MODEL OF THE OPTIMAL PARAMETERS CHOICE FOR THE CHARGED PARTICLES BEAM

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Abstract

Problem of the optimal parameters choice for the charged particles beam is considered. It is supposed that the beam is characterized by the set of quality characteristics and may be controlled by multiple parameters. It is assumed that in general case choice of the control parameters that is optimal for all criteria is not possible. In the article the optimization problem is formulated as the conflict control problem. The case is considered when parameters that should be optimized form the vector. Two cases are under consideration. In the first one fully optimal solution may be found. In the second case finding of the compromise solution is considered. Computing algorithms are proposed.

INTRODUCTION

The charged particles beams are used in different areas of technics, science, medicine etc. Accurate adjustment of the beam characteristics often plays crucial role in producing final results of a target irradiation. There are different approaches to the optimization of the beam parameters adjustment [1], [2], [3]. In many cases there are multiple criteria of the beam quality which should be taken into account. Some of these characteristics should be considered in tight connection with the target properties. For example in the hadron therapy among possible characteristics of the particles beam following ones may be mentioned: energy, intensity, biological efficiency, depth of the Bragg peak localization, influence of the fragmentation products and so on. Importance of the problem is confirmed by projects in hadron therapy [4], new facilities in high energy physics [5].

In the article [6] problem of the focusing system optimization was considered with the only one beam characteristic - beam divergence. The only control device was the accelerator's focusing system. In more general situations few beam characteristics may be important and few control channels may be used. For example, in stereotactic radiosurgery a set of up to few hundreds of irradiation channels are used which should be adjusted for more efficient action on tumor. In these case all optimization parameters may be considered as independent. On the other hand multiple quality characteristics may be dependent. In the hadron radiotherapy energy of the therapeutic beam is connected with its biological efficiency. But change of the beam energy influences depth and width of the Bragg peak localization. Shift of the depth-dose distribution maximum and its broadening will affect treatment effect. Also increasing of the beam energy will increase effects of fragmentation also affecting results of the hadron therapy. So some criteria of optimization may be contradictory. In this case choice of optimal parameters should be considered from the point of view of finding of a compromise of all characteristics of the particles beam.

At the present paper the problem of the beam optimization with multiple quality characteristics is considered as the problem of conflict control. Both cases of independent and conflicting characteristics are considered. Formalization of the problem and algorithms of its solution are proposed.

FORMALIZATION OF THE CONFLICT CONTROL PROBLEM

It is supposed that dynamic of the beam characteristics may be described by the system of differential equations:

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}, \mathbf{u}, \mathbf{v}), \tag{1}$$

with initial condition

$$\mathbf{X}(t = 0) = \mathbf{X}_0, t \in [0, T] \tag{2}$$

Here $\mathbf{X} \in \mathrm{R}^m$ is state vector, $\mathbf{u} \in U \subset \mathrm{R}^p$ are control parameters which should be adjusted to improve the beam's quality, and $\mathbf{v} \in V \subset \mathrm{R}^q$ are control parameters, associated with uncontrollable external actions. U and V are compact sets in Euclidean spaces R^p and R^q. It is supposed that vector function $\mathbf{f}(\mathbf{X}, \mathbf{u}, \mathbf{v})$ in (1) satisfies following conditions:

1. $\mathbf{f}$ is continuous on $(\mathbf{X}, \mathbf{u}, \mathbf{v}) \in R^m \times U \times V$;
2. $\mathbf{f}$ satisfies Lipschitz condition for $\mathbf{X}$ with constant A, i.e. for any $\mathbf{u} \in U$, $\mathbf{v} \in V$ and $\mathbf{X}, \overline{\mathbf{X}} \in R^m$ the following inequality holds:
$$\left|\mathbf{f}(\mathbf{X}, \mathbf{u}, \mathbf{v}) - \mathbf{f}(\overline{\mathbf{X}}, \mathbf{u}, \mathbf{v})\right| \leq A \left|\mathbf{X} - \overline{\mathbf{X}}\right|, \tag{3}$$
3. $\mathbf{f}$ is bounded, i.e. for any $\mathbf{u} \in U, \mathbf{v} \in V, \mathbf{X} \in R^m$:
$$|\mathbf{f}(\mathbf{X}, \mathbf{u}, \mathbf{v})| \leq B, \ \ (B > 0); \tag{4}$$
4. for any $\mathbf{X} \in R^m$ set
$$\{\mathbf{f}(\mathbf{X}, \mathbf{u}, \mathbf{v}) | \ \ \mathbf{u} \in U, \ \mathbf{v} \in V\}$$
is convex.

We assume that these conditions hold for a wide range of physical situations under consideration.

Definition. Measurable on the interval $[0, T]$ vector functions $\mathbf{u} = \mathbf{u}(t)$ $(\mathbf{v} = \mathbf{v}(t))$ which satisfies conditions

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$\mathbf{u}(t) \in U$ ($\mathbf{v}(t) \in V$) for any $t \in [0, T]$ are called *admissible controls*.

For any pair of admissible controls $\mathbf{u}(t),\ t \in [0, T]$ and $\mathbf{v}(t),\ t \in [0, T]$ conditions 1–3 guarantee existence and uniqueness of solution continuable to interval $[0, T]$ for the system (1), which satisfies initial condition (2).

At the moment T of achieving of required quality, process participant who makes control choice $\mathbf{v}$ gets payoff equal to $\mathbf{H}(\mathbf{X}(T))$, where $\mathbf{X}(T)$ is final system state, $\mathbf{H} : R^m \to R^n$ is uniformly continuous bounded vector-function which satisfies Lipschitz condition: for any $\mathbf{X}, \overline{\mathbf{X}} \in R^m$ there exists positive constant L, such as:

$$\left|\mathbf{H}(\mathbf{X}) - \mathbf{H}(\overline{\mathbf{X}})\right| \le L \left|\mathbf{X} - \overline{\mathbf{X}}\right|.$$

Operator's goal is getting the optimal solution for $\mathbf{H}(\cdot)$ by the choice of control $\mathbf{u}$. External factors may be considered as action of the other operator (let us call him "operator 2") with opposite goal.

Definition *Control strategy* $\varphi(\psi)$ of operator (operator 2) is called pair (σ_1, K_{σ_1}) $((\sigma_2, K_{\sigma_2}))$, where $\sigma_1 = \left\{t_0 = 0 < t_1 < \ldots < t_{N_{\sigma_1}} = T\right\}$ $(\sigma_2 = \left\{t_0 = 0 < t_1 < \ldots < t_{N_{\sigma_2}} = T\right\})$ is arbitrary partitioning of interval $[0, T]$ and $K_{\sigma_1}(K_{\sigma_2})$ is mapping that associates with state of information of operator (operator 2) at time's moments $t_i \in \sigma_1,\ i = 0, \ldots, N_{\sigma_1} - 1$ $(t_j \in \sigma_2,\ j = 0, \ldots, N_{\sigma_2} - 1)$ admissible controls $u_i(\tau),\ \tau \in [t_i, t_{i+1})$ $(v_j(\tau),\ \tau \in [t_j, t_{j+1}))$.

Set of control strategies of operator (operator 2) is denoted by $\Phi(\Psi)$. Trajectory $\chi(\varphi, \psi)$ is uniquely defined by the pair of strategies (φ, ψ) in the following way. Let $\sigma = \{t_0 < t_1 < ... < t_{N_\sigma}\}, \sigma = \sigma_1 \cup \sigma_2$ is arbitrary decomposition of the time interval $[0, T]$. At any subinterval $[t_k, t_{k+1}), k = 0, 1, ..., N_\sigma - 1$ images of maps K_{σ_1} and K_{σ_2} are continuous controls $\mathbf{u}(t)$ and $\mathbf{v}(t)$ so on subinterval $[t_0, t_1)$ eq.(1) has unique solution $\mathbf{X}(t) = \mathbf{X}(\mathbf{X}_0, \mathbf{u}(t), \mathbf{v}(t)), \mathbf{X}(t_0) = \mathbf{X}_0, t \in [t_0, t_1)$. After that eq.(1) should be solved with initial conditions $\mathbf{X}(t_1)$. Making iterations of the algorithm the unique trajectory $\chi(\varphi, \psi)$ of control parameters may be obtained.

Payoff function of operator for the situation (φ, ψ) is defined as follows:

$$\mathbf{K}(\mathbf{X}_0, \varphi, \psi) = \mathbf{H}(\chi(\varphi, \psi)(T)),$$

where $\chi(\varphi, \psi)(T) = \chi(\varphi, \psi)(t)\,|_{t=T}$ and $\chi(\varphi, \psi)(t)$ is trajectory of control corresponding to $(\varphi, \psi) \in \Phi \times \Psi$.

Due to antagonistic type of operators interaction payoff function of operator 2 is equal to $-\mathbf{K}(\mathbf{X}_0, \varphi, \psi)$.

In [7] it was demonstrated that ε-equilibrium exists in games in complete metric spaces, when dynamics of the game is defined by generalized dynamic system.

NUMERICAL SCHEMA FOR THE HAMILTON-JACOBY EQUATION

$\boldsymbol{\Omega}$ is uniform mesh in the configuration space of the system R^m. The mesh steps over spatial variables are $h_\alpha > 0$, $\alpha = 1, 2, \ldots, m$ and:

$$\omega_h = \left\{\mathbf{X}^j = (x_{1_{j_1}}, x_{2_{j_2}}, \ldots, x_{m_{j_m}}),\ \ x_{\alpha_{j_\alpha}} = j_\alpha h_\alpha,\ j_\alpha = 0, \pm 1, \pm 2, \ldots;\ h_\alpha = 1/M_\alpha,\ \alpha = 1, 2, \ldots, m\right\}$$

where $h = (h_1, h_2, \ldots, h_m)$, $j = (j_1, j_2, \ldots, j_m)$ and M_α are positive integers.

Let us introduce uniform mesh also on the time interval $[0, T]$ with step $\delta > 0$

$$\overline{\omega}_\delta = \{t_n = n\delta,\ t_0 = 0,\ t_{N_\sigma} = T,\ n = 0, 1, \ldots, N_\sigma\}$$

which coincides with partitioning σ.

Let us denote mesh function defined in mesh nodes $\omega_{h\delta}$ as $\overline{V}^n_{j_1, j_2, \ldots, j_m}$, where:

$$\omega_{h\delta} = \omega_h \times \overline{\omega}_\delta = \left\{(\mathbf{X}^j, t_n) |\ \mathbf{X}^j \in \overline{\omega}_\delta\right\}.$$

Change of the parameter h leads to mesh sequence $\{\omega_h\}$ which exhausts countable everywhere dense set in R^m. Let us denote this set as $X = \{\omega_h\}$.

The Hamilton-Jacoby or Bellman-Isaacs equation may be written for the value function $\overline{\mathbf{V}}(\cdot)$:

$$\frac{\partial \overline{\mathbf{V}}}{\partial \tau} = \min_{\{\mathbf{u}\}} \max_{\{\mathbf{v}\}} \left[\sum_{i=1}^{m} \frac{\partial \overline{\mathbf{V}}}{\partial x_i} \cdot f_i(\mathbf{X}, \mathbf{u}, \mathbf{v})\right] \tag{5}$$

with initial condition

$$\overline{\mathbf{V}}(\mathbf{X}, \tau)\big|_{\tau=0} = \mathbf{H}(\mathbf{X}(T)), \tag{6}$$

where $\tau = T - t, \tau \in [0, T]$.

Let us associate with the problem (5)-(6) following finite difference scheme on the mesh $\omega_{h\delta}$:

$$\begin{aligned} \overline{V}^n_{j_1, j_2, \ldots, j_m} &= \overline{V}^{n-1}_{j_1, j_2, \ldots, j_m} + \delta \min_{\{\mathbf{u}\}} \max_{\{\mathbf{v}\}} \\ &\left[\frac{\overline{V}^{n-1}_{j_1+1, j_2, \ldots, j_m} - \overline{V}^{n-1}_{j_1, j_2, \ldots, j_m}}{h_1} \cdot f_1(\mathbf{X}^j, \mathbf{u}, \mathbf{v}) + \ldots \right. \\ &\left. \ldots + \frac{\overline{V}^{n-1}_{j_1, j_2, \ldots, j_m+1} - \overline{V}^{n-1}_{j_1, j_2, \ldots, j_m}}{h_m} \cdot f_m(\mathbf{X}^j, \mathbf{u}, \mathbf{v})\right], \\ & j_i \in Z,\ i = \overline{1, m};\ n = 1, \ldots, N_\sigma, \\ \overline{V}^0_{j_1, j_2, \ldots, j_m} &= H_{j_1, \ldots, j_m},\ j_i \in Z,\ i = \overline{1, m};\ n = 0. \end{aligned} \tag{7}$$

Properties of the scheme (7) and amenity of its application to solve some control problems are considered in [8]-[10].

COMPROMISE SOLUTION OF THE BEAM CONTROL PROBLEM

Let us consider a situation when it is impossible to make one optimality parameter the best one without making any other optimality parameters worse off. Such kind of optimization should be considered in cases mentioned in the Introduction when improvement of some characteristic of the beam leads to worsening of other characteristics. In this case compromise solution of the beam control problem should be found.

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Definition. *Compromise solution* is defined as follows. Let us $X = \{x\}$ is the set of all admissible solutions then

$$C_H = \Big\{x \in X |\max_i(M_i - H_i(x)) \leq \leq \max_i(M_i - H_i(x'))\forall x' \in X\Big\}$$

Here

$$M_i = \max_{x \in X} H_i(x)$$

$\mathbf{H} = (H_1, H_2, ..., H_n)$ is vector-function which describes the set of optimization criteria and $H_i : X \to R^1$, $\mathbf{H} : X \to R^n$. Compromise solution C_H of the optimization problem may be found as a result of the following algorithm.

1. Find "ideal vector" $\mathbf{M} = (M_1, M_2, ..., M_n)$.
2. Fix $x \in X$ and define corresponding discrepancy vector:

   $$\mathbf{\Delta}_k = (M_1 - H_1(x), M_2 - H_2(x), ..., M_n - H_n(x)).$$

3. Rearrange vector's components on increasing:

   $$\mathbf{\Delta}'_k = (M'_1 - H'_1(x), M'_2 - H'_2(x), ..., M'_n - H'_n(x)).$$

4. Choose $x \in X$ which leads to

   $$C^I_H = arg \min_{x \in C^{I-1}_H} \max_{i \in I}\{M'_i - H_i(x)'\}$$

   where $I = \{1, 2, ..., n\}$.
5. If the set of compromise solutions includes few elements then steps should be repeated for the next beam characteristic and so on.

Usually the resulting control is the only solution of the optimization problem.

CONCLUSION

In the article problem of finding of the beam optimal parameters choice is formulated as control problem. It makes possible to use mathematical and computing techniques of dynamic programming. Computational method for the compromise solution is formulated. Some issues relating to mathematical aspects of conflict control are discussed in [11]-[14]. High-performance computing may be used to reduce computing time [15].

REFERENCES

[1] D.A. Ovsyannikov, N.V. Egorov, Mathematical Modelling of. Electron and Ion Beams Forming Systems, St.-Petersburg University. Press, 1998.

[2] E.M.Vinogradova,N.V.Egorov, Simulation of an electron gun with a field-effect cathode, Journal of Communications Technology and Electronics, v. 49, N 2, 2004, P.232.

[3] G.R.Brewer, Focusing of High-density electron beams in Focusing of Charged Particles, Ed. by A.Septier, v.11, 1967, p.23

[4] O.A.Kalatusha, O.V.Ruban, S.A.Nemnyugin, Computer simulation of radiation dose absorption in biological specimens, Mathematical Modelling and Geometry, v.4, N 1, 2016, P.41-50.

[5] S.P.Merts, S.V.Razin, O.V.Rogachevskii, Accumulation of space charge in the time-projection chamber of MPD detector, Pisma v zhurnal "Physika elementarnyh tchastic i atomnogo yadra", v.10, N 1, 2013, P.113 (in Russian).

[6] G. V. Alferov, O. A. Malafeyev, S. A. Nemnyugin. Charged Particles Beam Focusing with Uncontrollable Changing Parameters // 2014 2nd International Conference on Emission Electronics, ICEE 2014 Joined with 10th International Vacuum Electron Sources Conference, IVESC 2014, International Conference on Computer Technologies in Physical and Engineering Applications, ICCTPEA 2014, 20th International Workshop on Beam Dynamics and Optimization, BDO 2014 - Proceedings, 2014. — P. 25-27.

[7] O.A.Malafeyev. Situation of equilibrium in dynamic games, Kibernetika, N 3, 1974, P.111. (in Russian)

[8] O.A.Malafeyev, M.S.Troeva (Davydova), On the weak solution of the basic equation of differential game and its approximation. Voprosy mehaniki i processov upravleniya. 1991. N 14. P. 42 (in Russian).

[9] O.A.Malafeyev, Control in conflict dynamic systems. - Saint-Petersburg: Izdatelstvo SPbGU, 1993 (in Russian).

[10] O.A.Malafeyev, Stability of solutions of many-criterial optimization problems and conflict control processes. - Leningrad: Izdatelstvo LGU, 1990 (in Russian).

[11] V.N.Kolokoltsov, O.A.Malafeyev, Understanding Game Theory, New Jersey, London,World Scientific Publishing Company 2010.

[12] O.A.Malafeyev, A.J.Radchenko, A.F.Zubova, Idempotent analysis and many-agent interaction in sybchronization problems and production systems. Linear and non-linear systems dynamics, V.25, N 2, Works of ISI RAS, P.233.

[13] V.N.Kolokoltsov, O.A.Malafeyev, Dynamic competitive systems of many-agent interaction and their asymptotics, part 1, Vestnik of civil engineers, V.5., 2010, P.144.

[14] V.N.Kolokoltsov, O.A.Malafeyev, Dynamic competitive systems of many-agent interaction and their asymptotics, part 2, Vestnik of civil engineers, V.5, 2010, P.133.

[15] A.E.Basalaev, A.S.Bondarenko, S.P.Merts, S.A.Nemnyugin, A.N.Timofeev, Application of high-performance computing technologies in problems of high-energy physics for NICA/MPD project. Vestnik Yuzhno-Uralskogo Gosudarstvennogo Universiteta, v.2, N 1, 2013, p.17 (in Russian).

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