RADIATION OF A BUNCH FLYING FROM THE OPEN END OF A WAVEGUIDE WITH A DIELECTRIC LOADING*

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Abstract

In this paper we proceed with our investigation of Terahertz emission from beam moving in waveguide structures with dielectric layer [1]. Recently we have considered an open-ended waveguide (with uniform dielectric filling) placed inside regular vacuum waveguide of a larger radius and excited by a single incident waveguide mode [2]. Here we present analytical results for the case where the structure is excited by a moving charge. We also perform simulations using CST® PS code and compare results.

THEORY

Analytical methods for investigation of various waveguide discontinuities have been developed several decades ago [3, 4]. However, the number of problems analysed by rigorous methods is quite limited; as a rule, they belong to the situation where the structure is excited by a specified waveguide mode.

Here we apply the modified residue-calculus technique to investigation of radiation from a charge (or Gaussian bunch) moving along the axis of a semi-infinite cylindrical waveguide with uniform dielectric filling and having an open end. We consider this waveguide to be placed inside coaxial infinite vacuum waveguide. This problem is of interest in the context of development of Terahertz radiation source based on waveguide structures loaded with dielectric and excited by short electron bunch [5].

Geometry of the problem under consideration is shown in Fig. 1. A semi-infinite ideally conducting ($\sigma=\infty$) cylindrical waveguide with radius b filled with a homogeneous dielectric ($\varepsilon>1$) is put into a concentric infinite waveguide with radius $a>b$. A point charge q moves along the axis with constant velocity $\vec{V}=\beta c\vec{e}_z$. Fourier harmonic of the magnetic component of the incident field has the following form (cylindrical frame r,φ,z is used):

$$H_{\omega\varphi}^{(i)}=\frac{iq\tilde{s}}{2c}\left[H_1^{(1)}(r\tilde{s})-\frac{H_0^{(1)}(\tilde{r}\tilde{s})}{J_0(\tilde{r}\tilde{s})}J_1(r\tilde{s})\right]e^{i\omega z/V}, \qquad (1)$$

where $\tilde{r}=b$, $\tilde{s}=s=\sqrt{\omega^2V^{-2}(\varepsilon\beta^2-1)}$ for $z<0$, and $\tilde{r}=a$, $\tilde{s}=s_0=\sqrt{\omega^2V^{-2}(\beta^2-1)}$ for $z>0$ ($\operatorname{Im}\tilde{s}>0$). The reflected field in the domain (1) is

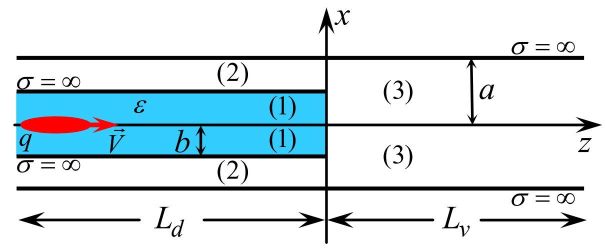

Figure 1: Geometry of the problem.

$$H_{\omega\varphi}^{(1)}=\sum\nolimits_{m=1}^{\infty}B_mJ_0(rj_{0m}/b)e^{\kappa_{zm}^{(1)}z}, \qquad (2)$$

where $J_0(j_{0m})=0$, $\kappa_{zm}^{(1)}=\sqrt{j_{0m}^2b^{-2}-k_0^2\varepsilon}$, $\operatorname{Re}\kappa_{zm}^{(1)}>0$, $k_0=\omega/c$. The fields generated in domains (2) and (3) can be presented by the following series:

$$H_{\omega\varphi}^{(3)}=\sum\nolimits_{m=1}^{\infty}A_mJ_0(rj_{0m}/a)e^{-\gamma_{zm}^{(3)}z}, \qquad (3)$$

$$H_{\omega\varphi}^{(2)}=C_0r^{-1}e^{\gamma_{z0}^{(2)}z}+\sum\nolimits_{m=1}^{\infty}C_mZ_m(r\chi_m)e^{\gamma_{zm}^{(2)}z}, \qquad (4)$$

where $\gamma_{zm}^{(3)}=\sqrt{j_{0m}^2a^{-2}-k_0^2}$, $\gamma_{z0}^{(2)}=-ik_0$, $\gamma_{zm}^{(2)}=\sqrt{\chi_m^2-k_0^2}$, $\operatorname{Re}\gamma_{zm}^{(2,3)}>0$,

$$Z_m(\xi)=J_1(\xi)-N_1(\xi)J_0(a\chi_m)N_0^{-1}(a\chi_m), \qquad (5)$$

χ_m is the solution of the dispersion relation for the domain (2),

$$J_0(b\chi_m)N_0(a\chi_m)-J_0(a\chi_m)N_0(b\chi_m)=0. \qquad (6)$$

Performing the matching of the components $H_{\omega\varphi}$ and $E_{\omega r}=c(i\omega\varepsilon)^{-1}\partial H_{\omega\varphi}/\partial z$ for $z=0$, and integrating these relations separately over $0<r<b$ with $J_0(rj_{0m}/a)$ and over $b<r<a$ with $Z_m(r\chi_m)$, after cumbersome calculations we obtain the following infinite systems for unknown coefficients:

$$\sum_{m=1}^{\infty}\left(\frac{\tilde{A}_m}{\gamma_{zm}^{(3)}-\gamma_{zp}^{(1)}}+\frac{\tilde{A}_m\rho_p}{\gamma_{zm}^{(3)}+\gamma_{zp}^{(1)}}\right)+\frac{q}{4j_{0p}cJ_1(j_{0p})}\times$$
$$\times\left[\left(\pi bs_0^2J_1(j_{0p})h_0-2ij_{0p}b^{-1}\right)\left(F_{vp}^{-}+\rho_pF_{vp}^{+}\right)+\right. \qquad (7)$$
$$\left.+F_{dp}^{+}+\rho_pF_{dp}^{-}\right]=0,$$

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$$\begin{gathered}\sum_{m=1}^{\infty}\left(\frac{\tilde A_m\rho_p}{\gamma_{zm}^{(3)}-\gamma_{zp}^{(1)}}+\frac{\tilde A_m}{\gamma_{zm}^{(3)}+\gamma_{zp}^{(1)}}\right)+\frac{q}{4j_{0p}cJ_1(j_{0p})}\times\\ \times\Big[\left(\pi b s_0^2 J_1(j_{0p})h_0-2ij_{0p}b^{-1}\right)\left(\rho_pF_{vp}^-+F_{vp}^+\right)+\\ +\rho_pF_{dp}^+ + F_{dp}^-\Big]=4\gamma_{zp}^{(1)}\kappa_{zp}^{(1)}\tilde B_p\Big/\left(\kappa_{zp}^{(1)}+\varepsilon\gamma_{zp}^{(1)}\right),\end{gathered}\tag{8}$$

$$\sum_{m=1}^{\infty}\frac{\tilde A_m}{\gamma_{zm}^{(3)}-\gamma_{zn}^{(2)}}+\frac{iq}{2c}\frac{s_0^2h_0}{\omega/(iV)-\gamma_{zn}^{(2)}}=0\,,\tag{9}$$

$$\sum_{m=1}^{\infty}\frac{\tilde A_m}{\gamma_{zm}^{(3)}+\gamma_{zn}^{(2)}}+\frac{iq}{2c}\frac{s_0^2h_0}{\omega/(iV)+\gamma_{zn}^{(2)}}=-2\tilde C_n\gamma_{zn}^{(2)}\,,\tag{10}$$

$$h_0=H_0^{(1)}(bs_0)-H_0^{(1)}(as_0)J_0(bs_0)\big/J_0(as_0)\,,\tag{11}$$

$$F_{dp}^{\pm}=\frac{2ij_{0p}}{\pi b}\left(\frac{\omega}{iV\varepsilon}\pm\gamma_{zp}^{(1)}\right)\left[s^2-(j_{0p}/b)^2\right]^{-1},\tag{12}$$

$$F_{vp}^{\pm}=2ij_{0p}(\pi b)^{-1}\left(\omega/(iV)\pm\gamma_{zp}^{(1)}\right)^{-1},\tag{13}$$

$$\rho_p=\left(\varepsilon\gamma_{zp}^{(1)}-\kappa_{zp}^{(1)}\right)\left(\varepsilon\gamma_{zp}^{(1)}+\kappa_{zp}^{(1)}\right)^{-1},\tag{14}$$

$$\frac{\tilde A_m}{A_m}=J_0\left(\frac{bj_{0m}}{a}\right)\frac{j_{0m}}{a}\,,\quad \frac{\tilde B_p}{B_p}=\frac{bJ_1(j_{0p})}{2},\tag{15}$$

$$\frac{\tilde C_0}{C_0}=\ln\left(\frac{a}{b}\right),\quad \frac{\tilde C_n}{C_n}=\frac{a^2}{2b}\frac{Z_n^2(a\chi_n)}{Z_n(b\chi_n)}-\frac{b}{2}Z_n(b\chi_n)\,.\tag{16}$$

According to the residue-calculus technique [2, 3], in order to solve the systems (7)–(10) one should construct the function $f(w)$ regular in the complex plane w (excluding first-order poles $w=\omega/(iV)$, $w=\gamma_{zm}^{(3)}$), having certain first-order zeros and satisfying condition $f(w)\xrightarrow[|w|\to\infty]{} w^{-(\tau+1/2)}$ with $\sin(\pi\tau)=(\varepsilon-1)/(2\varepsilon+2)$ (consequence of Meixner's edge condition [3] for $r=b$, $z\to+0$). Considering the integrals over the circle C_∞ with infinite radius

$$\oint_{C_\infty}\frac{f(w)}{w\pm\gamma_{zn}^{(2)}}dw=\oint_{C_\infty}\left(\frac{f(w)}{w\mp\gamma_{zp}^{(1)}}+\frac{\rho_pf(w)}{w\pm\gamma_{zp}^{(1)}}\right)dw=0\,,\tag{17}$$

calculating them using residue theorem and comparing the result with (7)–(10), we obtain

$$\tilde A_p=\operatorname{Res}f(\gamma_{zp}^{(3)})\,,\quad \tilde C_n=f(-\gamma_{zn}^{(2)})(2\gamma_{zn}^{(2)})^{-1},\tag{18}$$

$$\begin{gathered}\tilde B_p=\left(\varepsilon\gamma_{zp}^{(1)}+\kappa_{zp}^{(1)}\right)\Big/\left(4\gamma_{zp}^{(1)}\kappa_{zp}^{(1)}\right)\left(\frac{iq}{2cb}\left(\rho_pF_{dp}^++F_{dp}^-\right)-\right.\\ \left.-\rho_pF_{vp}^--F_{vp}^+-\rho_pf(\gamma_{zp}^{(1)})-f(-\gamma_{zp}^{(1)})\right).\end{gathered}\tag{19}$$

Function $f(w)=Pg(\omega)$, where

$$g(w)=\frac{(w-\gamma_{z0}^{(2)})\prod_{n=1}^{\infty}\left(1-\frac{w}{\gamma_{zn}^{(2)}}\right)}{\prod_{m=1}^{\infty}\left(1-\frac{w}{\gamma_{zn}^{(3)}}\right)}\prod_{s=1}^{\infty}{}^{(l)}\left(1-\frac{w}{\Gamma_s}\right)Q(w)\,,\tag{20}$$

$$Q(w)=\exp\left[-\frac{w}{\pi}\left(b\ln\left(\frac{b}{a-b}\right)+a\ln\left(\frac{a-b}{a}\right)\right)\right],\tag{21}$$

$P=iqs_0^2h_0[2cg(-i\omega/V)]^{-1}$ and shifted zeros $\Gamma_s=\gamma_{zs}^{(1)}+\pi\Delta_s/b$ are found by iterations from

$$\begin{gathered}f(\gamma_{zp}^{(1)})+\rho_pf(-\gamma_{zp}^{(1)})=\\ =iq\left(F_{dp}^++\rho_pF_{dp}^--F_{vp}^--\rho_pF_{vp}^+\right)\Big/\left(2cbJ_1(j_{0p})\right),\end{gathered}\tag{22}$$

which result in nonlinear infinite system

$$\frac{\pi}{b}\Delta_p=\frac{G_pu_p\left[\Gamma_p-\omega/(iV)\right]-2\gamma_{zp}^{(1)}\rho_p}{\upsilon_{p+}+\rho_p\upsilon_{p-}}\,,\tag{23}$$

$$u_p=g(w)\left.\frac{w-\omega/(iV)}{1-w/\Gamma_p}\right|_{w=\omega/(iV)},\quad \upsilon_{p\pm}=\frac{g(\pm\gamma_{zp}^{(1)})}{1\mp\gamma_{zp}^{(1)}/\Gamma_p}\,,\tag{24}$$

$$G_p=\left(F_{dp}^++\rho_pF_{dp}^--F_{vp}^--\rho_pF_{vp}^+\right)\Big/\left(bJ_1(j_{0p})s_0^2h_0\right).\tag{25}$$

Meixner's edge condition dictates that $\Delta_s\to\tau$ for $s\to\infty$, which gives convenient zero-order approximation for Δ_s and allows controlling the iteration process convergence. The described approach gives the modes excitation coefficients for the given frequency ω. It should be also noted that the substitution $q\to q\exp(-\omega^2\sigma_g^2V^{-2})$ in the formulas above gives the result for the case of Gaussian bunch of length σ_g.

RESULTS

The key problem of the described theoretical approach is the determination of the shifted zeros Γ_m from the relation (23). Iteration process is organized as follows. We fix quantity N of Δ_m to be found. For the zero-order approximation, we put $\Delta_m=\tau$ for all m and calculate first-order approximation for $\Delta_m, m=1,2,\ldots N$. Then we substitute found Δ_m in the right-hand side of (23) and calculate second-order approximation, etc. After iterations have converged, we compare Δ_N with τ: if $\Delta_N=\tau$ within accepted accuracy, process is stopped, otherwise we change N and/or accuracy of calculations and repeat.

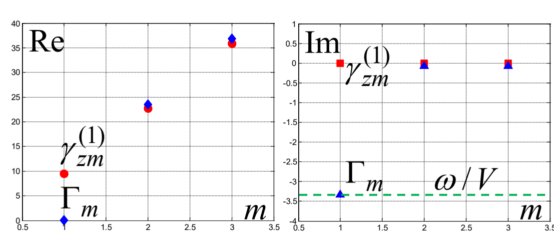

Figure 2: Shifting for first 3 zeros.

For the numerical simulation in CST® PS suite [6], we have chosen the following parameters: $b=0.24$cm, $\varepsilon=10$, $a=0.90$cm, $\beta=0.9999$, 1nC Gaussian bunch with

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$\sigma_g = 0.5\text{cm}$. In this case, the incident field (1) in dielectric waveguide contains only first Cherenkov mode with frequency $\omega_{1\text{Cher}} = 2\pi \cdot 16\,\text{GHz}$ (for this frequency $s = j_{01}/b$), other modes are suppressed by the factor $\exp(-\omega^2\sigma_g^2 V^{-2})$. The reflected (2) and transmitted (3), (4) fields contain a single propagating mode, other modes are evanescent. Analytical calculations have been performed for $\omega = \omega_{1\text{Cher}}$. We used $N = 80-100$, and system (23) got converged at around 10 iterations with 0.001 accuracy.

Figure 2 shows how the first 3 zeros Γ_m are shifted with respect to $\gamma_{zm}^{(1)}$: essential shift appears for Γ_1 only, Γ_1 is purely imaginary (while $\gamma_{z1}^{(1)}$ was purely real) and $\Gamma_1 \approx \omega/(iV)$. As one can show, $P \sim g^{-1}(\omega/(iV)) \sim (1-\omega/(iV\Gamma_1))^{-1}$, therefore frequency spectrum of coefficients A_m, B_m and C_m contains pole which corresponds to $\omega_{1\text{Cher}}$. Contribution of this pole describes reflected and transmitted Cherenkov wave.

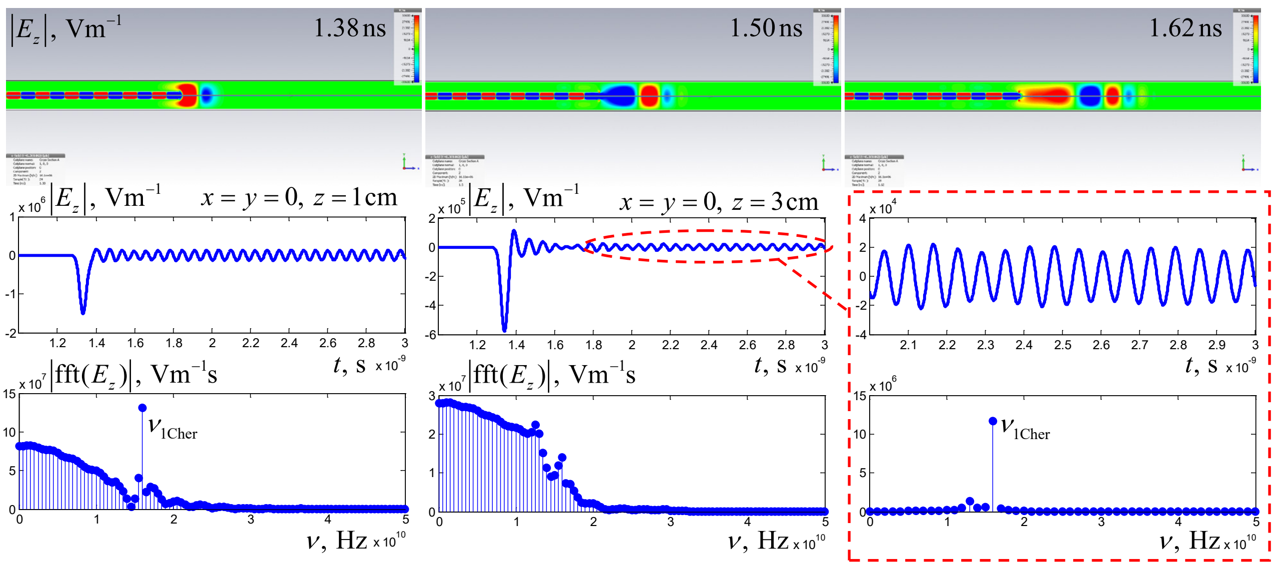

Figure 3: Simulated field distribution (top), signals from probes located on $z-$axis (middle) and spectra of these signals (bottom). Structure lengths for simulations are $L_d = 35\,\text{cm}$, $L_v = 50\,\text{cm}$. Full time of bunch propagation through the structure is $3\,\text{ns}$.

Figure 3 shows the results of simulations. From two-dimensional field distribution (upper row) we can see some wave process in the domain (3) (larger radius vacuum waveguide). Field dependencies obtained from two probes (middle row) show relatively weak quasi-harmonic signal after strong peak which presents Coulomb field. Fast Fourier transforms of these signals (lower row) show that a harmonic $\omega_{1\text{Cher}} = 2\pi \cdot \nu_{1\text{Cher}}$ is clearly expressed in the signal from the first probe. Signal from the second probe contains a lot of secondary harmonics, and Cherenkov harmonic can be clearly seen in the reduced signal (from 2ns to 3ns).

CONCLUSION

In this report, we have presented rigorous theory for describing radiation from the open end of a cylindrical waveguide with dielectric filling in the case where it is enclosed in vacuum waveguide with larger radius and is excited by the charge (or Gaussian bunch) passing along the structure axis. In particular, we have shown that spectra of transmitted and reflected modes contain frequency corresponding to Cherenkov radiation in dielectric waveguide. We have also performed direct numerical simulation of electromagnetic field excitation in such structure using CST® PS code. Results are in qualitative agreement since the simulated signal contains Cherenkov frequency obtained theoretically.

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