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OPTIMUM LUMINOSITY OF PROTON-ION COLLIDER

I.N. Meshkov, Joint Institute for Nuclear Research, 141980 Dubna, Russia and
St Petersburg University, 199034, St. Petersburg

Abstract

The problem of optimal parameters of proton-ion collider is considered. It is shown that the maximum luminosity of the p-i collider is achieved when the Laslett tune shifts of the ion and proton bunches are equal. As an example, the luminosity of the NICA collider in three different collision mode – ion-ion (symmetric), proton-ion (asymmetric) and proton-proton (symmetric) are presented. These calculations are based on the results of the article [1].

LUMINOSITY OF SYMMETRIC AND ASYMMETRIC CIRCULAR COLLIDERS

Luminosity of Symmetric Colliders

Luminosity of a circular collider in so called symmetric mode of the parameters of the colliding particles and the bunches are identical. Then, the collider luminosity is described by the well-known formula:

$$L = \frac{n_{bunch} N_1 N_2 f_0}{4\pi\sqrt{\varepsilon_x \varepsilon_y} B^*} \cdot F_{HG},\ F_{HG}(\alpha) = \frac{1}{\sqrt{\pi}} \int_0^\infty \frac{e^{-u^2} du}{1+(\alpha u)^2},\ \alpha = \frac{\sigma_s}{B^*} \quad (1)$$

Here $\varepsilon_{x,\,y}$ - x, y - emittances of the bunches of colliding beams, B* - value of the beta function at the meeting point, n_{bunch} - number of bunches in the beam, $N_{1,2}$ - number of particles in the cluster of the 1st and 2nd beams, σ_s is the length (σ-parameter) of a Gaussian bunch.

Luminosity of an Asymmetric Colliders with a Common Final Focus System

The luminosity of such a collider is described by the formula [1]

$$L = \frac{n_{bunch} N_1 N_2 f_0}{(2\pi)^2 \sigma_{s1} \sigma_{s2}} \cdot Int\,, \quad (2)$$

$$Int = \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\eta \frac{f_s(\xi,\eta)}{\sqrt{f_x(\xi,\eta) \cdot f_y(\xi,\eta)}},$$

where

$$f_s(\xi,\eta) = exp\left\{-\frac{1}{2}\left[\frac{\eta^2}{\sigma_{s1}^2} + \left(\frac{\eta + V\xi}{\sigma_{s2}}\right)^2\right]\right\}$$

$$f_x(\xi,\eta) = \left(\varepsilon_{x1} B_{x1}(\xi,\eta) + \varepsilon_{x2} B_{x2}(\xi,\eta)\right)$$

$$f_y(\xi,\eta) = \left(\varepsilon_{y1} B_{y1}(\xi,\eta) + \varepsilon_{y2} B_{y2}(\xi,\eta)\right)$$

Here $V = 1 + v_1 / v_2$. The indices 1 and 2 indicate the parameters of the bunches of the first and second colliding beams and the values of the beta functions $B_{x1,2}$ for them. The particle velocity of the second beam for colliding beams is negative, $v_2 < 0$, whereas for merging beams both velocities are positive, i.e. $v_{1,2} > 0$. The latter do not coincide if the lenses of the final focus of the collider are common for both beams, and the particles of the beams 1 and 2 differ in at least one of the parameters - charge, mass or energy. Beta functions depend on two parameters - the longitudinal coordinates of the colliding particles ξ and η in the moving system of one of the two colliding clusters (see details in [1]). Since the values of the betatron functions $B_{x1,2}$ are proportional to the magnetic rigidity of the particles, it is possible to introduce the relative magnetic stiffness parameter λ of the colliding particles:

$$\lambda_x = \frac{B_{x1}^*}{B_{x2}^*} = \frac{p_1}{Z_1} \cdot \frac{Z_2}{p_2}, \quad (3)$$

where $B^*_{x1,2}$ are the beta function values in the interaction points (IP). $p_{1,2}$ are the momenta of colliding particles, $Z_{1,2}$ are their charges in units of electron charge. It follows that

$$\lambda_y = \lambda_x \equiv \lambda.$$

Formula (3) is simplified if the perimeters of the rings are the same. Then from the collision synchronization condition it follows that $v_1 = v_2$ and

$$\lambda = \frac{p_1}{Z_1} \cdot \frac{Z_2}{p_2} = \frac{A_1 m_N \gamma_1 v_1}{Z_1} \cdot \frac{Z_2}{A_2 m_N \gamma_2 v_2} = \frac{A_1}{Z_1} \cdot \frac{Z_2}{A_2}, \quad \gamma_{1,2} = \frac{1}{\sqrt{1-\beta_{1,2}^2}},\ \beta_{1,2} = \frac{v_{1,2}}{c}, \quad (4)$$

Thus, the values of $\lambda_{x,\,y}$ do not depend on the velocity of the colliding particles.

We shall confine our attention to the case of identical values of the parameters of two beams and *focusing systems of two rings:*

$$C_1 = C_2 \equiv C_{Ring}, Q_{x1} = Q_{y1} = Q_{x2} = Q_{y2} \equiv Q. \quad (5)$$

$$v_1 = v_2, \varepsilon_{x1} = \varepsilon_{x2} = \varepsilon_{y1} = \varepsilon_{y2} \equiv \varepsilon,\ \sigma_{s1} = \sigma_{s2}. \quad (6)$$

The luminosity formula (2) for such an asymmetric collider takes the form (see Appendix):

$$L = \frac{n_{bunch} N_1 N_2 f_0}{4\pi^{3/2} \varepsilon B_1^*} \cdot Int_{asym}\,,$$

$$Int_{asym} == \int_{-\infty}^{\infty} \frac{e^{-\chi^2} \cdot d\chi}{\left[1+\left(\frac{\sigma_s}{B_1^*}\chi\right)^2\right] + \frac{1}{\lambda}\left[1+\left(\lambda \cdot \frac{\sigma_s}{B_1^*}\chi\right)^2\right]} \quad (7)$$

HOW TO FIND OPTIMAL VALUES OF THE COLLIDER PARAMETERS

Limitation of Collider Parameters by Space-Charge Effects

The choice of the parameters of the collider is determined to a large extent by the stability conditions of the beams circulating in the rings of the collider. The strongest limitations are the effects of the space charge of the beams - the so-called "Laslett effect" and the "beam-beam effect". Both of them lead to shifts in the frequencies of betatron oscillations of particles in the collider focusing system, bringing them closer to the

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frequencies of the nonlinear resonances These shifts are described by well-known formulas. For beams with a Gaussian distribution of the particle density along the transverse (x, y) and longitudinal (s) coordinates, this shift is (oscillations along the x-coordinate)

- for the Laslett effect

$$\Delta q_1 = \frac{Z_1^2}{A_1}\cdot N_1\cdot a_1\,,\quad a_1 = \frac{r_p}{4\pi\beta_1^2\gamma_1^3\varepsilon_{x1}}\cdot\frac{C_1}{\sqrt{2\pi}\sigma_{s1}}\,, \qquad (8)$$

- for the beam-beam effect

$$\xi_{12} = \frac{Z_1 Z_2}{A_1}\cdot N_2\cdot b_1,$$

$$b_1 = \frac{r_p}{(2\pi)^{3/2}\beta_1\gamma_1\varepsilon_{x2}}\cdot\frac{\langle B_1\rangle}{B_2^*}\cdot\frac{1+\beta_1\beta_2}{\beta_1+\beta_2}\cdot\left(1+\frac{\sigma_{s1}}{\sigma_{s2}}\right). \qquad (9)$$

Here r_p = 2.818 cm is the classical radius of the proton, β_1, β_2 is the particle velocity in terms of the speed of light, γ_1 is the Lorentz factor of the particle 1, ε_{x1}, ε_{x2} are the emittances of the particle clusters 1 and 2 in the x-coordinate, σ_{s1}, σ_{s2} - longitudinal "sigma" dimensions of their clots, $\langle B_{x1}\rangle$, $\langle B_{x2}\rangle$ -are the average values of betatron functions of collider rings:

$$\langle B_{x1,2}\rangle = \frac{C_{1,2}}{2\pi Q_{x1,2}}\,, \qquad (10)$$

$C_{1,2}$ are the circumferences perimeter of the rings, $Q_{x1,2}$ are their betatron numbers (number of betatron oscillations per revolution of the particle in the ring). The values of the parameters Δq_2 and ξ_{21} for the colliding particles are obtained by changing the indices 1 ⇄ 2.

The betatron shifts Δq and ξ depend on the energy of the particles through the Lorentz factors β and γ.

The sum of the shifts Δq + ξ = ΔQ usually serves to estimate the stability of the particle beam in the collider. It is well known from practice that the intense beam is stable (under other fulfilled conditions), if

$$\Delta Q \le 0.05 \qquad (11)$$

Optimization of the Luminosity of the Collider

When choosing the optimal values of the parameters of the collider beams, one could, apparently, write down two equations

$$\Delta q_1 + \xi_{12} = \Delta Q_1, \qquad (12)$$

$$\Delta q_2 + \xi_{21} = \Delta Q_2, \qquad (13)$$

and require condition (11) for both beams: $\Delta Q_1, \Delta Q_2 \le 0.05$. Then, substituting here the values of Δq (8) and ξ (9), we obtain a system of two algebraic equations with respect to the unknowns N_i and N_p:

$$\frac{Z_1^2}{A_1}a_1 N_1 + \frac{Z_1 Z_2}{A_1}b_1 N_2 = \Delta Q_1, \qquad (14)$$

$$\frac{Z_2^2}{A_2}a_2 N_2 + \frac{Z_1 Z_2}{A_2}b_2 N_1 = \Delta Q_2\,. \qquad (15)$$

Note that these equations are connected through the beam-beam effect - the parameters ξ_{12} and ξ_{21}. However, when trying to solve the "forehead" of this system of equations, it turns out that for certain values of the particle energy the determinant of the system drops to zero, which means that here no solution exists. Therefore, optimization of the parameters of the beams must be made, starting from *physical and obvious mathematical considerations.*

First of all, we assume that the collider rings and their focusing structures meet the conditions (5). We will also assume that the colliding beams 1 and 2 are "tuned" in such a way that they meet the conditions (6), and their Laslett shifts are equal each other:

$$\Delta q_1 = \Delta q_1 \equiv \Delta q\,. \qquad (16)$$

Then the number of particles in the bunches of these beams satisfies the equality (see (8))

$$\frac{Z_1^2}{A_1}N_1 = \frac{Z_2^2}{A_2}N_2. \qquad (17)$$

We note also that the ratio of the parameters ξ_{12} and ξ_{21} (see (3), (4), (17)) is equal to 1/λ:

$$\frac{\xi_{12}}{\xi_{21}} = \frac{A_2}{A_1}\cdot\left(\frac{N_2}{N_1}\right)\cdot\frac{B_1^*}{B_2^*} = \frac{Z_1}{A_1}\cdot\frac{A_2}{Z_2} \equiv \frac{1}{\lambda}, \qquad (18)$$

and the parameters Δq and ξ_{12} are not independent (see (8), (9)), but related by the equality

$$\frac{\Delta q}{\xi_{12}} = \lambda\frac{a_1}{b_2} = \frac{\pi Q}{\gamma^2(1+\beta^2)}\cdot\frac{B_1^*}{\sigma_s} \equiv \eta\,. \qquad (19)$$

Now equations (12), (13) are completed by equations (18), (19) and the whole system of four equations reduces to two:

$$\Delta q\left(1+\frac{1}{\eta}\right) = \Delta Q_1, \qquad (20)$$

$$\Delta q\left(1+\frac{\lambda}{\eta}\right) = \Delta Q_2\,. \qquad (21)$$

Hence it follows that the shifts ΔQ_1 and ΔQ_2 are not independent parameters!

Having chosen the value of ΔQ_1, we unambiguously determine the values of Δq and ΔQ_2:

$$\Delta q = \frac{\eta}{1+\eta}\Delta Q_1\,,\quad \Delta Q_2 = \frac{\lambda+\eta}{(1+\eta)}\Delta Q_1\,. \qquad (22)$$

When *Δq* is defined one can calculate the values of N_1 from (8) and then N_2 from (17). It allows too calculate luminosity *L* by Formula (7) (see the next paragraph).

NUMERICAL EXAMPLES

Below three examples of calculation of optimal parameter values of hadron colliders are presented. The parameter values of these collider rings are chosen according to the NICA project:

$$C_{\text{Ring}} = 503\text{ m}, Q_x = Q_y = 9.44,$$

$$\langle B\rangle = 8.48\, m, B_{ion}^* = 60\, cm, n_{\text{bunch}} = 22,$$

$$\sigma_s = 60\text{ cm}, \varepsilon_1 = 1.0\,\pi\cdot\text{mm}\cdot\text{mrad}$$

Symmetric Mode of the NICA Collider: Au×Au Beams

Ions: $^{197}Au^{79+}$, E_i = 1.0 – 4.5 GeV/u

$$\Delta Q_1 = 0.05 \text{ (the value is set)};$$

$$\Delta Q_2(E_i) = \Delta q\,(E_i) + \xi_{21}\,(E_i) \qquad (23)$$

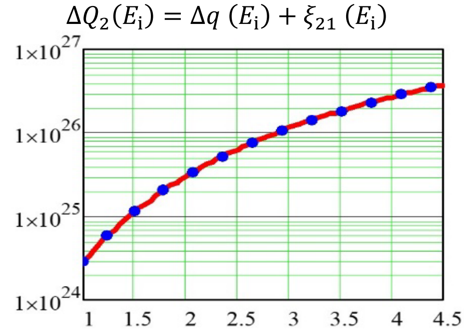

Figure 1: Collider Luminosity [$cm^{-2}\cdot s^{-1}$] vs ion energy [GeV/u] in Au x Au mode: solid line – L(E_i) calculated by Formula (7), and the dotted line – by Formula (1).

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In the coincidence of both lines in Fig. 1, there is nothing surprising because Formulas (1) and (7) at conditions (5) and (6) are identical.

Asymmetric Mode of the NICA Collider: Au×proton Beams

Ions: $^{197}Au^{79+}$, energy range 1.0 – 4.5 GeV/u

$\Delta Q_i = 0.05$ (the value is set).

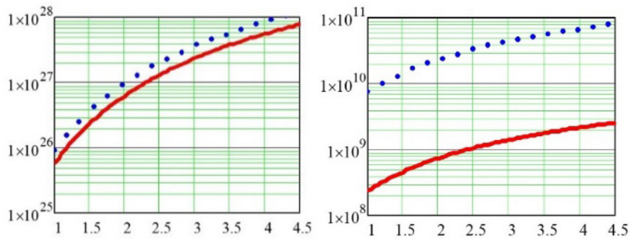

Figure 2: a) Collider Luminosity vs ion energy in Au x proton mode. Solid line: L(E_i) calculated by Formula (7), and the dotted line – by Formula (1). b) Particle number per bunch vs E_i . Solid line – $N_i(E_i)$, dotted line – $N_p(E_i)$.

As we see from Fig. 2a, Formula (1) overestimates the luminosity of such a ion-proton collider. This asymmetric mode is planned for the NICA. The estimates (Figs. 2a,b) show the reachability of the project value of the NICA Au x proton collider luminosity.

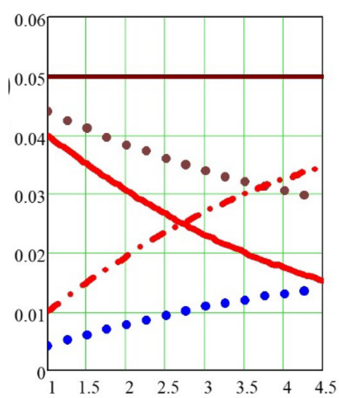

Figure 2c: The betatron shifts vs E_i: Solid dark red straight lines is ΔQ_i; dark red dotted line is $\Delta Q_p(E_i)$, solid red line is $\Delta q(E_i)$; dot-dashed red line is $\xi_{ip}(E_i)$ and dotted blue line is $\xi_{pi}(E_i)$.

The range to the left from cross point in Fig. 2c is so called "the space charge dominated regime", and to the right – "the beam-beam dominated regime".

Symmetric Mode of the NICA Collider, proton×proton Beams

The polarized proton-proton colliding beams in the energy range of 2 – 12 GeV are planned for the NICA project. The luminosity and proton number per bunch (Figs. 3a, b) do fit to project requirements of this collider mode.

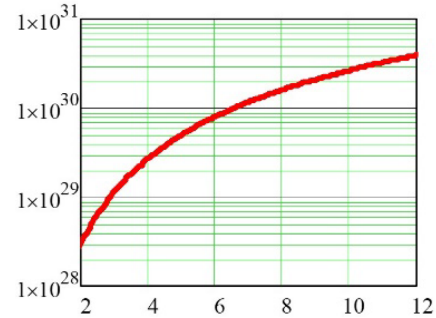

Figure 3a: Collider Luminosity [$cm^{-2} \cdot s^{-1}$] vs proton energy [GeV] in proton x proton mode.

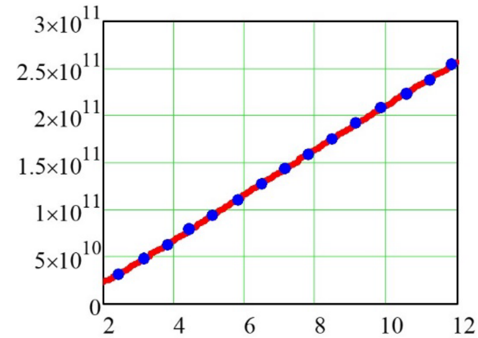

Figure 3b: Particle number per bunch vs E_p [GeV]. Solid line – $N_{p1}(E_i)$, dotted line – $N_{p2}(E_i)$.

CONCLUSION

The paper presents a self-consistent solution to the problem of choosing the values of the number of particles in the bunches of colliding beams, when their intensity is limited by the space charge of the bunches - the so-called Laslett and beam-beam effects.

The solution is obtained for the case of fixed emittance of the bunches and parameters of focusing system of the collider. The task of forming bunch emittances, taking into account the influence of IBS and, particularly, using cooling methods, is beyond the scope of this work.

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REFERENCES

[1] I. N. Meshkov Luminosity of a Collider with Asymmetric Beams, Phys.Part. Nucl. Lett., 2018, **15**, No. 5, pp. 506–509.