which are linearly dependant on  $\mathbf{X}$  form vector  $\mathbf{Y} = \mathbf{A}\mathbf{X}$ , where  $\mathbf{A}$  is the matrix with coefficients derived from the field generated by a cylindrical magnet.

The aim is to find vector  $\mathbf{X}$  (i.e combination of the dipole moments  $\mathbf{m}$ ) that provides  $\mathbf{Y} = \mathbf{A}\mathbf{X}$  (gradient  $\mathbf{G}$  at the reference points) as close to  $\mathbf{Y}_0$  (the required  $\mathbf{G}_0$ ) as possible. The optimal solution is searched through minimizing the functional

$$\Phi = \max_k |\mathbf{m}_k|^2 = \max_k \|X^{(k)}\|_2^2$$

at given

$$\Psi = \max_l |\mathbf{G}_l - \mathbf{G}_0|^2 = \max_l \|Y^{(l)} - Y_0^{(l)}\|_2^2 = \text{const}$$

The research and optimization lead to a set of 26 identical PM, 11mm × 11mm each, positioned around the aperture with different orientations, see Fig. 1. Fig. 2 shows calculated field deviations from the ideal distribution. Field gradient error in the elliptical region was found to be below 0.07%.

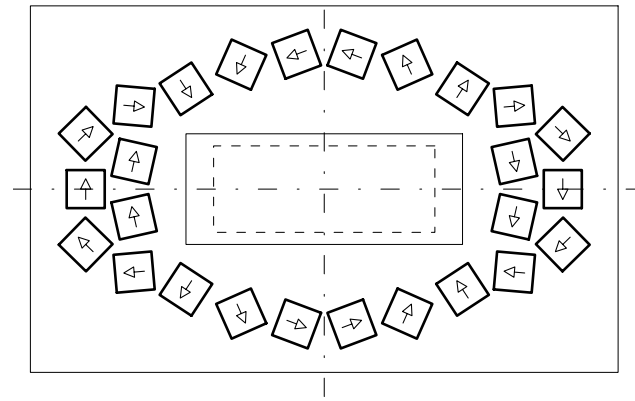


Figure 1: Quad formed with 26 identical 11 mm × 11 mm PM bricks each magnetized to 1.1402T. Arrows indicate PM orientations, dashed lines bound 64mm × 25mm working region, solid lines are for 80mm × 32mm aperture and 170mm × 106mm overall sizes.

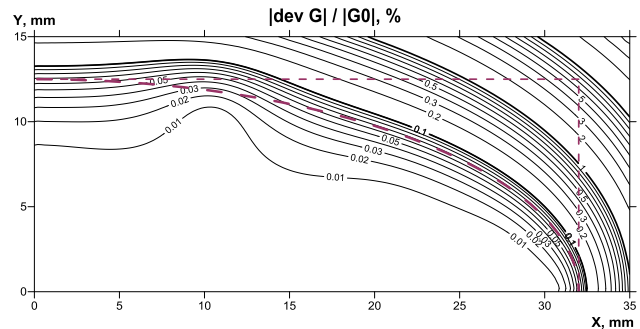


Figure 2: Relative gradient error  $\varepsilon_G = |\delta\mathbf{G}|/|\mathbf{G}_0|$  (%). Dashed lines indicate 64mm × 25mm working region with inscribed elliptical region.

### CORRECTION FOR REALISTIC PM

The selected configuration is then used in 2D and 3D field simulations, performed with KOMPOT and KLONDIKE codes [2, 3]. The quad model is fully parametrized and accommodated to influential parameters of actual PM blocks, primarily: realistic 3D geometry of PM, remanent field  $B_r$  and magnetic susceptibility  $\chi$  (anisotropic, if that is the case). Sensibility of the simulated field to various perturbing factors is presented in Table 3 in terms of the gradient error  $\varepsilon_G$ .

Table 3: Max Gradient Error  $\varepsilon_G$  in Elliptical Region

Perturbing factor	max $\varepsilon_G$
Unperturbed system	0.07%
Square PM cross-section	0.5%
Intrinsic PM degaussing ( $\kappa=0.1$ )	5%
Mutual PM magnetization ( $\kappa=0.1$ )	0.6%
Anisotropy $\kappa$ ( $\kappa_{  M}=0.1$ , $\kappa_{\perp M}=0$ )	0.3%
3D representation ( $\kappa=0.1$ )	0.17%

Simulations have demonstrated that the influence of any of the perturbing factor is described by a smooth, weakly variable function at low variations of the relevant criterion. This was used as a basis for iterative optimization of the quad design in order to neutralize the perturbing factors.

### Iterative Optimization of PMQ Design

At every iteration  $k$  ( $k = 0, 1, \dots$ ):

- The target gradient value  $G_{targ}$  is prescribed at the reference points:  $G_{targ,k} = G_0 - \Delta G_{corr,k-1}$  (at  $k = 0$ :  $G_{targ,0} = G_0$ ).
- Using  $G_{targ,k}$ , optimization with the simplified 2D model is performed to determine: (a) orientation and magnetic moments  $m_k$  for every PM, (b) optimized gradient map:  $G_{simp,k}$ .
- The obtained  $m_k$  are then used in 2D/3D simulation with perturbing factors involved. The simulated data represent expected gradient distribution  $G_k$ .
- Finally, influence of the perturbing factors is estimated as  $\Delta G_{corr,k} = G_k - G_{simp,k}$ .

It took 2 iterations to determine the PM parameters enabling the desired field quality accurate to  $\varepsilon_G = 0.08\%$ . In the calculations magnetic susceptibility of the Nd-Fe-B PM  $\kappa = 0.1$  was applied.

As a result, the optimized PM quad specification was generated (Table 4). The numerical study also defines tolerances for the PM geometry and orientation (Table 5).

The proposed design (Fig. 3) envisages assembly technologies and long-term vacuum operating conditions.

## CONCLUSIONS

The iterative numerical study has been used to optimize the PM quad design and assess sensitivity of its magnetic specification to various factors. However, any procured batch of PM may have fluctuations from the specified characteristics. The remanent field of commercial PM typically deviates up to 3%. Magnetic susceptibility and PM dimensions may also vary over the batch. This necessitates additional correction with respect to results of the acceptance inspection. The iterative optimization of the quad design should be repeated with the detected imperfections as inputs.

Temperature characteristics of the PM material (typically  $B_r - 0.1\%/^{\circ}\text{C}$ ) and possible long-term decay of magnetic properties must be kept in mind in the design. The final design will reflect all latest modifications.

Table 4: PM Parameters at Operating Temperature

Remanent field $B_r$	1.1853 T
Magnetic susceptibility $\kappa$	0.1
Linear piece of B-H curve	up to 1200 kA/m
Nd-Fe-B grade	N35SH, N35UH
PM cross-section	11 mm $\times$ 11 mm
PM length $L_{PM}$	300 mm
Total number of PM	26

Table 5: Geometrical and Magnetic Tolerances

PM dimensions	$\pm 0.05$ mm
Remanent field $ B_r $ :	
- average over batch	$\pm 3\%$
- single PM	$\pm 1-1.5\%$
Magnetization direction	$\pm 1^{\circ}$
Positioning:	
- X, Y, Z coordinates	$\pm 0.05$ mm
- orientation	$\pm 0.3^{\circ}$

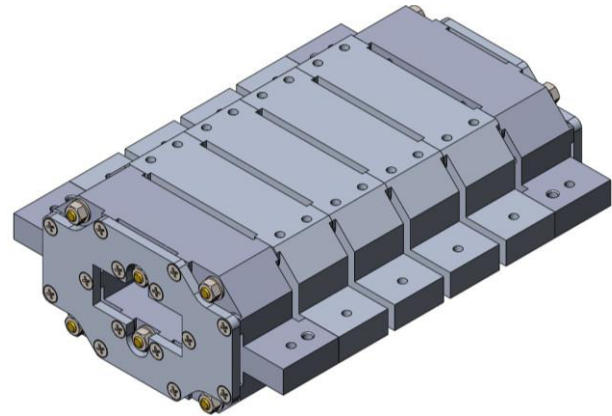


Figure 3: Proposed PM quad design.

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