NUMERICAL COMPUTATION OF DYNAMIC APERTURE FOR THE NICA BOOSTER

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Abstract
Modern accelerators are made with high-field superconducting dipoles, quadrupoles and sextupoles that exhibit unintentional imperfections of the guiding field-shape due to construction tolerances, persistent currents and magnetic saturation. This deviation from linearity has a profound influence on the beam dynamics. In this work the research of nonlinear beam dynamics for the NICA booster was carried out. Sextupole and octupole fields for dipole and quadrupole magnets, according to ongoing magnetic measurements, were taken into account, as well as adjustment errors for magnets. Numerical simulation in MADX program has been used to evaluate the dynamic aperture for the booster. The results showed that dynamic aperture is exceeded its geometrical acceptance at least in three times.

INTRODUCTION
The sextupole field increases quadratically with $x$ and while we compensate the chromaticities, these same sextupoles generate nonlinear, quadratic perturbations especially for particles with large betatron oscillation amplitudes. These perturbations are known as geometric aberrations. The art of accelerator design is then to correct the chromatic aberrations while keeping the geometric aberrations at a minimum. This can be achieved by distributing sextupoles along the orbit at properly selected locations. However, even with carefully distributing the sextupoles around the ring lattice, we still deal with a nonlinear problem and we cannot expect to get perfect compensation. There will always be a limit on the maximum stable betatron oscillation amplitude in the storage ring. This limit is called the dynamic aperture in contrast to the physical aperture defined by the vacuum chamber. There is no analytical solution for the dynamic aperture and it is determined by numerical particle tracking programs only [1].

NUMERICAL SIMULATION
The booster of the NICA project [2] is a heavy ion superconducting synchrotron [3]. Magnetic lattice of the booster consists of 4 quadrants, each of which includes 6 DFO-periods. Main parameters of the booster are shown at Table 1. The booster resonance diagram with marked operating point is shown at Fig.1. All simulation and modelling processes were made using MADX program [4]. Dynamic aperture was defined as stable motion region for particles after specified number of revolutions.

Despite the progress in explanation of nonlinear phenomena, there still is a gap between analytical, numerical prediction and reality. Especially, perturbation theory is limited near resonances. The resonant conditions can lead to continuously growing betatron amplitudes and rapidly, after at most a few thousand turns, particles will be lost on the booster vacuum chamber.

Table 1: Main Parameters of the Booster

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ions, max MeV/u</td>
<td>$^{197}$Au$^{31+}$, 578</td>
</tr>
<tr>
<td>Circumference, m</td>
<td>210.96</td>
</tr>
<tr>
<td>Maximum magnetic field, T</td>
<td>1.8</td>
</tr>
<tr>
<td>Maximum magnetic rigidity, T·m</td>
<td>25.2</td>
</tr>
<tr>
<td>Number of dipole magnets</td>
<td>40</td>
</tr>
<tr>
<td>Curvature radius for dipole magnets, m</td>
<td>14.09</td>
</tr>
<tr>
<td>Effective length of dipole magnets, m</td>
<td>2.2</td>
</tr>
<tr>
<td>Number of quadrupole magnets</td>
<td>48</td>
</tr>
<tr>
<td>Effective length of quadrupole magnets, m</td>
<td>0.47</td>
</tr>
<tr>
<td>Vacuum chamber, mm$^2$</td>
<td>129 x 68</td>
</tr>
</tbody>
</table>

Figure 1: Resonance diagram for the booster with lines up to 4th order.

Resonances of 3rd Order
There are 2 the closest resonances near the operating point: $Q_x = -2Q_y = -5$ and $3Q_x = 14$. Consider the first resonance with such betatron tunes: $Q_x=4.8$, $Q_y=4.85$. Dynamic aperture (see Fig.2) has no visible changes inside the vacuum chamber. The green line on the X-PX graph concerns to the wall of the vacuum chamber in X-
direction. The yellow line on the Y-PY graph concerns the wall of the vacuum chamber in Y-direction. There are also no visible changes (see Fig.3) for another set of tunes: Q_x=4.7, Q_y=4.85. It means that the resonance 1Q_x - 2Q_y = -5 has no significant impact to the ion beam of booster.

Figure 2: Booster dynamic aperture with Q_x=4.8, Q_y=4.9.

Figure 3: Booster dynamic aperture with Q_x=4.7, Q_y=4.85.

Now let’s consider the second resonance 3Q_x = 14. The particle changes inside the ion beam go to the centre of the vacuum chamber. Dynamic aperture (see Fig.4, 5) in this case has betatron tune-amplitude dependence. This process needs to be studied more detailed in future works.

Figure 4: Booster dynamic aperture for Q_x=4.662, Q_y=4.85.

Figure 5: Booster dynamic aperture for Q_x=4.667, Q_y=4.85.

Parametric Resonances and Half-Widths of Resonance Bands

Effective length spread of quadrupole magnets can leads to excitation of parametric resonances. The first resonance is 2Q_x = 10. There are beam losses near the wall of the vacuum chamber (see Fig.6) at Q_x=4.97, Q_y=4.85. So, the half-width of resonance band is 0.03.

Second resonance is 2Q_y = 10. Dynamic apertures for this case are shown at Fig.7, 8. The half-width is 0.01.

Figure 6: Booster dynamic aperture Q_x=4.97, Q_y=4.85.

Figure 7: Booster dynamic aperture Q_x=4.8, Q_y=4.99.
Dynamic Aperture with Alignment Errors and Field Harmonics

All previous figures with dynamic aperture were analysed only with the impact of field harmonics up to 4th order including. But the case with all errors (field harmonics and alignment errors) has important information value. Alignment errors are following:

- Effective length spread of dipole magnets: $5 \cdot 10^{-4}$.
- Rotation around longitudinal axis of dipole magnets: $5 \cdot 10^{-4}$ mrad.
- Effective length spread of quadrupole magnets: $5 \cdot 10^{-4}$.
- Rotation around longitudinal axis of quadrupole magnets: $10^{-3}$ mrad.
- Transverse displacement of quadrupole magnets: 0.1 mm.

That values were taken from the technical requirements on superconducting magnets production because magnetic measurements are still running. The final dynamic aperture with all errors are shown at Fig.9 as provided by the size of the booster vacuum chamber. As can be seen from Fig.9 the particle motion is stable along the full cross section of the vacuum chamber. But for the correct comparison the Courant-Snyder invariant must be calculated. This invariant is the area of the ellipse mapped turn by turn in the phase space where the motion is stable. The horizontal value of invariant is $780 \pi \text{ mm} \cdot \text{mrad}$ and vertical is $220 \pi \text{ mm} \cdot \text{mrad}$. The required acceptance of the beam is $\varepsilon_x = 150 \pi \text{ mm} \cdot \text{mrad}$ and $\varepsilon_y = 60 \pi \text{ mm} \cdot \text{mrad}$. So, essential requirement, that dynamic aperture must be bigger than geometrical acceptance at least in two times, is being implemented.

CONCLUSIONS

Research of nonlinear beam dynamics for the $^{197}\text{Au}^{31+}$ ion beam was carried out with the MADX program. The analysis of neighbouring resonance $3Q_x = 14$ around the operating point showed the betatron tune-amplitude dependence. Alignment errors and multipolar fields (up to octupoles including) for dipole and quadrupole magnets were taken into account to evaluate the dynamic aperture for the booster. The results showed that the dynamic aperture is exceeded its geometrical acceptance at least in three times.

REFERENCES