COUPLED BUNCH INSTABILITIES IN THE STORAGE RINGS*

A.S. Smygacheva#, V.N. Korchuganov, Y.A. Fomin
NRC «Kurchatov Institute», Moscow, Russia

Abstract

Coherent instabilities of the bunched beam are one of the reasons that limit a total beam current in the storage rings. Although there are solutions of this problem, the estimation and reduction of the wake-fields influence on the longitudinal beam dynamics remain important things. In the article we return to the subject of coherent instabilities of the unevenly-filled bunches in the storage rings.

INTRODUCTION

The interaction of the bunched beam with its wake-fields in the vacuum chamber of the storage ring causes the coherent single-bunch and coupled-bunch instabilities. The growth of the coherent instabilities contributes to an increase of the longitudinal and transverse emittances and the energy spread of the single bunch. Also it leads to the partial losses and to complete losses of the bunch particles in some cases. The result of this process is the limitation of the maximum synchrotron radiation brightness of a facility.

Since most of the modern storage rings operate in a multi-bunch mode, a primary task is to cure the coupled-bunch instabilities. To dump the coherent oscillations of a bunch sequence the feedback systems are used [1]. But it's not a single way to solve the problem. To increase the instability threshold and the total beam current in the storage ring, it requires the reducing the wake-fields influence. In view of this fact the RF cavities with the HOM dumping or with a good HOM frequency control and stabilization, the smoothing of a vacuum chamber structure and the using the harmonic RF cavities for Landau damping have place at the accelerators [2].

The review of bunched beam coherent instabilities can be found in [3, 4, 5, 6, 7]. In most cases authors considered the interaction of the symmetrically disposed point charge bunches with wake-fields. Whereas the operation with the non-symmetrical beam and unevenly-filled bunches allows to increase the instability threshold and the total beam current. The attempts to determine the wake-field contribution to the longitudinal dynamics and the bunch sequence have led to the development of the several calculation schemes [8, 9, 10]. But these are special cases of a symmetrically-filled ring, and in some of them estimation results not always agreed to the experiment data.

The coherent frequencies of the non-symmetrical bunched beam were found following a basic approach that uses a notion of the beam spectrum and an impedance function to describe the beam-chamber interaction. This analytical solution allows to estimate the influence of each field mode on the coherent oscillations of bunches with known mode parameters (the resonant frequency, the shunt impedance, the quality factor), the given bunch sequence and the Gauss distribution of particles in the phase plane.

COUPLED BUNCH INSTABILITIES

The longitudinal dynamics of the bunched beam under the influence of the external RF fields and its own wake-fields is presented in this article.

The M electron bunches circulate in the accelerator with an angular revolution frequency ω_o. Bunches fill the orbit in the arbitrary order. Maximum number of bunches corresponds with the separatrix number of the ring, which is equal to the ratio of the RF frequency to the revolution frequency.

The appearance of the coherent oscillations adds to stationary distribution the components of the density perturbation. Then the electron distribution function can be written as:

$$\begin{cases} \Psi(\hat{\tau},\varphi,t) = \Psi_o(\hat{\tau}) + \sum_{m\neq 0}\Psi_m(\hat{\tau})e^{jm\varphi}e^{j\omega_{sm}t} \\ \tau = \hat{\tau}\cos\varphi\,,\ \frac{\dot{\tau}}{\omega_s} = \hat{\tau}\sin\varphi \\ \varphi = \omega_s t + \varphi_o \end{cases}, \quad (1)$$

where $\Psi_o(\hat{\tau})$ – the stationary distribution function of particles in the bunch, $\Psi_m(\hat{\tau})$ – the amplitude of the density perturbation component for the *m*-mode of oscillations, ω_{sm} – the coherent angular frequency of the *m*-mode of oscillations, τ – the time deviation of the particle from the reference particle place, $\hat{\tau}$ и φ – the amplitude and phase of oscillations in polar coordinates, ω_s – the incoherent synchrotron frequency taking into account the potential well distortion effect [11], φ_o – the initial phase of oscillations.

The longitudinal dynamics of electrons in the *k*-bunch is described by a synchrotron motion equation of the single particle and the Vlasov equation for the distribution function of particles in the bunch [3]. For small oscillations the linearized equations are:

$$\ddot{\tau} + \omega_{so}^2\tau = -2\pi\frac{e\eta}{\beta^2 T_o E}\sum_{p,m,i} j^{-m}\cdot I_b^i\cdot Z(p\omega_o+\omega_{sm})\cdot F_{pm}^i\cdot e^{-jp\omega_o(i-k)\frac{T_o}{h}}\cdot e^{jp\omega_o\tau}\cdot e^{j\omega_{sm}t}, \quad (2)$$

$$\left(\frac{\partial}{\partial t} - \omega_s\frac{\partial}{\partial\varphi}\right)\sum_m \Psi_m^k(\hat{\tau})e^{jm\varphi}e^{j\omega_{sm}t} = -(\ddot{\tau}+\omega_s^2\tau)\cdot\frac{\sin\varphi}{\omega_s}\cdot\frac{\partial\Psi_o^k(\hat{\tau})}{\partial\hat{\tau}}, \quad (3)$$

where

*The reported study was funded by RFBR according to the research project No. 16-32-00335 мол_а.
#sasmyga@mail.ru

ISBN 978-3-95450-181-6

$$F_{pm}^{i} = \int_0^{+\infty} \hat{\tau}\Psi_m^i(\hat{\tau}) J_m(p\omega_o\hat{\tau})d\hat{\tau},$$

p,m=0,±1,±2,±3…, i=0,1,2..h-1, $h = \frac{\omega_{rf}}{\omega_o}$ – the RF harmonic number, ω_{so} – the incoherent synchrotron frequency, $\beta = {}^{v}/_{c}$ – the relativistic velocity factor, T_o – the revolution time period, $\eta = \alpha - 1/\gamma^2$, α – the momentum compaction factor, γ – the relativistic factor, E – the particle energy, $I_b = \frac{Ne}{T_o}$ – the bunch current, N – the number of electrons in the bunch, e – the elementary charge, $Z(\omega)$ – the longitudinal impedance function, $J_m(p\omega_o\hat{\tau})$ – the first order Bessel function.

The main cause of the rise of the longitudinal bunched beam instabilities is the long-range electromagnetic fields excited by bunches in the resonant structures of the vacuum chamber. As a rule these structures are the accelerating RF cavities that have them own resonant frequency spectrum, given by the cavity design.

The impedance function $Z(\omega)$ of the structures can be represented as a sum of resonant impedances of the RLC circuits with the resonant frequency ω_r, shunt impedance $R_{sh.r}$ and quality factor Q_r. In addition, if the resonant structures of the ring separate from each other as far as the electromagnetic relation is not existed, that the impedances are additive. The impedance of the structure at the beam current harmonic with coherent frequency is:

$$Z(p\omega_o + \omega_{sm}) = \sum_r -j\frac{R_{sh.r}\cdot h_r}{Q_r}\frac{p+\nu_{sm}}{(p-p_{1.r})(p-p_{2.r})}, \quad (4)$$

where $p_{1,2.r} = -\nu_{sm} + j\frac{h_r}{2Q_r} \pm \frac{h_r}{2Q_r}\sqrt{4Q_r^2 - 1}$; $h_r = \frac{\omega_r}{\omega_o}$; $\nu_{sm} = \frac{\omega_{sm}}{\omega_o}$, for small bunch current $\nu_{sm} \approx m\nu_s = \frac{m\omega_s}{\omega_o}$, the sum over r is the sum over the impedance frequency spectrum.

Rewrite the equation (3) for components of the distribution function with the view of the coherent mode coupling is absent:

$$j(\omega_{sm} - m\omega_s)\cdot\Psi_m^k(\hat{\tau}) = -\frac{e\eta}{\beta^2 E}\frac{m}{\omega_s\hat{\tau}}\frac{\partial\Psi_o^k(\hat{\tau})}{\partial\hat{\tau}}\cdot\sum_{p,i} I_b^i\cdot F_{pm}^i\cdot \frac{Z(p\omega_o+\omega_{sm})}{p}\cdot e^{-jp\frac{2\pi}{h}(i-k)}\cdot J_m(p\omega_o\hat{\tau}). \quad (5)$$

Multiply both parts of (5) by $\left(\frac{\hat{\tau}}{T_o}\right)^m$ and integrate respect to $\hat{\tau}$:

$$j(\omega_{sm} - m\omega_s) = -\frac{e\eta}{\beta^2 E}\cdot\frac{m}{\omega_s}\cdot\sum_{p,i}\frac{Z(p\omega_o+\omega_{sm})}{p}\cdot I_b^i\cdot e^{-jp\frac{2\pi}{h}(i-k)}\cdot\frac{F_{pm}^i\cdot G_{pm}^k}{F_m^k}, \quad (6)$$

where

$$G_{pm}^k = \int_0^{+\infty}\frac{\partial\Psi_o^k(\hat{\tau})}{\partial\hat{\tau}}\left(\frac{\hat{\tau}}{T_o}\right)^m J_m(p\omega_o\hat{\tau})d\hat{\tau}.$$

$$F_m^k = \int_0^{+\infty}\hat{\tau}\Psi_m^k(\hat{\tau})\left(\frac{\hat{\tau}}{T_o}\right)^m d\hat{\tau}$$

The first order Bessel function is

$$J_m(p\omega_o\hat{\tau}) = \sum_{n=0}^{+\infty}\frac{(-1)^n}{n!\cdot\Gamma(m+n+1)}\left(\frac{p\omega_o\hat{\tau}}{2}\right)^{2n+m}.$$

The most typical particle distribution for electron bunched beams is a Gaussian distribution. So the stationary distribution of electrons in the k-bunch is

$$\Psi_o^k(\hat{\tau}) = \frac{1}{2\pi\sigma_{\tau k}^2}e^{-\frac{\hat{\tau}^2}{2\sigma_{\tau k}^2}}, \quad (7)$$

$\sigma_{\tau k}$ – the RMS bunch length. The initial bunch length is set by the potential well distortion effect, the microwave instability, the Touschek effect, the quantum fluctuations and the radiation damping in the storage rings. The first free effects dominate at the low energies, and so the bunch length depends on the bunch current. The quantum fluctuations and the radiation damping determine the bunch length at the high energies. Therefore we can suppose the bunch length is independent of the current at the high energies.

We have for the Gaussian distribution

$$G_{pm}^k = -\frac{\sigma_{\tau k}^{2m-2}\cdot(2\pi p)^m\cdot e^{-\frac{(p\omega_o\sigma_{\tau k})^2}{2}}}{2\pi\cdot T_o^{2m}},$$

and

$$j(\omega_{sm} - m\omega_s) = \frac{e\eta}{2\pi\beta^2 E}\cdot\frac{m}{\omega_s}\cdot\sum_{p,i,n}\frac{(-1)^n\cdot T_o^{2n}}{n!\cdot\Gamma(n+m+1)\cdot 2^n\cdot\sigma_{\tau k}^{2n+2}}\cdot\frac{F_{nm}^i}{F_m^k}\cdot I_b^i\cdot\frac{Z(p\omega_o+\omega_{sm})}{p}\cdot\left(\frac{p\omega_o\sigma_{\tau k}}{\sqrt{2}}\right)^{2m+2n}\cdot e^{-\frac{(p\omega_o\sigma_{\tau k})^2}{2}}\cdot e^{-jp\frac{2\pi}{h}(i-k)},$$

$$F_{nm}^i = \int_0^{+\infty}\hat{\tau}\Psi_m^i(\hat{\tau})\left(\frac{\hat{\tau}}{T_o}\right)^{2n+m}d\hat{\tau}.$$

The substitution of the infinite upper limit in the integrals has place due to the density perturbation components are bounded in the phase space.

Using the method presented by B. Zotter in [12], we can find the effective impedance of the resonant structure that surrounds the beam. The final equation for the coherent frequencies of coupled bunches having the Gaussian distribution of particles is:

$$(\omega_{sm} - m\omega_s) = -\frac{e\eta}{2\pi\beta^2 E}\cdot\frac{m}{\omega_s}\cdot\sum_{r,n}\frac{(-1)^n\cdot T_o^{2n}}{n!\cdot\Gamma(n+m+1)\cdot 2^n\cdot\sigma_{\tau k}^{2n+2}}\cdot\frac{R_{sh.r}\cdot h_r}{Q_r\cdot(p_{2.r}-p_{1.r})}\cdot\left(I_b^k\cdot\frac{F_{nm}^k}{F_m^k}\cdot[S_{02} - S_{01}] + \sum_{i\neq k} I_b^i\cdot\frac{F_{nm}^i}{F_m^k}\cdot[S_{\alpha 2} - S_{\alpha 1}]\right), \quad (8)$$

where

ISBN 978-3-95450-181-6

$$S_{01,2} = \left(\left(x_{1,2}\right)^{2(m+n)} \cdot R_{01,2} + \sum_{l=0}^{m+n-1} \Gamma\left(l+\frac{1}{2}\right) \cdot \left(x_{1,2}\right)^{2(m+n)-2l-1}\right) + \nu_{sm} \frac{\omega_o \sigma_{\tau k}}{\sqrt{2}} \cdot \left(\left(x_{1,2}\right)^{2(m+n)} \cdot R_{01,2} + \sum_{l=0}^{n+m-1} \Gamma\left(l+\frac{1}{2}\right) \cdot \left(x_{1,2}\right)^{2(m+n)-2l-2}\right),$$

$$S_{\alpha 1,2} = \left(\left(x_{1,2}\right)^{2(m+n)} \cdot R_{\alpha 1,2} + \sum_{l=0}^{n+m-1} \Gamma\left(l+\frac{1}{2}\right) \cdot \left(x_{1,2}\right)^{2(m+n)-2l-1}\right) + \nu_{sm} \frac{\omega_o \sigma_{\tau k}}{\sqrt{2}} \cdot \left(\left(x_{1,2}\right)^{2(m+n)} \cdot R_{\alpha 1,2} + \sum_{l=0}^{n+m-1} \Gamma\left(l+\frac{1}{2}\right) \cdot \left(x_{1,2}\right)^{2(m+n)-2l-2}\right),$$

$$R_{01,2} = -\pi e^{-(x_{1,2})^2} \cot \pi p_{1,2.r} + j\pi\left(w(x_{1,2}) - e^{-(x_{1,2})^2}\right),$$

$$R_{\alpha 1,2} = -j2\pi e^{-(x_{1,2})^2} \frac{e^{-jp_{1,2.r}2\pi\frac{i-k}{h}}}{1-e^{-j2\pi p_{1,2.r}}} + j\pi\left(w(x_{1,2}) - e^{-(x_{1,2})^2}\right),$$

$$x_{1,2} = \frac{p_{1,2.r}\omega_o \sigma_{\tau k}}{\sqrt{2}},$$

$w(x) = e^{-x^2}\left(1 + \frac{2j}{\sqrt{\pi}}\int_0^x e^{t^2} dt\right)$ – a complex error function.

The ratio $\frac{F_{nm}^i}{F_m^k}$ is unknown in the (8). Let us assume the amplitude of particle oscillations is small as compared with the wave-length of the electromagnetic wake-field ($\frac{p\omega_o \hat{\tau}}{2} \ll 1$) to define the ratio. That in (5)

$$J_m(p\omega_o\hat{\tau}) \approx \frac{1}{\Gamma(m+1)} \cdot \left(\frac{p\omega_o\hat{\tau}}{2}\right)^m.$$

Multiplied both parts equation (5) by $\left(\frac{\hat{\tau}}{T_o}\right)^m$ and integrated respect to $\hat{\tau}$, we get

$$j(\omega_{sm} - m\omega_s) = -\frac{e\eta}{\beta^2 E} \cdot \frac{1}{\omega_s} \cdot \frac{F_{om}^k}{F_m^k} \cdot \sum_{p,i} \frac{m\cdot(\pi p)^{2m}}{p\cdot\Gamma^2(m+1)} \cdot I_b^i \cdot F_m^i \cdot Z(p\omega_o + \omega_{sm}) \cdot e^{-jp\frac{2\pi}{h}(i-k)}, \quad (9)$$

where

$$F_{om}^k = \int_0^{+\infty} \frac{\partial \Psi_0^k(\hat{\tau})}{\partial \hat{\tau}} \left(\frac{\hat{\tau}}{T_o}\right)^{2m} d\hat{\tau} = -\frac{2^m \cdot \Gamma(m+1)\cdot \sigma_{\tau k}^{2m-2}}{2\pi \cdot T_o^{2m}}.$$

Put equation (9) in (5) and find the approximate solution for functions $\Psi_m^k(\hat{\tau})$ for $\frac{p\omega_o\hat{\tau}}{2} \ll 1$:

$$\Psi_m^k(\hat{\tau}) = \frac{1}{\hat{\tau}} \cdot \frac{\partial \Psi_0^k(\hat{\tau})}{\partial \hat{\tau}} \cdot \left(\frac{\hat{\tau}}{T_o}\right)^m \cdot \frac{F_m^k}{F_{om}^k}. \quad (10)$$

Put (10) in the ratio $\frac{F_{nm}^i}{F_m^k}$, we can rewrite equation (8)

$$(\omega_{sm} - m\omega_s) = -\frac{e\eta}{2\pi\beta^2 E} \cdot \frac{m}{\omega_s} \cdot \sum_{r,n} \frac{(-1)^n}{n!\cdot\Gamma(m+1)\cdot\sigma_{\tau k}^{2n+2}} \cdot \frac{R_{sh.r}\cdot h_r}{Q_r\cdot(p_{2.r}-p_{1.r})} \cdot \left(I_b^k \cdot \sigma_{\tau k}^{2n} \cdot [S_{02} - S_{01}] + \sum_{i\neq k} I_b^i \cdot \frac{F_m^i}{F_m^k} \cdot \sigma_{\tau i}^{2n} \cdot [S_{\alpha 2} - S_{\alpha 1}]\right), \quad (11)$$

The ratio $\frac{F_m^i}{F_m^k}$ is unknown in the (11) also. Suppose here that each bunch has the same distribution function and the RMS bunch length is independent of the self-bunch current. Hence, the RMS bunch lengths are equal each other and the ratio $\frac{F_m^i}{F_m^k}$ is equal 1 for bunches oscillating in the same potential well, formed by the external RF field and wake-fields. As a result, the coherent frequencies are

$$(\omega_{sm} - m\omega_s) = -\frac{e\eta}{2\pi\beta^2 E} \cdot \frac{m}{\omega_s} \cdot \sum_{r,n} \frac{(-1)^n}{n!\cdot\Gamma(m+1)\cdot\sigma_{\tau k}^2} \cdot \frac{R_{sh.r}\cdot h_r}{Q_r\cdot(p_{2.r}-p_{1.r})} \cdot \left(I_b^k \cdot [S_{02} - S_{01}] + \sum_{i\neq k} I_b^i \cdot [S_{\alpha 2} - S_{\alpha 1}]\right). \quad (12)$$

CONCLUSIONS

The equation (12) is the approximate solution of the coherent frequencies. But it already allows to estimate the wake-fields influence and the coherent oscillations stability of the bunched beam. Also the solution for single-bunch instabilities can be found taking just term for *i*=*k* in the equation (12).

REFERENCES

[1] M. Lonza, “Multi-bunch feedback systems”, CERN-2008-003, 2008, p. 285
[2] A. Mosnier, “Cures of coupled bunch instabilities”, Proc. of the 1999 Part. Acc. Conf., New York, pp. 628-632
[3] F. Sacherer, “Methods for computing bunched-beam instabilities”, CERN/SI-BR/72-5, 1972
[4] J.M. Wang, “Longitudinal symmetric coupled bunch modes, Brookhaven National Laboratory”, Report BNL 51302, 1980
[5] B. Zotter, “Longitudinal stability of bunched beams. Part III: Mode coupling and the microwave instability”, CERN SPS/81-20 (DI), 1981
[6] J.L. Laclare, “Bunched beam coherent instabilities”, CERN 87-03, p. 264, 1987
[7] A. Wu Chao, *Physics of Collective beam instabilities in high energy accelerators*, John Wiley&Sons Inc., 1993
[8] J. Scott Berg, “Bounds on multibunch growth rates when the bunches currents are not identical”, CERN SL Note 97-72 (AP), 1997
[9] R.D. Kohaup, “On multi-bunch instabilities for fractionally filled rings”, DESY 85-139, 1985
[10] S. Prabhakar, “New diagnostics and cures for coupled-bunch instabilities”, Proc. of the 2001 Part. Acc. Conf., Chicago, pp. 300-304
[11] B. Zotter, “Potential-well bunch lengthening”, CERN SPS/81-14 (DI), 1981
[12] B. Zotter, “The effective coupling impedance for instabilities of Gaussian bunches”, CERN/ISR-TH/80-03, 1980

ISBN 978-3-95450-181-6