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CONTROL THEORY MODEL FOR RFQ CHANNEL OPTIMIZATION

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Abstract

The common approach for RFQ channel design [1] allows to find geometric parameters of sections of the channel including cell lengths, apertures and coefficients of modulation of electrodes forming the channel. According to that approach, the phase of so called synchronous particle changes slowly. Our previous computations [2] indicate that quality of beam in this channel can be better, if synchronous phase changes more quickly with some oscillations.

To find opportunity for improving the structure, we propose a control theory model for charged particle beam in the RFQ channel. The problem includes some model particle distribution, and dynamics equations depending on control functions. The synchronous phase is chosen as one of the control functions. Great attention is paid to choice of the quality functional.

A numerical solution of such problem can be achieved by an iterative method based on computation of the functional variations. In real problems, we can parametrize the control functions. Then functional variations can be approximated by derivatives of functional on control functions parameters.

INTRODUCTION

The present report is devoted to the simple control model, which allows to solve difficult problem of optimization of the accelerating channel with radio frequency quadrupoles (RFQ) [1]. This model is based on the general model for dynamical systems with two sets of dynamical variables: phase variables of longitudinal and transverse motion with assumption that the longitudinal motion does not depend of the transverse motion.

The control problem is formulated as problem of finding of three control functions which describe geometry of the channel: the accelerating efficiency, the modulation coefficient and derivative of the synchronous phase. Method of the numerical solution based on previously known methods is considered.

BEAM DYNAMICS CONTROL

Consider a beam describing by particle distribution density $\varrho(x)$ in the phase space Ω, $x \in \Omega$ [3, 4, 5, 6]. Let particle trajectories described by the differential equation

$$\frac{dx}{dt} = f(t, x, u),$$

where t is trajectory parameter, u — control function, $u(t) \in U \in R^r$. Assume that vector f is defined in a domain $[t_0, T] \times \Omega \times U$, and the solution of the Cauchy problem for this equation with initial condition $x(t_0) = x_0$ uniquely exists for any x_0 under consideration.

Let at the initial moment t_0 the particle distribution density is given on some $m-$ dimensional surface S $\varrho(t_0, x) = \varrho_{(0)}(x) = \underline{\varrho_{(0)\,1\ldots m}}(x)\, dx^1 \wedge \ldots \wedge dx^m$, $m \leq dim\Omega$, where x^i, $i = \overline{1, m}$, are coordinates on S which can be taken also as some of coordinates in the phase space. The phase density satisfy to the Liouville equation or the Vlasov equation [3, 4, 5, 6]

$$\varrho(t + \delta t, F_{f,\,\delta t} x) = F_{f,\,\delta t}\, \varrho(t, x),$$

Here $F_{f,\,\delta t}$ denotes Lie dragging along vector field f by parameter increment δt [7].

Let introduce functional charachterizing quality of the controlled process

$$\Phi(u) = \int\limits_{\Omega} g(x_T) \varrho(T, x_T).$$

The problem of minimizing of the functional on control function u from U will be called the terminal problem of beam control with account of particle distribution density.

BEAM MODEL

Consider RQF channel which is sequence of cells with quadrupole symmetric modulation of four electrodes. Let L is the cell length, which is slowly rises along the channel. Take approximate expression for cartesian components of electric field E in some cell

$$E_x = u_0 (\frac{2\varkappa}{a^2} x + \frac{2k^2 T}{\pi} x \sin\eta) \cos\omega t,$$

$$E_y = u_0 (-\frac{2\varkappa}{a^2} y + \frac{2k^2 T}{\pi} y \sin\eta) \cos\omega t,$$

$$E_z = u_0 \frac{4kT}{\pi} \cos\eta \cos\omega t.$$

Here $2u_0$ is intervane voltage, ω is frequency of the field oscillations, a is aperture of the cell, $k = \pi/L$ η is the phase of electrode modulation, $\eta(z) = \int\limits_{z_0}^{z} k(z')\, dz'$, T is acceleration efficiency, m is modulation coefficient.

As longitudinal motion in this model does not depends of transverse motion, consider particle distribution in the phase space of longitudinal motion. As coordinates take

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reduced energy γ and phase of the particle $\varphi = \omega t$. Take initial distribution in the phase space of longitudinal motion in the form

$$\varrho_\varphi = (2\pi)^{-1}, \qquad \varphi - \varphi_0 \in [-2\pi, 0], \qquad \gamma = \gamma_0.$$

Here ϱ_φ is $\varphi-$component of the distribution density, φ_0 γ_0 are initial phase and energy of a particle.

Take initial distribution in the phase space of the transverse motion in the form

$$x = R_x\sigma\cos\varphi_x\cos\theta, \quad v_x = V_x\sigma\sin\varphi_x\cos\theta,$$

$$y = R_y\sigma\cos\varphi_y\sin\theta, \quad v_y = V_y\sigma\sin\varphi_y\sin\theta,$$

$\varphi_x, \varphi_y \in [0, 2\pi]$, $\theta \in [0, \frac{\pi}{2}]$, $\sigma \in [0, 1]$.

The particles lie on inside a four-dimensional ellipsoid

$$\frac{x^2}{R_x^2} + \frac{v_x^2}{V_x^2} + \frac{y^2}{R_y^2} + \frac{v_y^2}{V_y^2} = 1.$$

For particles on the surface of this ellipsoid $\sigma = 1$. Let set such distribution that particle are distributed uniformely in coordinates x, y, v_x, v_y inside this ellipsoid. It can be shown that

$$\varrho_{\varphi_x\varphi_y sp} = \mathrm{const}, \quad \sigma = s^{1/4}, \quad \theta = \arcsin\sqrt{p},$$

$s \in [0, 1]$, $p \in [0, 1]$, give such uniform distribution.

Assume that $R_x = R_y = R$, $V_x = V_y = V$ and take dimensionless coordinates

$$\overline{x} = x/R, \quad \overline{v}_x = v_x/V, \quad \overline{y} = y/R, \quad \overline{v}_y = v_y/V.$$

At initial moment

$$\overline{x}(0) = \sigma\cos\varphi_x\cos\theta, \qquad \overline{v}_x(0) = \sigma\sin\varphi_x\cos\theta, \quad (1)$$

$$\overline{y}(0) = \sigma\cos\varphi_y\sin\theta, \qquad \overline{v}_y(0) = \sigma\sin\varphi_y\sin\theta. \quad (2)$$

The equations of transverse motion are linear and can be written in the form

$$\frac{d\overline{x}}{d\zeta} = \frac{V_x}{R_x}\frac{2\pi}{\beta\omega}\overline{v}_x,$$

$$\frac{d\overline{v}_x}{d\zeta} = \frac{\pi e U_0 R_x}{m\lambda^2 V_x\omega}\frac{1}{\sqrt{\gamma^2-1}}\left(\frac{2\varkappa}{\overline{a}^2} + \frac{2\overline{k}^2 T}{\pi}\sin\eta\right)\overline{x}\cos\varphi,$$

$$\frac{d\overline{y}}{d\zeta} = \frac{V_y}{R_y}\frac{2\pi}{\beta\omega}\overline{v}_y,$$

$$\frac{d\overline{v}_y}{d\zeta} = \frac{\pi e U_0 R_y}{m\lambda^2 V_y\omega}\frac{1}{\sqrt{\gamma^2-1}}\left(-\frac{2\varkappa}{\overline{a}^2} + \frac{2\overline{k}^2 T}{\pi}\sin\eta\right)\overline{y}\cos\varphi$$

(β is reduced longitudinal velocity). This system have 4 linear independent solutions. Choose them with the next initial conditions

$$\overline{x}_1(0) = 1, \quad \overline{v}_{x1}(0) = 0, \quad \overline{y}_1(0) = 0, \quad \overline{v}_{y1}(0) = 0,$$

$$\overline{x}_2(0) = 0, \quad \overline{v}_{x2}(0) = 1, \quad \overline{y}_2(0) = 0, \quad \overline{v}_{y2}(0) = 0,$$

$$\overline{x}_3(0) = 0, \quad \overline{v}_{x3}(0) = 0, \quad \overline{y}_3(0) = 1, \quad \overline{v}_{y3}(0) = 0,$$

$$\overline{x}_4(0) = 0, \quad \overline{v}_{x4}(0) = 0, \quad \overline{y}_4(0) = 0, \quad \overline{v}_{y4}(0) = 1.$$

Then solutions of the system with initial conditions (1), (2) can be written in the form

$$\overline{x}(t) = \overline{x}_1(t)\sigma\cos\varphi_x\cos\theta + \overline{x}_2(t)\sigma\sin\varphi_x\cos\theta,$$

$$\overline{v}_x(t) = \overline{v}_{x1}(t)\sigma\cos\varphi_x\cos\theta + \overline{v}_{x2}(t)\sigma\sin\varphi_x\cos\theta,$$

$$\overline{y}(t) = \overline{y}_3(t)\sigma\cos\varphi_y\sin\theta + \overline{y}_4(t)\sigma\sin\varphi_y\sin\theta,$$

$$\overline{v}_y(t) = \overline{v}_{y3}(t)\sigma\cos\varphi_y\sin\theta + \overline{v}_{y4}(t)\sigma\sin\varphi_y\sin\theta.$$

FUNCTIONAL AND CONTROL FUNCTIONS

Let introduce functional

$$\Phi = \alpha_l\Phi_l + \alpha_{tr}\Phi_{tr}, \; \Phi_l = \int g_l(x)\varrho(x), \; \Phi_{tr} = \int g_{tr}(x)\varrho(x)$$

where

$$\Phi_l = \int g_l(x)\varrho(x), \quad \Phi_{tr} = \int g_{tr}(x)\varrho(x).$$

Here $\varrho(x)$ denotes the phase density in the phase space of the longitudinal motion,

$$g_{tr} = \sum_{i=1}^{2} x_i + \sum_{i=3}^{4} y_i,$$

α_l, α_{tr} – are coefficients.

To write Φ_l assume that spatial modulation of the electrodes is such that there exists a particle for which

$$d\varphi/\zeta = \overline{k}, \quad \overline{k} = \lambda k,$$

at least in some last sections of the structure. Let us call this particle synchronous particle and mark it by index s. Choose g_l as follows

$$g_l(\varphi,\gamma) = \begin{cases} 0, & H(\varphi,\gamma) \le H_0, \\ (H(\varphi,\gamma) - H_0)^2, & H(\varphi,\gamma) > H_0, \end{cases}$$

where functions

$$H(\varphi,\gamma) = (\varphi - \varphi)^2 - \frac{\pi\gamma^3}{4C_L\overline{k}T\beta_s^3\sin\varphi_s}(\gamma-\gamma_s)^2$$

conserves for particles which are close to the synchronous particle.

Take difference between phase of synchronous particle and the phase of space modulations of electrodes

$$\Phi_c = \phi_c - \int \overline{k}\, d\zeta, \qquad \frac{d\eta}{d\zeta} = 2\pi\gamma_c(\gamma_c^2-1)^{-1/2} - u_1.$$

Take as the control functions $u_1(\zeta) = d\Phi_c/d\zeta$, $u_2(\zeta) = T(\zeta)$, $u_3(\zeta) = \frac{2\varkappa(\zeta)}{\overline{a}^2(\zeta)}$. Rewrite system of dynamics equations in the form

$$\frac{d\gamma}{d\zeta} = C_L(2\pi\gamma_c(\gamma_c^2-1)^{-1/2} - u_1)u_2\cos\eta\cos\varphi, \quad (3)$$

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$$\frac{d\overline{x}}{d\zeta} = C_{1Tx}\beta\overline{v}_x, \tag{4}$$

$$\frac{d\overline{v}_x}{d\zeta} = C_{2Tx}\beta\gamma(u_3 + \frac{2}{\pi}(\frac{2\pi}{\beta_c} - u_1)^2 u_2 \sin\eta)\overline{x}\cos\varphi, \tag{5}$$

$$\frac{d\overline{y}}{d\zeta} = C_{1Ty}\beta\overline{v}_y, \tag{6}$$

$$\frac{d\overline{v}_y}{d\zeta} = C_{2Tx}\beta\gamma(-u_3 + \frac{2}{\pi}(\frac{2\pi}{\beta_c} - u_1)^2 u_2 \sin\eta)\overline{y}\cos\varphi, \tag{7}$$

where

$$C_L = \frac{2eU_0}{\pi mc^2}, \quad C_{1Tx} = \frac{2\pi}{\omega}\frac{V_x}{R_x}, \quad C_{2Tx} = \frac{\pi eU_0 R_x}{m\lambda^2 V_x \omega},$$

$$C_{1Ty} = \frac{2\pi}{\omega}\frac{V_y}{R_y}, \quad C_{2Ty} = \frac{\pi eU_0 R_y}{m\lambda^2 V_y \omega}.$$

NUMERICAL OPTIMIZATION

To solve the problem we can apply one of known numerical methods [8]. For the method of gradient descent we should integrate the system of equations conjugated to system.

According to method of macroparticles replace all integrals in functional can be replaced by sums over all particles. For example, differential equation for ψ_η takes the form

$$\frac{d\psi_\eta}{d\zeta} = \sum_{i\in I}\psi_{\gamma i} C_L \overline{k} u_2 \sin\eta\cos\varphi_i$$

$$-\frac{2}{\pi}\overline{k}^2 u_2\cos\eta\Big[\sum_{i\in I_x}\frac{1}{\sqrt{\gamma_i^2-1}}C_{2Tx}\Psi_{vxi}\overline{x}_i\cos\varphi_i$$

$$+\sum_{i\in I_y}\frac{1}{\sqrt{\gamma_i^2-1}}C_{2Ty}\Psi_{vyi}\overline{y}_i\cos\varphi_i\Big],$$

where I, I_x, I_y denotes set of all particle indices, and sets of particle indices for particles with $x \neq 0$, and $y \neq 0$ correspondingly.

Further, assume that u_k are piecewise-constant vector-function:

$$u_k = u_k^i, \quad \zeta \in [t_{i-1}, t_i), \quad i = \overline{1, M}, \quad \zeta_M = T.$$

Then the functional can be considered as a function of $3M$ parameters. The derivatives of the functional on these parameters are

$$\frac{\partial\Phi}{\partial u_i^k} = \int\limits_{t_{i-1}}^{t_i}\sum_j \psi(\zeta, x_{(j)}))\frac{\partial\delta_u f(\zeta, x_{(j)})}{\partial u_i^k}\, d\zeta,$$

$i = \overline{1, M}$, $k = \overline{1, 3}$.

The optimization procedure consists in successive steps in directions of decreasing of the functional.

REFERENCES

[1] I.M. Kapchinsky, *Theory of Resonance Linear Accelerators*, (New York: Harwood Academic Publishers, 1985).

[2] O.I. Drivotin, D.A. Ovsyannikov, "Modeling and Optimization of the Dynamics of High Density Beam in RFQ Channel", 6th Int. Workshop "Beam Dynamics and Optimization" (BDO'99), Saratov, Sept 6-9, p. 31 (1999).

[3] O.I. Drivotin, "Covariant Formulation of the Vlasov Equation", IPAC'2011, San-Sebastian, Sept. 4-9, 2011, p. 2277 (2011).

[4] O.I. Drivotin, "Degenerate Solutions of the Vlasov Equation", RuPAC'2012. St.Petersburg, 2012., p. 2277 (2012).

[5] O.I. Drivotin, "Covariant Description of Phase Space Distributions", Vestnik of St.Petersburg Univ. Ser.10. iss.3 (2016) 39.

[6] O.I. Drivotin, D.A. Ovsyannikov, "Stationary Self-Consistent Distributions for a Charged Particle Beam in the Longitudinal magnetic Field", Phys. of Particles and Nuclei. 47 (2016) 884.

[7] O.I. Drivotin, *Matematicheskie osnovy teorii polya [Mathematical Foundations of the Field Theory]*, (St.Petersburg: Publ. Comp. of Saint Petersburg State University, 2010), 168. (Russ.)

[8] D.A. Ovsyannikov, O.I. Drivotin, "Modeling of High Intensity Charge Particle Beams", (St.Petersburg: Publishing Company of Saint-Petersburg State University, 2003), 176. (Russ.).