THE BEHAVIOR OF POWERFUL RELATIVISTIC ELECTRON BEAM WITH ELLIPTICAL CROSS-SECTION IN LONGITUDINAL MAGNETIC FIELD

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Abstract

The behavior of relativistic intense electron beam with elliptical cross-section moving in a longitudinal magnetic field is investigated with the help of self-consistent model. The solutions for the beam envelopes are obtained in the case of the beam current differed from Alfven limit and the beam charge neutralized. The conditions of stationary beam propagation are determined, however it is discovered that for the case of non-zero self-consistent magnetic field the stationary beam propagation is violated, the partial emittance oscillations being observed. The found time- dependence of the partial emittances and the beam envelopes illustrates the effect of emittance transfer caused by the coupled particle motion in magnetic field.

INTRODUCTION

In [1,2,3] studied the behavior of the electron beam in a quadrupole system, the aim of this study was the possibility of compression - reducing the area of the cross-section of the beam when changing quadrupole forces. If the [1] to the transverse emittance were considered equal, in [2], these values are considered to be different, but continuing when the beam moves. In this paper we study the distribution of the electron beam decompensated with an elliptical cross-section in the absence of external quadrupole system and in the presence of a longitudinal magnetic field. The presence of the longitudinal magnetic field greatly complicates the situation - there are not two independent integrals of motion. Emittance can be converted (pumped). It is not a conserved quantity as the product of these values.

EQUATIONS

Consider a beam whose charge is compensated by the secondary particles. Lateral movement can be separated from the lengthwise when the current satisfies the following condition:

$J << J_A$, where $J_A = mc^3\gamma_0\beta_0/e$. The distribution function can be written as: $F = \delta(\beta_z - \beta_0) f(\vec{r}_\perp, z, \vec{v}_\perp)$. In this paraxial approximation the longitudinal velocity of the particles can be considered constant and equal for all particles. The stationary problem instead of the time you can use a coordinate z. In accordance with the invariant that defines the movement, should depend on $x(z), y(z), x' = \frac{dx}{dz}, y' = \frac{dy}{dz}, z$. We derive the equations of motion of particles in the laboratory frame. Consider that in the system connected to the main beam axes(x_1, y_1) have their own self-compression force directed to the beam axis: $F_{x_1} = -\frac{2ix_1}{R_x(R_x+R_y)}, F_{y_1} = -\frac{2iy_1}{R_y(R_x+R_y)}$. Here $i = J/J_A$ beam current related to Alfven,, $R_x(z), R_y(z)$ - the value of the semi-axes of the elliptic beam cross section). Calculating further, $F_x = F_{x_1}\cos\theta - F_{y_1}\sin\theta, F_y = F_{x_1}\sin\theta + F_{y_1}\cos\theta$, which should be considered $x_1 = x\cos\theta + y\sin\theta, y_1 = -x\sin\theta + y\cos\theta$, and $\theta(z)$ - angle of rotation of the principal axes of the ellipse relative to fixed axes, the equation can be obtained:

$$x'' = \omega_H y' - \alpha(z)x + \beta(z)y,\ y'' = -\omega_H x' + \beta(z)x - \gamma(z)y, \quad (1)$$

where

$$\alpha = \frac{i}{R_x R_y}\big(1 - \frac{R_x - R_y}{R_x + R_y}\cos 2\theta\big)$$

$$\beta = \frac{-i}{R_x R_y}\frac{R_x - R_y}{R_x + R_y}\sin 2\theta$$

$$\gamma == \frac{i}{R_x R_y}\big(1 + \frac{R_x - R_y}{R_x + R_y}\cos 2\theta\big)$$

In equations (1) are also taken into account the presence of an external longitudinal magnetic field, and $\omega_H = \frac{eH}{mc^2\gamma_0\beta_0}$, the dimension of this magnitude - the inverse length.Invariant system (1) can be represented as:

$$I = A_1(z)x'^2 + 2A_2(z)x'x + A_3(z)x^2 + B_1(z)y'^2 + 2B_2(z)y'y + B_3(z)y^2 + C_1(z)x'y' + C_2(z)x'y + C_3(z)xy' + C_4(z)xy \quad (2)$$

From condition $\frac{dI}{dz} \equiv 0$ using (1) we obtain:

$$\begin{aligned} A_1' &= -2A_2 + \omega_H C_1,\\ A_2' &= -A_3 + A_1\alpha(z) + 0.5\omega_H C_3 - 0.5C_1\beta(z),\\ A_3' &= 2A_2\alpha(z) - C_3\beta(z),\\ B_1' &= -2B_2 - \omega_H C_1\\ B_2' &= -B_3 + B_1\gamma(z) - 0.5\omega_H C_2 - 0.5C_1\beta(z),\\ B_3' &= 2B_2\gamma(z) - C_2\beta(z),\\ C_1' &= -C_2 - C_3 + 2\omega_H(B_1 - A_1),\\ C_2' &= -C_4 + C_1\gamma(z) - 2A_1\beta(z) + 2\omega_H B_2,\\ C_3' &= -C_4 + C_1\alpha(z) - 2A_2\omega_H - 2B_1\beta(z),\\ C_4' &= C_2\alpha(z) + C_3\gamma(z) - 2B_2\beta(z) - 2A_2\beta(z). \end{aligned} \quad (3)$$

Converting, furter, I. Instead x', y' introduce variables ξ, η.

$$x' = \xi\cos\alpha + \eta\sin\alpha, \quad y' = -\xi\sin\alpha + \eta\cos\alpha \quad (4)$$

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Suppose that $\tan(2\alpha) = C_1/(B_1 - A_1)$. If then assume $\xi_1 = \xi + (x\cos\alpha(A_2 + C_2/2) - y\sin\alpha(B_2 + C_3/2))/P, \eta_1 = \eta + (x\sin\alpha(A_2 + C_2/2) + y\cos\alpha(B_2 + C_3/2))/Q$, one should obtain:

$$I = P\xi_1^2 + Q\eta_1^2 + Ax^2 + By^2 + Cxy \tag{5}$$

where

$$P = A_1 + B_1 - \sqrt{(A_1 - B_1)^2 + C_1^2},$$
$$Q = A_1 + B_1 - \sqrt{(A_1 - B_1)^2 + C_1^2},$$
$$A = A_3 + K/\Lambda, B = B_3 + L/\Lambda,$$
$$C = C_4 + M/\Lambda, \Lambda = A_1B_1 - C_1^2/4,$$
$$K = A_2^2B_1 - 0.5A_2C_1C_3 + 0.25A_1C_3^2,$$
$$L = A_1B_2^2 - 0.5B_2C_1C_2 + 0.25B_1C_2^2,$$
$$M = A_1B_2C_3 + A_2B_1C_2 - A_2B_2C_1 - 0.25C_1C_2C_3. \tag{6}$$

If we assume that the distribution function is: $f = \delta(I - 1)$, the integral should be calculated for the density of the particles:

$$\int d\xi_1 d\eta_1 \delta\big(P\xi_1^2 + Q\eta_1^2 - Ax^2 - By^2 - Cxy - 1\big) =$$
$$\frac{\pi}{\sqrt{PQ}}\sigma\big(1 - Ax^2 - By^2 - Cxy\big),$$

where $\sigma(x)$ Heaviside function, $\sigma(x) = 1, x > 0, \sigma(x) = 0, x < 0$.To bring to the principal axes of the bunch should take advantage of the relations, expressing x_1, y_1 wia x, y. Supposing $\tan 2\theta = \frac{C}{B-A}$, we obtain

$$Ax^2 + By^2 + Cxy = \frac{x_1^2}{R_x^2} + \frac{y_1^2}{R_y^2},$$
$$R_x^2 = \frac{2}{A + B - \sqrt{(A-B)^2 + C^2}},$$
$$R_y^2 = \frac{2}{A + B + \sqrt{(A-B)^2 + C^2}}.$$

There remains the phase volume: $V = \Lambda(AB - C^2/4) \equiv const = \Lambda(A_3B_3 - C_4^2/4) - B_3K - A_3L + (C_4/2)M + (A_2B_2 - C_2C_3/4)^2$.Important characteristic beam are rms values. You can express the mean values through coefficient quadratic form I_{xy}. For example,

$$\overline{x'^2} = \frac{B_1}{2\Lambda}, \overline{x^2} = \frac{B}{2(AB - C^2/4)},$$
$$\overline{y'^2} = \frac{A_1}{2\Lambda}, \overline{y^2} = \frac{A}{2(AB - C^2/4)}. \tag{7}$$

Determing values:

$$E_x(z) = B_1(z)B(z)/$$
$$V, E_y(z) = A_1(z)A(z)/V,$$

beam emittance are in x and y direction. To solve the system (3) to set the initial conditions - the values of functions A_i, B_j, C_k if $z = 0$.

SOLUTION OF THE EQUATIONS

To find out the conditions under which the conditions of possible solutions for the stationary beam envelopes. Put $\omega_H = 0$ and all $C_j \equiv 0$. So invariant I mai be represented as sum of $I = I_1 + I_2$, where $I_1 = A_1x'^2 + 2A_2x'x + A_3x^2, I_2 = B_1y'^2 + 2B_2y'y + B_3y^2$. Constant solutions for R_x, R_y may be obtained if $A_2 \equiv B_2 \equiv 0$. Except initial conditions $A_2(0) = 0, B_2(0) = 0$ must be derivatives vanish at the starting point. From (3) follows: $A_3(0) = A_1(0)\alpha(0)$, $B_3(0) = B_1(0)\gamma(0)$. One can obtain:

$$A_1 = \frac{A_3}{2i\sqrt{B_3}}(1/\sqrt{B_3} + 1/\sqrt{A_3}),$$
$$B_1 = \frac{B_3}{2i\sqrt{A_3}}(1/\sqrt{B_3} + 1/\sqrt{A_3}).$$

Putting in this equalities $A_3(0) = 0.9, B_3(0) = 0.1$. Then, in the case of a force field in the solution of system (3) can be constant values$R_x \equiv \sqrt{10}, R_y \equiv \sqrt{1/0.9}, B_1(0) = 2/9i, A_1(0) = 6/i$. This is a very characteristic change of solutions when defining a small field($\omega_H = 0.003$), as shown in Figure 1.

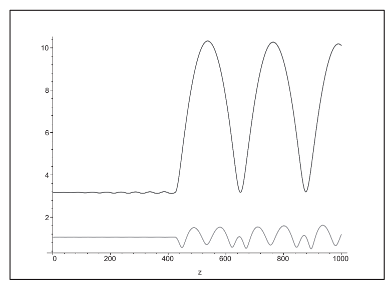

Figure 1: The dependence of the beam size $R_x(z), R_y(z)$ from coordinate.

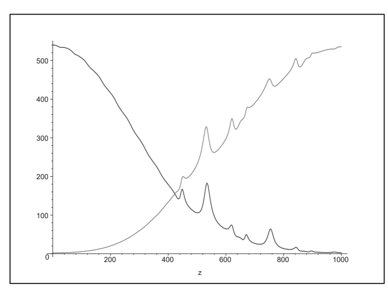

Figure 2: The dependence of emittance $E_x(z), E_y(z)$ from coordinate.

From Fig.1. It shows that the beam dimensions remain substantially constant up to a certain point (i.e. range of relatively small amplitude) after reaching these sizes vary at a constant frequency and a substantially greater amplitude. From Fig.2. You can see what is happening pumping emittance, and it was after reaching a point where the emittance compared begin intense oscillations of the transverse

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dimensions of the beam. The above initial conditions under which the $A_1(0) \neq B_1(0)$ does not correspond to the conventional cathode elliptical shape whose $R_x \neq R_y$, and the spread of the initial transverse velocities are the same in the directions (ie, must be made equal $A_1(0) = B_1(0)$). Here are the results of a solution of (3) in the case of an isotropic distribution of initial velocities, assuming that $A_1(0) = B_1(0) = 25$.

In Fig. 3 dependences are given sizes $R_x(z)$ and $R_y(z)$ at isotropic distribution of initial velocities, in Fig.4 dependences are given emittances $E_x(z)$ and $E_y(z)$ at isotropic distribution of initial velocities.

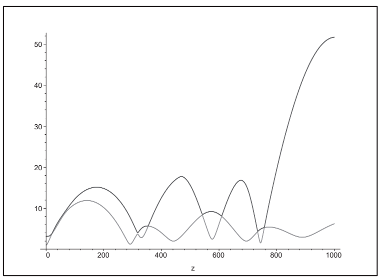

Figure 3: The dependence of the beam size $R_x(z), R_y(z)$ from the coordinates of an isotropic distribution of initial velocities.

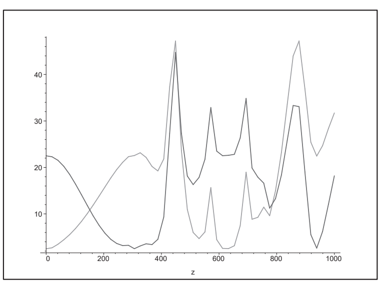

Figure 4: The dependence of emittance $E_x(z), E_y(z)$ from the coordinates of an isotropic distribution of initial velocities.

CONCLUSION

In this case, it should be noted quite chaotic dependence of the beam characteristics of the longitudinal coordinate. At the initial stage can be seen pumping emittance, and subsequently deprived of regular structure vibrations, the beam sizes also vary. Thus, in the work of the model Kapchinsky obtained system of equations that allows to study characteristics beam compensated charge with an elliptical cross-section in a constant longitudinal magnetic field. Obtained partial solutions, describing the state of the beam. It seems likely that the management performance characteristics of the beam should be used alternating field, which is supposed to study in the future.

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