MODELING OF ADSR DYNAMICS WITH PROTON LINAC IN MULTI-POINT APPROXIMATION

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Abstract

The mathematical model of multi-point kinetics is proposed in the paper. The transients in subcritical reactor driven by proton linac taking into account the fuel and coolant temperature feedbacks are analyzed using this model. In contrast to the widely used point kinetics model, the proposed model makes it possible to more accurately take into account the heterogeneity of the material composition in the core. That is the one of the main features of transmutation systems with accelerator-driver.

INTRODUCTION

Accelerator Driven System (ADS) is a combination of high-power electronuclear neutron source with subcritical reactor [1]. In such systems external neutrons comes from the interaction of high energy proton beam with a heavy atom nucleus (spallation). ADS is of interest nowadays due to its prospects in long-living radionuclides transmutation [2], also subcritical condition in ADS provides advantages from safety standpoint in comparison with regular critical nuclear reactors.

Due to development of subcritical reactor technology for transmutation of long-living radionuclear waste, reactor cores with significant fuel spacial inhomogeneity are widely considered. Cascade system consisting of fast neutron section (plutonium fuel) and thermal neutron section (uranium fuel) can be treated as an example of such core [3, 4]. During dynamical processes analysis in the reactor with fuel inhomogeneity it is necessary to calculate kinetics characteristics (prompt mean lifetime, delayed-neutron fraction) correctly [5]. These parameters depend on fission nuclides and neutron spectrum in the reactor core [6]. In this paper multi-point kinetics model is proposed. This model allows to estimate the influence of fuel inhomogeneity in the reactor core on the kinetics characteristics better, than the well-known point kinetics model [7]. It is compared with well-known point kinetics model in dynamics analysis of subcritical reactor with homogeneous and heterogeneous fuel composition driven by proton linac.

MULTI-POINTS KINETICS EQUATIONS DERIVATION

In the general case nonstationary neutron distribution in the reactor core is described by the following set of integro-differential equations [8]

$$
\begin{aligned}
&\frac{1}{v}\frac{\partial F(\mathbf{r},E,\mathbf{n},t)}{\partial t} = -\mathbf{n}\nabla F(\mathbf{r},E,\mathbf{n},t)+ \\
&\int\limits_0^{E_0} dE' \int (1-\beta)\nu_{E'}(\mathbf{r})\Sigma_f(\mathbf{r},E')\frac{\chi(E)}{4\pi}F(\mathbf{r},E,\mathbf{n},t)d\Omega' - \\
&\Sigma_{ais}(\mathbf{r},E)F(\mathbf{r},E,\mathbf{n},t) + \sum_{i=1}^{6}\lambda_i C_i(\mathbf{r},t)\frac{\chi_{di}(E)}{4\pi} + \qquad (1)\\
&\int\limits_0^{E_0} dE' \int \omega(E,E',\mathbf{n}\cdot\mathbf{n}',\mathbf{r})F(\mathbf{r},E,\mathbf{n},t)d\Omega' + q(\mathbf{r},E,\mathbf{n},t), \\
&\frac{\partial C_i(\mathbf{r},t)}{\partial t} = \int\limits_0^{E_0} dE' \int \beta_i\Sigma_f(\mathbf{r},E')F(\mathbf{r},E,\mathbf{n},t)\nu_{E'}(\mathbf{r})d\Omega' - \\
&\lambda_i C_i(\mathbf{r},t).
\end{aligned}
$$

Here $F(\mathbf{r},E,\mathbf{n},t)$ — neutron flux, $C_i(\mathbf{r},t)$ — concentration of the i-th energy group delayed neutron precursors, $q(\mathbf{r},E,\mathbf{n},t)$ — external source, λ_i — decay constant of i-th energy group delayed neutrons, β_i — i-th energy group delayed neutron fraction, $\beta = \sum\limits_{i=1}^{6}\beta_i$, $\Sigma_{ais}(\mathbf{r},E)$ — macroscopic loss cross-section, $\Sigma_f(\mathbf{r},E)$ — macroscopic fission cross-section, $\nu_{E'}(\mathbf{r})$ — average number of neutrons produced per fission under the interaction of neutrons with energy E', $\omega(E,E',\mathbf{n}\cdot\mathbf{n}',\mathbf{r})$ — probability of neutrons of state $(E',\mathbf{n}')$ to move to state $(E,\mathbf{n})$ in elastic and inelastic scattering. Functions $\chi(E)$ and $\chi_i(E)$ describe normed delayed and prompt neutrons energy respectively.

Function $F(\mathbf{r},E,\mathbf{n},t)$ and $C_i(\mathbf{r},t)$ satisfy the following initial and boundary conditions

$$
\begin{aligned}
F(\mathbf{r}_\mathrm{b},E,\mathbf{n}_\mathrm{in},t) = 0, \quad C_i(\mathbf{r}_\mathrm{b},t) = 0,\\
F(\mathbf{r},E,\mathbf{n},0) = \tilde{F}_0(\mathbf{r},E,\mathbf{n}), \quad C_i(\mathbf{r},0) = c_{i0}(\mathbf{r}). \qquad (2)
\end{aligned}
$$

Widespread point kinetics method supposes separation of spatial and temporal variables:

$$F(\mathbf{r},E,\mathbf{n},t) \approx \tilde{F}(\mathbf{r},E,\mathbf{n})\phi(t),$$

where $\tilde{F}(\mathbf{r},E,\mathbf{n})$ is a solution of corresponding stationary equation with boundary condition like Eq. (2).

Point kinetics equations are the following:

$$
\begin{aligned}
\frac{d\phi(t)}{dt} &= \frac{\phi}{l}\left(\frac{k_\mathrm{eff}-1}{k_\mathrm{eff}} - \beta_\mathrm{eff}\right) + \sum_{i=1}^{6}\lambda_i C_{\mathrm{eff}i}(t) + q_\mathrm{eff}(t),\\
\frac{dC_{\mathrm{eff}i}(t)}{dt} &= \frac{\beta_{\mathrm{eff}i}\phi(t)}{l} - \lambda_i C_{\mathrm{eff}i}(t), \qquad (3)
\end{aligned}
$$

where l — prompt average lifetime, β_eff — effective delayed neutron fraction, $C_{\mathrm{eff}i}(t)$ — effective concentration of the i-th group delayed neutron precursors, $q_\mathrm{eff}(t)$ — effective external neutron source intensity [9].

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Multi-point kinetics equation can be derived using "method of coupled zones" [10], based on the idea that neutron field in the reactor core is considered as a superposition of neutron fields produced by fission neutrons in inner and outer sections.

If we split the reactor core into n zones with fissile materials, then due to linearity the Eq. (1) can be presented as a system of n equations ($k, j = 1..n$, $k \neq j$) [7]

$$\frac{1}{v}\frac{\partial F_k(\mathbf{r},E,\mathbf{n},t)}{\partial t} = -\mathbf{n}\nabla F_k(\mathbf{r},E,\mathbf{n},t) +$$
$$\int_0^{E_0} dE' \int (1-\beta_k)\nu_{E'k}(\mathbf{r})\Sigma_{fk}(\mathbf{r},E')\frac{\chi_k(E)}{4\pi}F_k(\mathbf{r},E,\mathbf{n},t)d\Omega' -$$
$$\Sigma_{ais}(\mathbf{r},E)F_k(\mathbf{r},E,\mathbf{n},t) + \sum_{i=1}^{6}\lambda_i C_{ki}(\mathbf{r},t)\frac{\chi^k_{di}(E)}{4\pi} + \tag{4}$$
$$\int_0^{E_0} dE' \int (1-\beta_k)\nu_{E'k}(\mathbf{r})\Sigma_{fk}(\mathbf{r},E')\frac{\chi_k(E)}{4\pi}F_j(\mathbf{r},E,\mathbf{n},t)d\Omega' +$$
$$\int_0^{E_0} dE' \int \omega(E,E',\mathbf{n}\cdot\mathbf{n}',\mathbf{r})F_k(\mathbf{r},E,\mathbf{n},t)d\Omega' + q_k(\mathbf{r},E,\mathbf{n},t),$$
$$\frac{\partial C_{ki}(\mathbf{r},t)}{\partial t} = \int_0^{E_0} dE' \int \beta_{ki}\Sigma_{fk}(\mathbf{r},E')F_k(\mathbf{r},E,\mathbf{n},t)\nu_{E'k}(\mathbf{r})d\Omega' +$$
$$\int_0^{E_0} dE' \int \beta_{ki}\Sigma_{fk}(\mathbf{r},E')F_j(\mathbf{r},E,\mathbf{n},t)\nu_{E'k}(\mathbf{r})d\Omega' - \lambda_i C_{ki}(\mathbf{r},t).$$

where $F_k(\mathbf{r},E,\mathbf{n},t)$ — neutron field in the reactor core, generated by neutrons $\nu_{E'_k}(\mathbf{r})\Sigma_{fk}$, produced in k-th zone. Then Eq. (1) solution can be represented in the following form

$$F(\mathbf{r},E,\mathbf{n},t) = \sum_{k=1}^{n} F_k(\mathbf{r},E,\mathbf{n},t).$$

The same representation can be done for stationary Boltzman equation solution: $\tilde{F}(\mathbf{r},E,\mathbf{n}) = \sum_{k=1}^{n}\tilde{F}_k(\mathbf{r},E,\mathbf{n})$.

The main assumption of two-point kinetics model (deducted by analogy with point kinetics model) is the possibility of phase and time variables separation for function $F(\mathbf{r},E,\mathbf{n},t)$:

$$F_k(\mathbf{r},E,\mathbf{n},t) \approx \tilde{F}_k(\mathbf{r},E,\mathbf{n})\phi_k(t). \tag{5}$$

The system of equations adjoint to Eq. (4) can be written in the following form:

$$0 = \mathbf{n}\nabla\tilde{F}^+_k(\mathbf{r},E,\mathbf{n}) - \Sigma_{ais}(\mathbf{r},E)\tilde{F}^+_k(\mathbf{r},E,\mathbf{n}) +$$
$$+\int_0^{E_0} dE' \int \omega(E,E',\mathbf{n}\cdot\mathbf{n}',\mathbf{r})\tilde{F}^+_k(\mathbf{r},E,\mathbf{n})d\Omega' + \tag{6}$$
$$+\frac{1}{k_k}\int_0^{E_0} dE' \left((1-\beta_k)\frac{\chi_k(E')}{4\pi} + \sum_{i=1}^{6}\beta_{ki}\frac{\chi^k_{di}(E')}{4\pi}\right)\cdot$$
$$\cdot\int \nu_{E'_k}(\mathbf{r})\Sigma_{fk}(\mathbf{r},E')\tilde{F}^+_k(\mathbf{r},E,\mathbf{n})d\Omega'.$$

Here k_k — multiplication factor in k-th zone. Functions $\tilde{F}^+_k(\mathbf{r},E,\mathbf{n})$ satisfy the boundary conditions

$$\tilde{F}^+_k(\mathbf{r}_\mathrm{b},E,\mathbf{n}_\mathrm{out}) = 0. \tag{7}$$

The traditional procedure, described in [8], is used to derive two-point kinetics equations: Eq. (4) are multiplied by $\tilde{F}^+_k(\mathbf{r},E,\mathbf{n})$ and Eq. (6) — by $F_k(\mathbf{r},E,\mathbf{n},t)$ correspondingly, then subtract the obtained expressions and integrate the result over the entire reactor volume, energy and neutrons velocity directions. Through the conjugacy of Eqs. (1) and (6), the terms taking into account neutron absorption and moderation are reduced and through boundary conditions (2) and (7) — terms describing neutron transport. As a result, taking into account assumption Eq. (5) two-point equations in similar to Eq. (3) form can be obtained ($k, j = 1..n$, $k \neq j$):

$$\frac{d\phi_k(t)}{dt} = \frac{\phi_k}{l_k}\left(\frac{k_k-1}{k_k} - \beta^k_\mathrm{eff}\right) + \frac{\phi_j(t)}{l_k}\left(\varepsilon_{kj} - \beta^{kj}_\mathrm{eff}\right) +$$
$$+\sum_{i=1}^{6}\lambda_i c_{ki}(t) + Q_k(t), \tag{8}$$
$$\frac{dc_{ki}(t)}{dt} = \frac{\beta^k_{\mathrm{eff}i}\phi_k(t)}{l_k} + \beta_{\mathrm{eff}}{}_i^{kj}\frac{\phi_j(t)}{l_k} - \lambda_i c_{ki}(t).$$

Introduced in the Eq. (8) parameters are defined by these expressions

$$l_k = \frac{1}{\mathrm{FNI}_k}\int_0^{E_0}\int\int dE\, d\Omega\, dV \frac{\tilde{F}_k(\mathbf{r},E,\mathbf{n})\tilde{F}^+_k(\mathbf{r},E,\mathbf{n})}{v},$$

$$Q_k(t) = \frac{1}{\mathrm{FNI}_k l_k}\int_0^{E_0}\int\int dE\, d\Omega\, dV \tilde{F}^+_k(\mathbf{r},E,\mathbf{n})q(\mathbf{r},E,\mathbf{n},t),$$

$$Q_k(t) = \frac{1}{\mathrm{FNI}_k l_k}\int_0^{E_0}\int\int dE\, d\Omega\, dV\ \tilde{F}^+_k(\mathbf{r},E,\mathbf{n})C_{ki}(\mathbf{r},t)\frac{\chi^k_{di}}{4\pi},$$

$$\varepsilon_{kj} = \frac{1}{\mathrm{FNI}_k}\int_0^{E_0} dE \int d\Omega \int dV\ \tilde{F}^+_k(\mathbf{r},E,\mathbf{n})\frac{\chi_k(E)}{4\pi}\cdot$$
$$\cdot\int_0^{E_0} dE' \int \Sigma_{fk}(\mathbf{r},E')\nu_{E'_k}(\mathbf{r})\tilde{F}_j(\mathbf{r},E',\mathbf{n}')d\Omega',$$

$$\beta^{kj}_{\mathrm{eff}i} = \frac{1}{\mathrm{FNI}k}\int_0^{E_0} dE \int d\Omega \int dV\ \tilde{F}^+_k(\mathbf{r},E,\mathbf{n})\frac{\chi^k_{di}(E)}{4\pi}\cdot$$
$$\cdot\int_0^{E_0} dE' \int \beta_{ki}\Sigma_{fk}(\mathbf{r},E')\nu_{E'_k}(\mathbf{r})\tilde{F}_j(\mathbf{r},E',\mathbf{n}')d\Omega',$$

$$\beta^{kj}_\mathrm{eff} = \sum_{i=1}^{6}\beta^{kj}_{\mathrm{eff}i},$$

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$$\mathrm{FNI}_k = \int\limits_0^{E_0} \int \int dE\, d\Omega\, dV \tilde{F}_k^+(\mathbf{r}, E, \mathbf{n}) \cdot$$

$$\cdot \int\limits_0^{E_0} dE' \left((1-\beta_k) \frac{\chi_k(E')}{4\pi} + \sum_{i=1}^{6} \beta_{ki} \frac{\chi_{di}^k(E')}{4\pi} \right) \cdot$$

$$\cdot \int \Sigma_{fk}(\mathbf{r}, E') \nu_{E'_k}(\mathbf{r}) \tilde{F}_k(\mathbf{r}, E', \mathbf{n}') d\Omega'.$$

Function $\tilde{F}_k(\mathbf{r}, E, \mathbf{n})$ and $\tilde{F}_k^+(\mathbf{r}, E, \mathbf{n})$ here are the solutions of stationary Boltzman equation and Eq. (6). These equations are solved numerically (Monte-Carlo method for multidimentional case or discrete ordinates method for one-dimensional). In this paper as an example one-dimensional multi-group diffusion transport model is used.

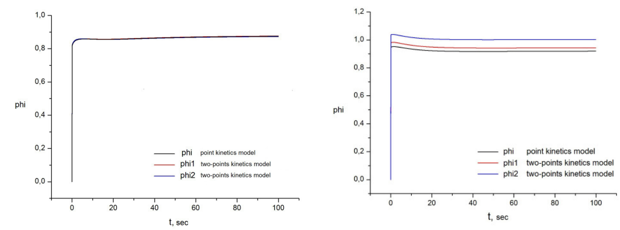
RESULTS AND ANALYSIS

To illustrate the proposed multi-point kinetics model, dynamics of ADS with homogeneous and heterogeneous reactor cores was calculated in two-point and point [11] approximation. Homogeneous reactor core with ^{235}U-fuel without moderator was divided into two sections with the same fuel composition. Heterogeneous reactor core already consists of two sections with different fuel: the inner section — ^{239}Pu-fuel and the outer one — ^{235}U-fuel with moderator ($\rho(^{12}\mathrm{C})/\rho(^{235}\mathrm{U}) = 10$). Physical characteristics of considered reactor cores are presented in Table 1 (index 0 corresponds to the whole reactor core, and indexes 1, 2 are related to the respective sections).

Table 1: Reactor Cores Main Physical Characteristics

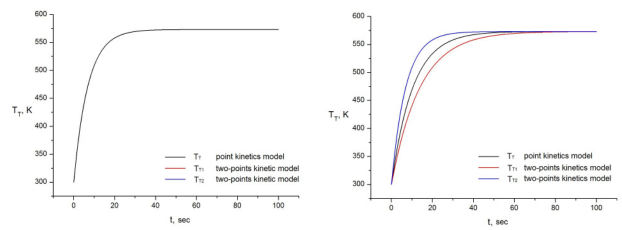
Characteristics	Homogeneous	Heterogeneous
k_{eff}	1.0013	1.0013
k_1	0.625	0.613
k_2	0.610	0.646
β_{eff}	0.0068	0.0045
β_{eff1}	0.0068	0.00215
β_{eff2}	0.0068	0.0068
l, sec	$6.4 \cdot 10^{-7}$	$8.5 \cdot 10^{-6}$
l_1, sec	$9 \cdot 10^{-7}$	$4 \cdot 10^{-6}$
l_2, sec	$9 \cdot 10^{-7}$	$2 \cdot 10^{-5}$

In Fig. 1 functions ϕ, ϕ_1, ϕ_2 change in time for homogeneous and heterogeneous reactor core respectively using point and two-point kinetics models is shown. Figure 2 illustrates average fuel temperature change in time. From calculation result it could be seen that for homogeneous reactor cores two-point kinetics model has no advantages over point model, but in case of heterogeneous reactor cores significant difference as in flux time-component as in average temperature is observed. It can be explained by the



a) b)

Figure 1: Time dependence of functions ϕ, ϕ_1, ϕ_2 for reactor cores with a) homogeneous b) heterogeneous fuel composition.



a) b)

Figure 2: Time dependence of average fuel temperature for reactor cores with a) homogeneous b) heterogeneous fuel composition.

fact, that in two-points model kinetics parameters averaging is fulfilled more accurately than in the point model (see Table 1).

CONCLUSION

Multi-point kinetics model, that can describe non-stationary processes in the reactor cores with spacial irregularity as in fuel composition as in energy neutron spectrum, is proposed. Homogeneous and heterogeneous reactor cores dynamics calculated via point and two-point kinetics models are compared. Good results agreement in case of homogeneous reactor core and significant disagreement in case of heterogeneous core are shown. The mentioned difference is explained by insufficiently correct averaged values of kinetics parameters in point model. The proposed multi-point kinetics model can be recommended for dynamical processes calculation in hybrid cascade subcritical reactors.

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