Russian Particle Accelerator Conference RuPAC'2018

# Optimum Luminosity of Proton-Ion Collider

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October 1 – 5, 2018

# Outlook

**Part I. Luminosity of a symmetric and an asymmetric colliders**

**Part II. How to find optimal values of collider parameters**

1. Limitation of collider parameters by beam space charge effects
2. Optimization of luminosity of a collider

**Part III. Three numerical examples: NICA collider in the mode of**

- 3-1. proton-ion colliding beams (asymmetric mode)
- 3-2. Au × Au colliding beams (symmetric mode)
- 3-3. proton-ion colliding beams (symmetric mode)

**Conclusion**

## 1. Luminosity of a symmetric collider (well-known formula):

$$L = \frac{n_{bunch} N_1 N_2 f_0}{4\pi\sqrt{\varepsilon_x \varepsilon_y} B^*} \cdot F_{HG},$$

$$F_{HG}(\alpha) = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{e^{-u^2} du}{1 + (\alpha u)^2}, \qquad \alpha = \frac{\sigma_s}{B^*} \tag{1}$$

Here we are using:

$\varepsilon_{x,y}$ – *x*, *y*-emittances of two bunched colliding beams of gaussian density distribution,

$B^*$ - beta-function value in IP (Interaction Point),

$n_{bunch}$ – number of bunches in each of the two beams,

$N_{1,2}$ – particle number in each bunch of both beams

$\sigma_s$ – bunch length ($\sigma$-parameter) of both beams,

$F_{HG}$ – parameter of so called *Hour glass effect* (accounting relation of $B^*$ и $\sigma_s$ ).

## 2. Luminosity of asymmetric collider with common system of final focus*):

$$L = \frac{n_{bunch} N_1 N_2 f_0}{(2\pi)^2 \sigma_{s1} \sigma_{s2}} \cdot Int \,, \tag{2}$$

$$Int = \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta \frac{exp\left\{-\frac{1}{2}\left[\frac{\eta^2}{\sigma_{s1}^2} + \left(\frac{\eta + V\xi}{\sigma_{s2}}\right)^2\right]\right\}}{\sqrt{\left(\varepsilon_{x1} B_{x1}(\xi,\eta) + \varepsilon_{x2} B_{x2}(\xi,\eta)\right)\left(\varepsilon_{y1} B_{y1}(\xi,\eta) + \varepsilon_{y2} B_{y2}(\xi,\eta)\right)}}$$

Here $V = 1 + (v_1/v_2)$, $v_1 > 0$, $v_2 < 0$ for colliding beams and $v_{1,2} > 0$ for merging beams . Indices 1, 2 indicate parameters of the bunches of the 1st and 2nd colliding beams and values of the beta-functions for them. The last ones differ if the lenses of the final focus are common for both colliding beams and the particles differ by one, at least, parameter like charge, mass, momentum, etc.

*) See details in the Reference

[1] I. N. Meshkov, Luminosity of a Collider with Asymmetric Beams, Phys.Part. Nucl. Lett., 2018, 15, No. 5, pp. 506–509.

## 2. Luminosity of asymmetric collider with common system of final focus (continued)

Since beta-functions are proportional to the particle magnetic rigidity, one can introduce the *parameter of relative magnetic rigidity*

$$\lambda_x = \frac{B^*_{x1}}{B^*_{x2}} = \frac{p_1}{Z_1} \cdot \frac{Z_2}{p_2} \ . \qquad (3)$$

Here $p_{1,2}$ are particle momenta, $Z_{1,2}$ are their charge numbers.

From second ratio in Formula (3) follows the equality:

$$\lambda_x = \lambda_y$$

We note, the parameter λ depends, generally speaking, on particles energy (momenta).

Beta-functions $B_{1,2}(\xi,\eta)$ depend on two parameters – longitudinal coordinates of colliding particles $\xi$ and $\eta$ in the moving reference system of one of colliding bunches (see Ref.[1]).

## 2. Luminosity of asymmetric collider with common system of final focus (continued)

Formula (3) is simplified if the circumferences of the rings are equal. Then, from the synchronization condition follows:

$$v_1 = v_2 .$$

Therefore the λ is constant (does not depend on energy):

$$\lambda = \frac{p_1}{Z_1} \cdot \frac{Z_2}{p_2} = \frac{A_1 m_N \gamma_1 v_1}{Z_1} \cdot \frac{Z_2}{A_2 m_N \gamma_2 v_2} = \frac{A_1}{Z_1} \cdot \frac{Z_2}{A_2}, \qquad (4)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \ \beta = \frac{v}{c}.$$

## 2. Luminosity of asymmetric collider with common system of final focus (continued)

Further we consider a simplified case of similar parameters of two colliding beams and lattice of two rings:

$$C_1 = C_2 \equiv C_{Ring}\,,\ \ Q_{x1} = Q_{y1} = Q_{x2} = Q_{y2} \equiv Q \quad . \tag{5}$$

$$v_1 = v_2,\ \ \varepsilon_{x1} = \varepsilon_{x2} = \varepsilon_{y1} = \varepsilon_{y2} \equiv \varepsilon,\ \ \sigma_{s1} = \sigma_{s2} \tag{6}$$

In this case, the formula luminosity for asymmetric collider (2) takes the form:

$$L = \frac{n_{bunch} N_1 N_2 f_0}{4\pi^{3/2} \varepsilon B_1^*} \int_{-\infty}^{\infty} e^{-\chi^2} \frac{d\chi}{\left[1 + \left(\frac{\sigma_s}{B_1^*} \cdot \chi\right)^2\right] + \frac{1}{\lambda}\left[1 + \left(\lambda \cdot \frac{\sigma_s}{B_1^*} \cdot \chi\right)^2\right]} \tag{7}$$

# Part II. How to find optimal values of collider parameters

## 1. Limitation of Collider Parameters by Beam Space Charge Effects

**The choice of the parameters of the collider is determined to a large extent by the stability conditions of the beams circulating in the rings of the collider. The strongest limitations are the effects of the space charge of the beams - the so-called "Laslett effect" and the "beam-beam effect". Both of them lead to shifts in the frequencies of betatron oscillations of particles in the collider focusing system, bringing them closer to the frequencies of the nonlinear resonances of particles 1 colliding with a bunch of particles of the colliding beam 2.**

**For beams with a Gaussian distribution of the particle density along the transverse (x, y) and longitudinal (s) coordinates, this shift $\Delta q_1$ is (oscillations in the x-coordinate) for *the Laslett effect*:**

$$\Delta q_1 = \frac{Z_1^2}{A_1} \cdot N_1 \cdot a_1 , \qquad a_1 = \frac{r_p}{4\pi\beta_1^2\gamma_1^3\varepsilon_{x1}} \cdot \frac{C_1}{\sqrt{2\pi}\sigma_{s1}} \qquad \textbf{(8)}$$

**and for *the beam-beam effect*:**

$$\xi_{12} = \frac{Z_1 Z_2}{A_1} \cdot N_2 \cdot b_1, \quad b_1 = \frac{r_p}{(2\pi)^{3/2}\beta_1\gamma_1\varepsilon_{x2}} \cdot \frac{\langle B_1 \rangle}{B_2^*} \cdot \frac{1+\beta_1\beta_2}{\beta_1+\beta_2} \cdot \left(1 + \frac{\sigma_{s1}}{\sigma_{s2}}\right).$$

**(9)**

## 1. Limitation of a Collider Parameters by Beam Space Charge Effects (Contnd)

In Formulas (9), (10) $r_p$ = 2.818 cm is the classical radius of the proton, $\beta_1$, $\beta_2$ is the particle velocity in terms of the speed of light, $\gamma_1$ is the Lorentz factor of the particle 1, $\varepsilon_{x1}$, $\varepsilon_{x2}$ are the emittances of the bunches 1 and 2 in the x-coordinate, $\sigma_{s1}$, $\sigma_{s2}$ are the longitudinal "sigma" dimensions of their clots, $\langle B_{x1} \rangle$, $\langle B_{x2} \rangle$ are average values of betatron functions of collider rings:

$$\langle B_{x1,2} \rangle = \frac{C_{1,2}}{2\pi Q_{x1,2}}, \qquad (10)$$

$C_{1,2}$ are the circumferences of the rings, $Q_{x1,2}$ are their betatron numbers (number of betatron oscillations per revolution of the particle in the ring). The values of the parameters $\Delta q_2$ and $\xi_{21}$ for the entercounting particles are obtained by changing the indices 1 ⇄2.

The betatron shifts Δq and ξ depend on the energy of the particles through the Lorentz factors β and γ.

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The betatron shifts Δq and ξ depend on the energy of the particles through the Lorentz factors β and γ.

**The sum of the shifts $\Delta q + \xi = \Delta Q$ usually serves to estimate the stability of the particle beam in the collider. It is well known from practice that the intense beam is stable (under other fulfilled conditions), if**

$$\Delta Q \leq 0.05 \qquad (11)$$

# Part II. How to find optimal values of collider parameters

## 2. Optimization of Luminosity of a Collider

When choosing the optimal values of the parameters of the collider beams, one could, apparently, write down two equations

$$\Delta q_1 + \xi_{12} = \Delta Q_1 \,, \qquad (12)$$

$$\Delta q_2 + \xi_{21} = \Delta Q_2 \,, \qquad (13)$$

and require to meet condition (11) for both beams: $\Delta Q_1, \Delta Q_2 \leq 0.05$.

Then, substituting here the values of Δq (9) and ξ (10), we obtain a system of two algebraic equations with respect to the unknowns $N_i$ and $N_p$:

$$\frac{Z_1^2}{A_1} a_1 N_1 + \frac{Z_1 Z_2}{A_1} b_1 N_2 = \Delta Q_1 \,, \qquad (14)$$

$$\frac{Z_2^2}{A_2} a_2 N_2 + \frac{Z_1 Z_2}{A_2} b_2 N_1 = \Delta Q_2 \,. \qquad (15)$$

(Note that these equations are iconnected through the beam-beam effect parameters $\xi_{12}$ and $\xi_{21}$.) However, when trying to solve the "forehead" of this system of equations, it turns out that for certain valuesof the particle energy the determinant of the system drops down to zero, which means that here no solution exists. Therefore, optimization of the parameters of the beams must be made, starting from

***physical and obvious mathematical considerations*.**

# Part II. How to find optimal values of collider parameters

## 2. Optimization of Luminosity of a Collider (Contnd)

First of all, we assume that the collider rings and their focusing structures meet the conditions (5). We will also assume that the colliding beams 1 and 2 are "tuned" in such a way that they meet the conditions (6), and their Laslett shifts are equal each other:

$$\Delta q_1 = \Delta q_2 \equiv \Delta q \qquad (16)$$

Then the number of particles in the bunches of these beams satisfies the equality (see (8))

$$\frac{Z_1^2}{A_1} N_1 = \frac{Z_2^2}{A_2} N_2 \qquad (17)$$

We note also that the ratio of the parameters $\xi_{12}$ and $\xi_{21}$ (see (3), (4), (17)) is equal to $\lambda$:

$$\frac{\xi_{12}}{\xi_{21}} = \frac{A_2}{A_1} \cdot \left(\frac{N_2}{N_1}\right) \cdot \frac{B_1^*}{B_2^*} = \frac{Z_1}{A_1} \cdot \frac{A_2}{Z_2} \equiv \frac{1}{\lambda} \qquad (18)$$

and the parameters $\Delta q$ and $\xi_{12}$ are not independent, but (see (8), (9)) related by the equality

$$\frac{\Delta q}{\xi_{12}} = \lambda \frac{a_1}{b_2} = \frac{\pi Q}{\gamma^2 (1 + \beta^2)} \cdot \frac{B_1^*}{\sigma_s} \equiv \eta \qquad (19)$$

# Part II. How to find optimal values of collider parameters

## 2. Optimization of Luminosity of a Collider (Contnd)

Now equations (12), (13) are completed by equations (18), (19) and the whole system of four equations reduces to two:

$$\Delta q\left(1+\frac{1}{\eta}\right)=\Delta Q_1, \tag{20}$$

$$\Delta q\left(1+\frac{\lambda}{\eta}\right)=\Delta Q_2\ . \tag{21}$$

Hence it follows that the shifts $\Delta Q_1$ and $\Delta Q_2$ are not independent parameters!

Having chosen the value of $\Delta Q_1$, we unambiguously determine the values of $\Delta q$ and $\Delta Q_2$:

$$\Delta q=\frac{\eta}{1+\eta}\Delta Q_1\ ,\quad \Delta Q_2=\frac{\lambda+\eta}{(1+\eta)}\Delta Q_1\ . \tag{22}$$

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Hence it follows that the shifts $\Delta Q_1$ and $\Delta Q_2$ are not independent parameters!

When $\Delta q$ is defined one can calculate

the values of $N_1$ from (8) and then $N_2$ from (17).

It allows to calculate luminosity $L$ by Formula (7) (see the next paragraph).

# PART III. NUMERICAL EXAMPLES

## Three examples of calculation of optimal parameter values of hadron colliders

**The parameter values of these collider rings chosen according to the NICA project:**

$$C_{\mathrm{Ring}} = 503\ \mathrm{m}, Q_x = Q_y = 9.44, \langle B \rangle = 8.48\ m, B^*_{ion} = 60\ cm, n_{\mathrm{bunch}} = 22,$$
$$\sigma_s = 60\ \mathrm{cm}, \varepsilon = 1.0\ \pi \cdot \mathrm{mm} \cdot \mathrm{mrad}$$

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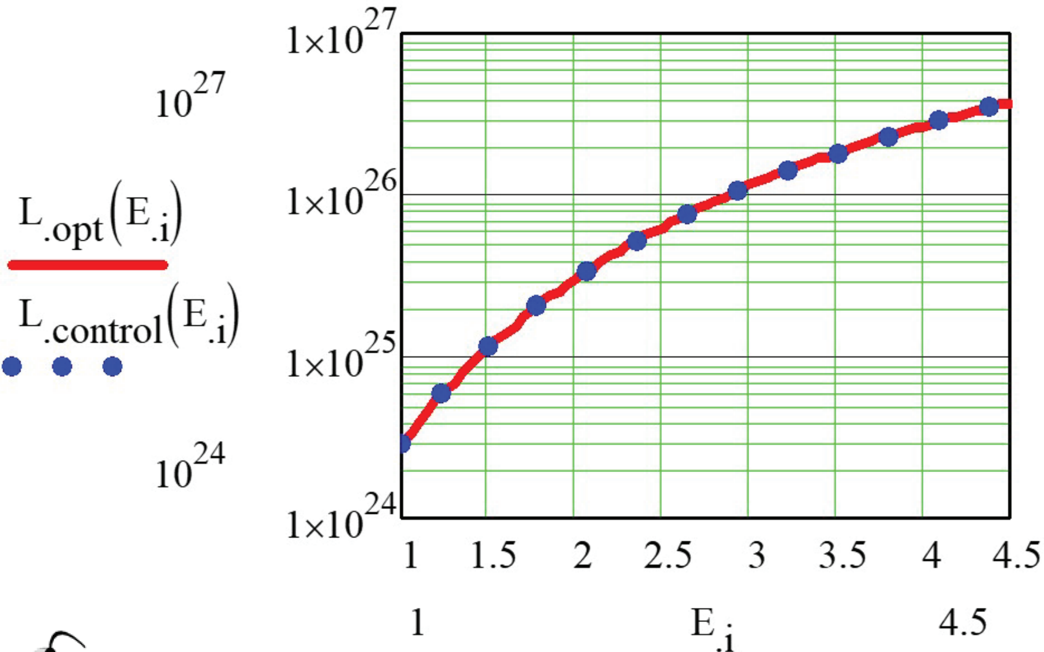
### SYMMETRIC MODE OF THE NICA COLLIDER: Au×Au BEAMS

**Ions: $^{197}Au^{79+}$, $E_i$ = 1.0 – 4.5 GeV/u , $\Delta Q_1 = 0.05$ (the value is set);**

$$\Delta Q_2(E_{\mathrm{i}}) = \Delta q\ (E_{\mathrm{i}}) + \xi_{21}\ (E_{\mathrm{i}})$$




In this coincidence of both lines, there is nothing surprising because Formulas (1) and (7) at conditions (5) and (6) are identical.

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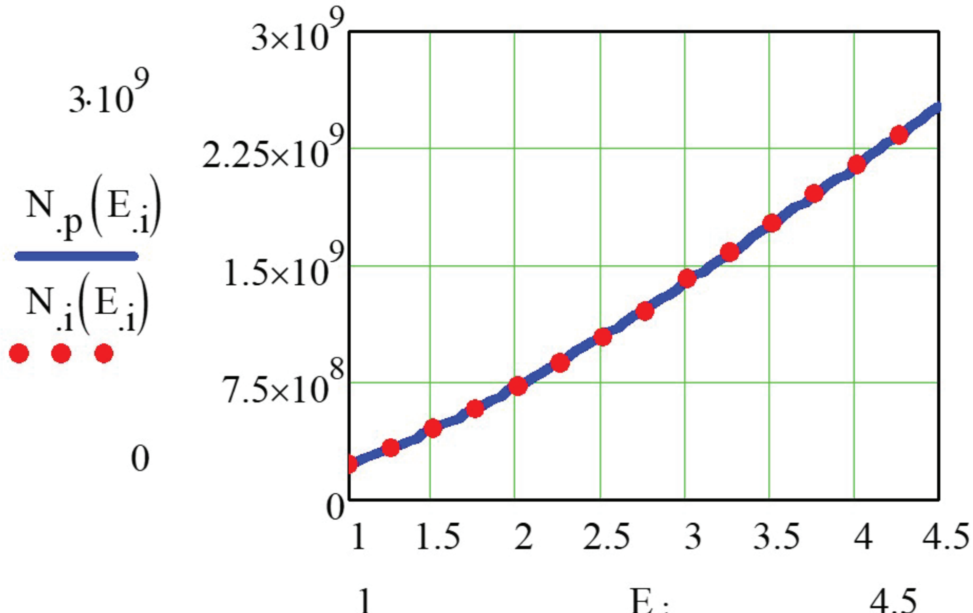


**Particle number per bunch vs $E_i$.**

**Solid line – $N_{i1}(E_i)$, dotted line – $N_{i2}(E_i)$.**

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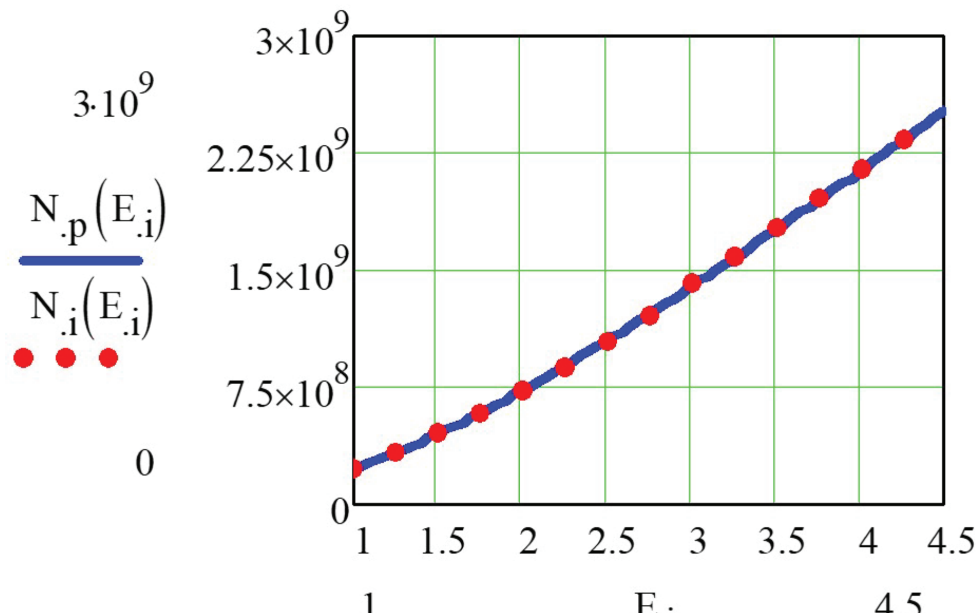


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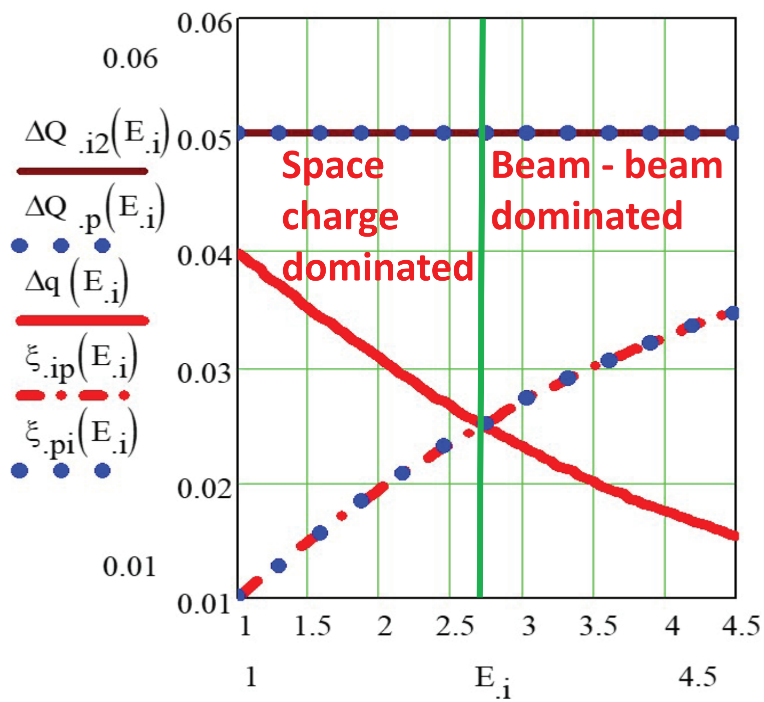

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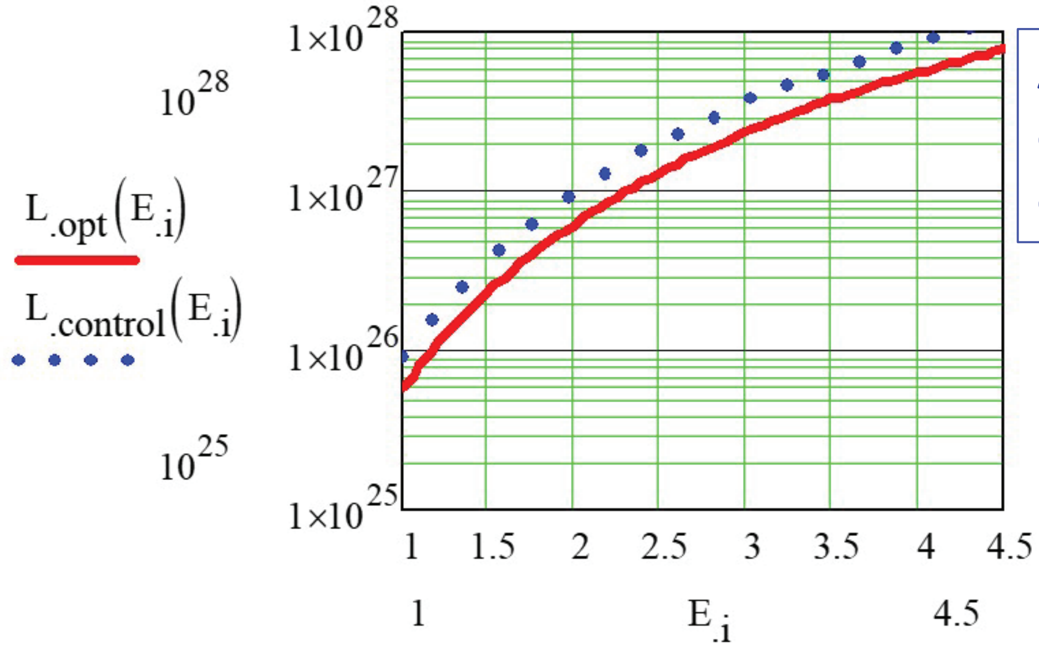


**As can be seen, Formula (1) overestimates the luminosity of such a ion-proton collider.**

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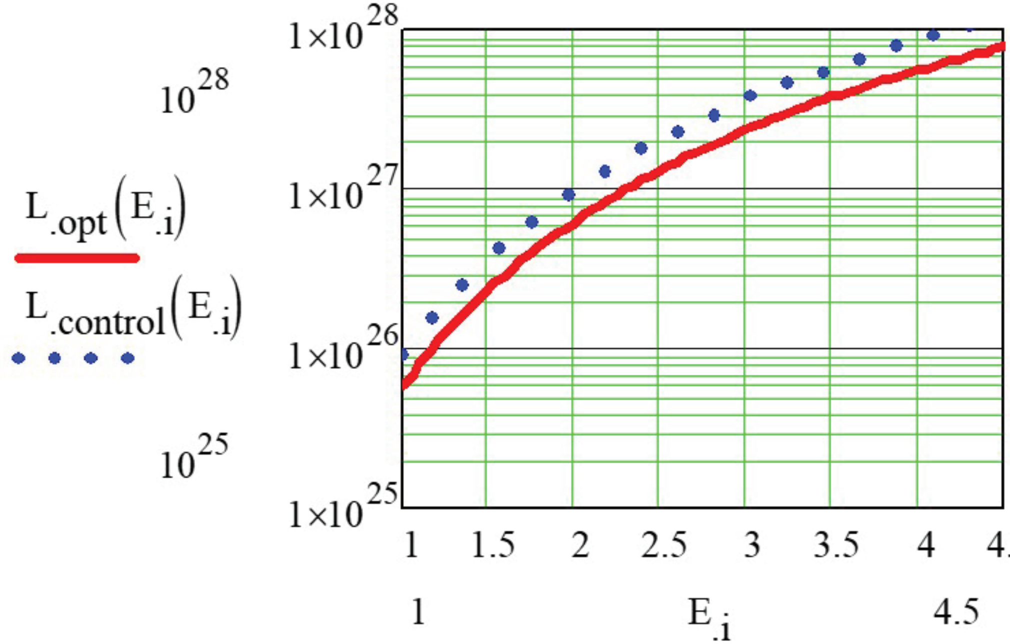


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This asymmetric mode is planned for the NICA project.
The estimates show the project value can be reached in the NICA project.

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**Proton energy $E_p$ = 2.0 – 12 GeV , $\Delta Q_1 = 0.05$ (the value is set);**

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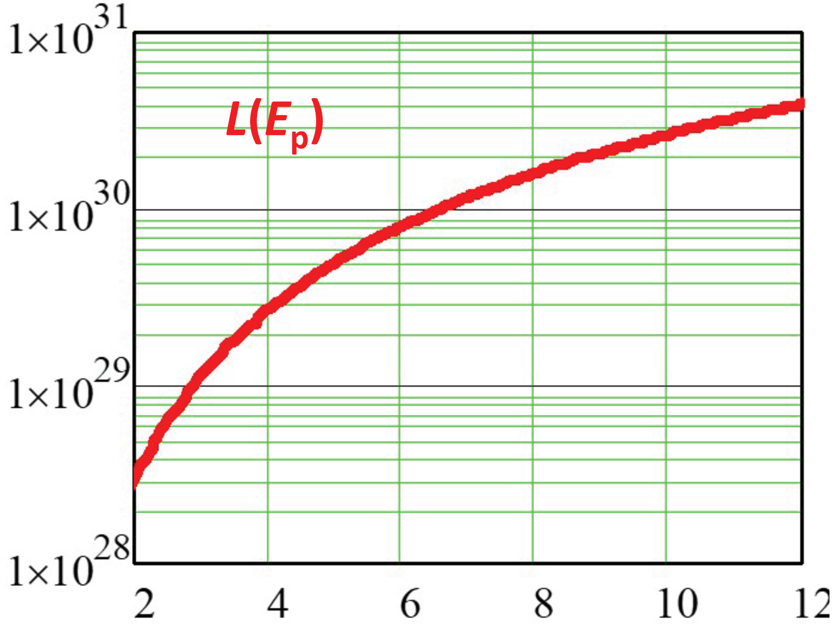


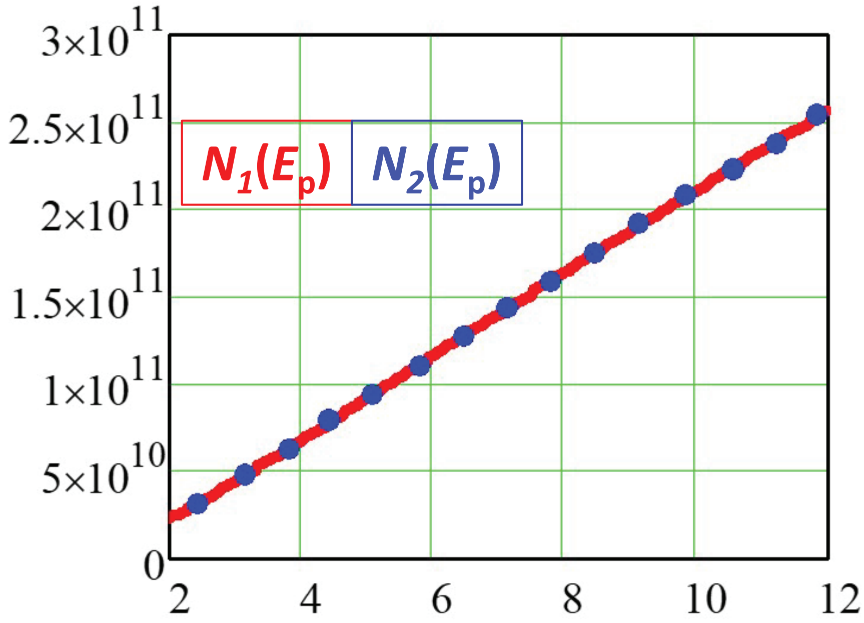

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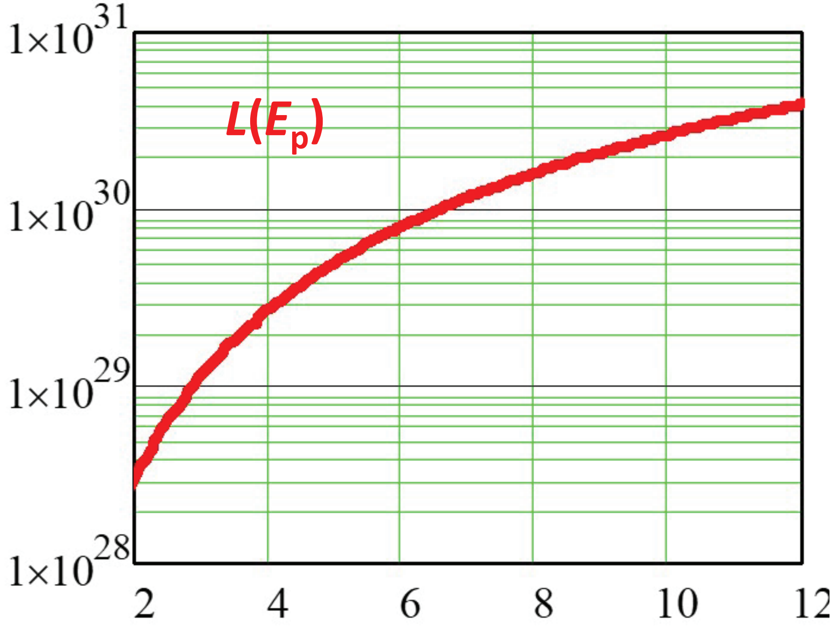


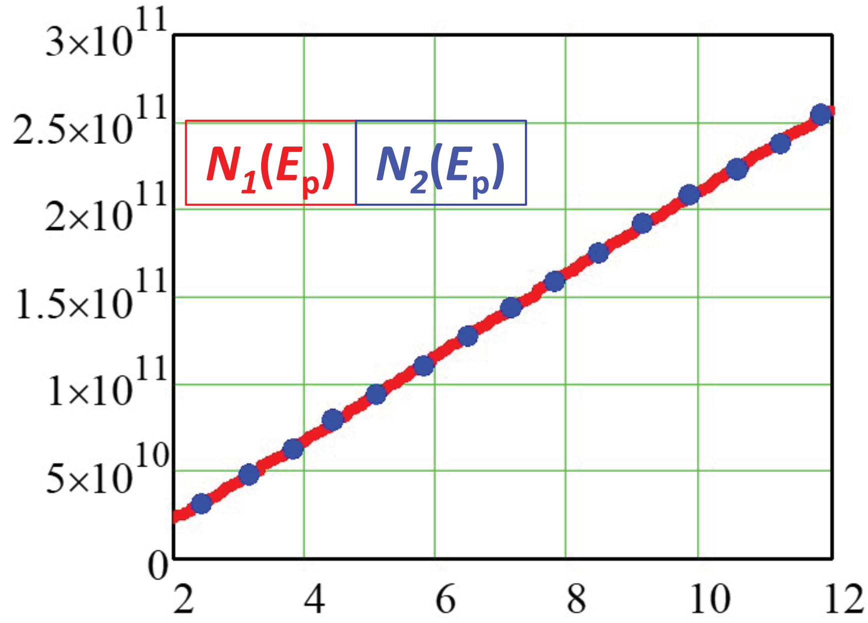


**This symmetric mode is planned for the NICA project at polarized protons.**

# Conclusion

**The paper presents a self-consistent solution to the problem of choosing the values of the number of particles in the bunches of colliding beams, when their intensity is limited by the space charge of the bunches - the so-called *Laslett* and *beam-beam effects.***

**The solution is obtained for the case of fixed emittance of the bunches and parameters of focusing system of the collider. The task of forming bunch emittances, taking into account the influence of IBS and, particularly, using cooling methods, is beyond the scope of this work.**

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**ACKNOWLEDGEMENTS**

**The author is grateful to Dr. J. Maltseva for fruitful discussions of the mathematical aspects of the work.**

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There is something to think about, is not it?

**Thank you for your attention!**