Numeric analysis of effective spectral width for the synchrotron radiation

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Outline

1. Radiated classical particle

2. Effective spectral width of SR polarization components

Objectives

- Introducing and computation effectife width of spectrum for SR (not exceeding 100 harmonics) in classical theory.

Radiated classical particle

- Spectral-angular distribution of SR polarization components [1, 2, 3]

$$W_s = W \sum_{\nu=1}^{\infty} \int_0^{\pi} f_s(\beta;\, \nu,\, \theta) \sin\theta d\theta; \qquad (1)$$

- Total radiated power

$$W = \frac{2}{3} \frac{e^4 H^2 (\gamma^2 - 1)}{m_0^2 c^3}; \qquad (2)$$

- Spectral-angular densities

$$\begin{aligned}
f_2(\beta;\, \nu,\, \theta) &= \frac{3\nu^2}{2\gamma^4} J_\nu'^{\,2}(x), \quad x = \nu\beta\sin\theta; \\
f_3(\beta;\, \nu,\, \theta) &= \frac{3\nu^2}{2\gamma^4} \frac{\cos^2\theta}{\beta^2 \sin^2\theta} J_\nu^2(x); \\
f_{\pm 1}(\beta;\, \nu,\, \theta) &= \frac{3\nu^2}{4\gamma^4} \left[J_\nu'(x) \pm \varepsilon \frac{\cos\theta}{\beta\sin\theta} J_\nu(x) \right]^2; \\
f_0(\beta;\, \nu,\, \theta) &= f_2(\beta;\, \nu,\, \theta) + f_3(\beta;\, \nu,\, \theta)\,.
\end{aligned} \qquad (3)$$

- Spectral densities of radiation in upper half-space

$$F_s^{(+)}(\beta;\,\nu) = \int_0^{\pi/2} f_s(\beta;\,\nu,\,\theta)\sin\theta d\theta\,; \qquad (4)$$

$$F_2^{(+)}(\beta;\,\nu) = \frac{3\nu}{4\,\gamma^{\,4}\beta^3}\left[2\beta^2 J'_{2\nu}(2\nu\beta) + \beta^2\int_0^{2\nu\beta} J_{2\nu}(x)dx - 2\nu\beta\int_0^{2\nu\beta}\frac{J_{2\nu}(x)}{x}dx\right],$$

$$F_3^{(+)}(\beta;\,\nu) = \frac{3\nu}{4\,\gamma^{\,4}\beta^3}\left[2\nu\beta\int_0^{2\nu\beta}\frac{J_{2\nu}(x)}{x}dx - \int_0^{2\nu\beta} J_{2\nu}(x)dx\right],$$

$$F_0^{(+)}(\beta;\,\nu) = \frac{3\nu}{4\,\gamma^{\,4}\beta^3}\left[2\beta^2 J'_{2\nu}(2\nu\beta) - (1-\beta^2)\int_0^{2\nu\beta} J_{2\nu}(x)dx\right],$$

$$F_{\pm1}^{(+)}(\beta;\,\nu) = \frac{1}{2}F_0^{(+)}(\beta;\,\nu) \pm \frac{3\nu J_\nu^{\,2}(\nu\beta)}{4\,\gamma^{\,4}\beta^2}.$$

- Spectral sums of radiation in upper half-space

$$\Phi_s^{(+)}(\beta) = \sum_{\nu=1}^{\infty} F_s^{(+)}(\beta;\,\nu); \qquad (5)$$

$$\Phi_2^{(+)}(\beta) = \frac{6+\beta^2}{16}, \quad \Phi_3^{(+)}(\beta) = \frac{2-\beta^2}{16}, \Phi_{\pm1}^{(+)}(\beta) = \frac{1}{4}\left[1 \pm \frac{3}{4}\chi_1(\beta)\right].$$ [4]

Effective spectral width of SR polarization components

- Partial spectrum densities [5]

$$P_s(\beta;\nu) = \frac{F_s^{(+)}(\beta;\,\nu)}{\Phi_s^{(+)}(\beta)}\,; \tag{6}$$

- Effective width of the spectrum

$$\Lambda_s(\beta) = \nu_s^{(2)}(\beta) - \nu_s^{(1)}(\beta) + 1; \tag{7}$$

$$\sum_{\nu=\nu_s^{(1)}(\beta)}^{\nu_s^{(2)}(\beta)} P_s(\beta;\nu) - \frac{1}{2} \geq 0. \tag{8}$$

Table 1: Effective spectral width Λ_s and initial harmonic boundaries $\nu_s^{(1)}$, $\tilde{\nu}_s^{(1)}$ of linear-polarizated components of SR for range $\Lambda_s = 1$ till $\Lambda_s = 10$

Λ	$\nu_2^{(1)}$	γ_2	$\tilde{\nu}_2^{(1)}$	$\tilde{\gamma}_2$	$\nu_3^{(1)}$	γ_3	$\tilde{\nu}_3^{(1)}$	$\tilde{\gamma}_3$
1	1	1.1434	1	1.1434	1	1.2363	1	1.2363
2	1	1.2955	1	1.2955	1	1.4519	1	1.4519
3	1	1.4179	2	1.3237	1	1.6126	1	1.6126
4	1	1.5215	2	1.4740	1	1.7435	1	1.7435
5	1	1.6121	2	1.5827	1	1.8554	1	1.8554
6	1	1.6932	2	1.6741	1	1.9540	2	1.8592
7	1	1.7669	3	1.6998	1	2.0428	2	1.9638
8	1	1.8347	3	1.7860	1	2.1238	2	2.0562
9	1	1.8977	3	1.8610	1	2.1986	2	2.1394
10	1	1.9566	3	1.9286	1	2.2680	2	2.2161

Table 2: Effective spectral width Λ_s and initial harmonic boundaries $\nu_s^{(1)}$, $\tilde{\nu}_s^{(1)}$ of linear-polarizated components of SR for range $\Lambda_s = 1$ till $\Lambda_s = 100$

Λ	$\nu_2^{(1)}$	γ_2	$\tilde{\nu}_2^{(1)}$	$\tilde{\gamma}_2$	$\nu_3^{(1)}$	γ_3	$\tilde{\nu}_3^{(1)}$	$\tilde{\gamma}_3$
1	1	1.1434	1	1.1434	1	1.2363	1	1.2363
11	1	2.0120	3	1.9906	1	2.3329	2	2.2868
21	2	2.4538	5	2.4215	1	2.8422	2	2.8217
31	3	2.7731	6	2.7499	1	3.2053	3	3.1740
41	3	3.0316	7	3.0121	1	3.4998	3	3.4793
51	4	3.2515	8	3.2342	1	3.7528	3	3.7383
61	4	3.4450	9	3.4289	1	3.9700	4	3.9502
71	5	3.6191	10	3.6039	1	4.1678	4	4.1513
81	5	3.7776	11	3.7631	1	4.3432	4	4.3295
91	6	3.9247	12	3.9107	1	4.5039	4	4.4919
100	6	4.0482	12	4.0381	1	4.6374	4	4.6263

Table 3: Effective spectral width Λ_s and initial harmonic boundaries $\nu_s^{(1)}$, $\tilde{\nu}_s^{(1)}$ of circle-polarizated components of SR for range $\Lambda_s = 1$ till $\Lambda_s = 10$

Λ	$\nu_{-1}^{(1)}$	γ_{-1}	$\tilde{\nu}_{-1}^{(1)}$	$\tilde{\gamma}_{-1}$	$\nu_1^{(1)}$	γ_1	$\tilde{\nu}_1^{(1)}$	$\tilde{\gamma}_1$
1	1	1.1062	1	1.1062	1	1.1712	1	1.1712
2	1	1.2348	1	1.2348	1	1.3411	1	1.3411
3	1	1.3440	2	1.3233	1	1.4737	1	1.4737
4	2	1.4394	2	1.4394	1	1.5843	2	1.4987
5	2	1.5323	3	1.4808	1	1.6802	2	1.6210
6	2	1.6125	3	1.5854	1	1.7657	2	1.7211
7	2	1.6841	3	1.6686	1	1.8431	2	1.8082
8	2	1.7493	4	1.7015	1	1.9142	2	1.8860
9	2	1.8094	4	1.7786	1	1.9800	2	1.9570
10	2	1.8655	4	1.8455	1	2.0416	2	2.0225

Table 4: Effective spectral width Λ_s and initial harmonic boundaries $\nu_s^{(1)}$, $\tilde{\nu}_s^{(1)}$ of circle-polarizated components of SR for range $\Lambda_s = 1$ till $\Lambda_s = 100$

Λ	$\nu_{-1}^{(1)}$	γ_{-1}	$\tilde{\nu}_{-1}^{(1)}$	$\tilde{\gamma}_{-1}$	ν_1	γ_1	$\tilde{\nu}_1^{(1)}$	$\tilde{\gamma}_1$
1	1	1.1062	1	1.1062	1	1.1712	1	1.1712
11	3	1.9197	5	1.8721	1	2.0995	3	2.0448
21	4	2.3402	7	2.3139	1	2.5522	4	2.5154
31	5	2.6449	9	2.6233	2	2.8828	4	2.8659
41	6	2.8911	11	2.8712	2	3.1505	5	3.1319
51	8	3.1007	13	3.0814	2	3.3781	6	3.3585
61	9	3.2851	14	3.2714	3	3.5781	6	3.5653
71	10	3.4505	16	3.4361	3	3.7581	7	3.7434
81	11	3.6013	17	3.5901	3	3.9219	7	3.9114
91	12	3.7403	19	3.7281	3	4.0727	8	4.0609
100	13	3.8570	20	3.8465	4	4.1997	9	4.1860

Summary

- γ, corresponding to given effective spectral width Λ_s, is limited by

$$\gamma_s(\Lambda_s - 1, \nu_s^{(1)}(\Lambda_s)) < \gamma \leqslant \gamma_s(\Lambda_s, \nu_s^{(1)}(\Lambda_s))\,; \qquad (9)$$

$$\gamma_s(\Lambda_s, k) > \gamma_s(\Lambda_s, n)\,, \quad \nu_s^{(1)}(\Lambda_s) \leqslant k < n \leqslant \tilde{\nu}_s^{(1)}(\Lambda_s). \qquad (10)$$

- At a fixed values Λ_s the corresponding values γ_s obey the inequalities

$$\gamma_3 > \gamma_1 > \gamma_0 > \gamma_2 > \gamma_{-1}\,. \qquad (11)$$

- At a fixed γ, the corresponding values of Λ_s are restricted by

$$\Lambda_3 < \Lambda_1 < \Lambda_0 < \Lambda_2 < \Lambda_{-1}\,. \qquad (12)$$

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