ANALYSIS OF THE PARTICLE DYNAMICS STABILITY IN THE PENNING-MALMBERG-SURKO TRAP

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Abstract

Problem of stability of charged particle dynamics in the Penning-Malmberg-Surko trap is considered. It is shown that, magnetron motion is unstable for sufficiently small value of parameter a (which is the amplitude related parameter of the Rotating Wall (RW) electric dipole field). This contradicts the conclusion of the article [1] that there is a possibility of the compression of magnetron motions in the case of $|a| \rightarrow 0$. So it may indicate that the simplified model of the dynamics used by the author of the article is not accurately enough to describe the dynamics of the original system.

INTRODUCTION

Present report refers to the problem of the study of charged particle dynamics in the Penning-Malmberg-Surko trap. Various models of particle dynamics describing the magnetron and cyclotron motions are considered. The problems of the stability of the magnetron motion are investigated. In articles [1,2] the compression and expansion rates of the magnetron radius are discussed. Compression rates have been presented. The particle does not leave the area of the rotating field, and the bunch is compressed. These results are questionable.

PROBLEM STATEMENT

We consider a charged particle in the field of the potential

$$\Phi(z) = \frac{m}{q} \cdot \frac{\omega_z^2}{2} \cdot (z^2 - \frac{r^2}{2}) + \frac{m}{q} a \cdot z \cdot r \cdot \cos(\theta + \omega_r t), \quad (1)$$

and homogeneous longitudinal magnetic field $\vec{B} = \vec{e_z}B$. Here m and q are the mass and the charge of the particle, ω_z is the frequency of the particle longitudinal oscillations in the axially symmetric electric field of the trap electrodes, a and ω_r is amplitude related parameter and the frequency of the Rotating Wall (RW) electric dipole field asymmetric in the z-direction, z and r are the axial and radial coordinates with the axis coinciding with symmetry axis of the trap electrodes. The magnitude of the parameter a can be estimated as

$$a = \frac{q}{m} \cdot \frac{U_r}{2RL},\tag{2}$$

where U_r is the maximum of the potential difference between the segmented electrode plates, 2L is the length of the dipole RW field and R is the curvature radius of the cylindrical plates.

The charged particle motion in these fields is described by the following system of equations:

$$\ddot{x} = \frac{\omega_z^2}{2} \cdot x - a \cdot z \cdot \cos(\omega_r t) + \Omega_c \dot{y} - k\dot{x}, \qquad (3)$$

$$\ddot{y} = \frac{\omega_z^2}{2} \cdot y + a \cdot z \cdot \sin(\omega_r t) - \Omega_c \dot{x} - k \dot{y}, \qquad (4)$$

$$\ddot{z} = -\omega_z^2 \cdot z - k\dot{z} - a(x \cdot \cos(\omega_r t) - y \cdot \sin(\omega_r t)).$$
(5)

Here $\Omega_c = qB/m$ is the particle cyclotron frequency, the parameter k presents the friction force related to the particle scattering by the trap buffer gas molecules.

Further, we transform the system (3-5) to the complex form. Multiplying the equation (4) by imaginary unit *i* and adding it with equation (3) we come to the equation for the complex function

$$\xi(t) = x + iy, \tag{6}$$

$$\ddot{\xi} + i\Omega_c \dot{\xi} + k\dot{\xi} - \frac{\omega_z^2}{2} \cdot \xi = -a \cdot z \exp(-i\omega_r t).$$
(7)

The equation (7) together with the equation (5) describes the particle motion in the trap.

FREE PARTICLE MOTION IN THE TRAP

The general solution of the homogeneous differential equation in (7) is the following

$$\xi(t) = A_1 \cdot \exp(i\omega_+ t) + A_2 \cdot exp(i\omega_- t), \qquad (8)$$

where

$$\omega_{\pm} = -\frac{\Omega_c - ik}{2} \mp \frac{1}{2}\sqrt{(\Omega_c - ik)^2 - 2\omega_z^2}.$$
 (9)

The constants A_1 , A_2 should be found using the initial conditions.

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At the typical values of the trap parameters we have the relations between the frequencies:

$$|\omega_{+}| \sim \Omega_{c} \gg \omega_{z} \gg |\omega_{-}| \sim \omega_{m} \equiv \omega_{z}^{2}/(2\Omega_{c}) \gg k.$$
(10)

Here ω_m is so called magnetron frequency: the frequency of the particle rotation in the crossed fields of the trap - longitudinal magnetic field B and radial component of the electric field (1) at a = 0. The relations (10) allow us to find the approximate values of the frequencies ω_{-} (lower sign in (9)) and ω_+ (upper sign in (9)).

Rewrite the equation (9) as following:

$$\omega_{\mp} = -\frac{\Omega_c - ik}{2} \pm \frac{\Omega_c}{2} \sqrt{1 + \left(-\frac{2ik}{\omega_c} - \frac{k^2}{\Omega_c^2} - 2\frac{\omega_z^2}{\Omega_c^2}\right)}.$$
 (11)

One can show that

$$\omega_{+} \approx -(1-\alpha)\Omega_{c} + i(1+\alpha)k \approx -\Omega_{c} + ik, \quad (12)$$

$$\omega_{-} \approx -\alpha \Omega_{c} - i\alpha k \approx -\omega_{m} - i\frac{k\omega_{m}}{\Omega_{c}}, \qquad (13)$$
$$\alpha = \frac{\omega_{z}^{2}}{2\Omega_{c}^{2}}.$$

Then the solution (8) can be presented as following:

$$\xi(t) = A_1 \exp(-i\Omega_c t - kt) + A_2 \exp(-i\omega_m t + (14) + (k\omega_m/\Omega_c)t),$$

$$A_1 \approx 2\omega_m \xi_0 - \frac{\dot{\xi}_0}{\Omega_c}, \quad A_2 \approx \xi_0 - \frac{\dot{\xi}_0}{\Omega_c}.$$

The first term in (14) describes a free cyclotron rotation of the particle in the field \vec{B} damped with the decrement k. The second term describes the particle rotation around the trap axis with the angular frequency ω_m , i.e. the magnetron rotation. Its amplitude increases in time with the increment $k\omega_m/\Omega_c$ the magnetron orbit expansion related to the particle scattering by the buffer gas molecules (the parameter *k*).

PARTICLE MOTION STABILITY IN THE **TRAP WITH RW DIPOLE FIELD**

In case of time-dependent linear differential system to investigate a stability of system (3-5) one should consider the set of characteristic indices of the system (which are the same in stationary case as real parts of characteristic numbers of the system with a = 0). Obviously, if at least one of characteristic indices of the system is positive then it is unstable in the Lyapunov sense.

In view of the stability of the characteristic indices of linear system (3-5) with respect to small perturbations [3], the system (3-5) has a positive characteristic index for sufficiently small $a \neq 0$ because it has the characteristic number with a positive real part for a = 0 (as it was shown above) and should be unstable in the Lyapunov sense at least for

$$\mid a \mid < k \frac{\omega_m \omega_z}{\Omega_c \sqrt{2}}.$$
 (15)

It should be noted that the stability of the characteristic indices of the system (3-5) means their continuity respectively to a parameter a.

DECOUPLING THE MAGNETRON AND CYCLOTRON MOTIONS

The authors [1,2] perform the change of variables in the equations (3-5) defined by the equality

$$\boldsymbol{V}^{\pm} = \frac{dr}{dt} + \omega_{\mp} \hat{\boldsymbol{z}} \times \boldsymbol{r}, \tag{16}$$

where
$$\hat{z} = \begin{pmatrix} 0\\0\\1 \end{pmatrix}$$
 is a unit vector, $r = \begin{pmatrix} x\\y\\0 \end{pmatrix}$, $V^{\pm} = \begin{pmatrix} V_x^{\pm}\\V_y^{\pm}\\0 \end{pmatrix}$, $\omega_{\pm} = \frac{1}{2}(\Omega_c \pm \sqrt{\Omega_c^2 - 2\omega_z^2})$.
Then, the equations (3-5) take the following form:

$$\dot{V_x^{\pm}} = \omega_{\pm} V_y^{\pm} - k \left(V_x^{\pm} + \frac{\omega_{\mp}}{\omega_{+} - \omega_{-}} (V_x^{+} - V_x^{-}) \right) - (17) \\ -a \cdot z \cdot \cos(\omega_r t), \\ \dot{V_y^{\pm}} = -\omega_{\pm} V_x^{\pm} - k \left(V_y^{\pm} + \frac{\omega_{\mp}}{\omega_{+} - \omega_{-}} (V_y^{+} - V_y^{-}) \right) - (18) \\ -a \cdot z \cdot \sin(\omega_r t), \\ \ddot{z} = -\omega_z^2 z - k \dot{z} - \frac{a}{\omega_{+} - \omega_{-}} \left[(V_y^{-} - V_y^{+}) \cos(\omega_r t) - (19) \\ - (V_x^{-} - V_x^{+}) \sin(\omega_r t) \right].$$

Transforming of the system (17-19) by introducing the complex variables

$$V_{c}^{+} = V_{x}^{+} + iV_{y}^{+};$$

$$V_{c}^{-} = V_{x}^{-} + iV_{y}^{-}.$$
(20)

$$\dot{V}_c^+ = -\left(k\frac{\omega_+}{\omega_+ - \omega_-} + i\omega_+\right)V_c^+ + k\frac{\omega_-}{\omega_+ - \omega_-}V_c^- - (21)$$
$$-a \cdot z \cdot \exp(-i\omega_r t),$$

$$\begin{split} V_c^+ &= V_x^+ + iV_y^+;\\ V_c^- &= V_x^- + iV_y^-. \end{split} \tag{20}$$

$$\dot{V_c^+} &= -\left(k\frac{\omega_+}{\omega_+ - \omega_-} + i\omega_+\right)V_c^+ + k\frac{\omega_-}{\omega_+ - \omega_-}V_c^- - (21) \\ &\quad -a \cdot z \cdot \exp(-i\omega_r t),\\ \dot{V_c^-} &= -k\frac{\omega_+}{\omega_+ - \omega_-}V_c^+ + \left(k\frac{\omega_-}{\omega_+ - \omega_-} - i\omega_-\right)V_c^- - (22) \\ &\quad -a \cdot z \cdot \exp(-i\omega_r t),\\ \frac{d^2}{dt^2}z &= -\omega_z^2 z - k\frac{dz}{dt} - \frac{a}{\omega_+ - \omega_-} \times (23) \\ &\quad \times Re\left[i(V_c^+ - V_c^-)\exp(i\omega_r t)\right]. \end{split}$$
The characteristic equation of system (21,22) for $a = 0$

The characteristic equation of system (21,22) for a = 0has the form

$$\lambda^{2} + (k + i(\omega_{+} + \omega_{-}))\lambda - \omega_{+}\omega_{-} = 0.$$
 (24)

Defining the roots $\lambda_{1,2} = i\omega_{1,2}$ via the "frequencies" $\omega_{1,2}$, we obtain

$$\omega_{1,2} = -\frac{(\omega_{+} + \omega_{-}) - ik}{2} \pm \frac{1}{2} \sqrt{((\omega_{+} + \omega_{-}) - ik)^{2} - 4\omega_{+}\omega_{-}} = -\frac{\Omega_{c} - ik}{2} \pm \frac{1}{2} \sqrt{(\Omega_{c} - ik)^{2} - 2\omega_{z}^{2}}.$$
 (25)

Let us note that characteristic indices of the systems (3-4) and (21-22) for a = 0 are the same (do compare (11, 12, 13, 25)). Indeed $\omega_+ + \omega_- = \Omega_c$ and $\omega_+\omega_- = \omega_z^2/2$ so formulae (25) and (9) are the same. So it is obvious that the system (21-22) at a = 0 has characteristic root with a positive real part and thus is unstable ($Re(i\omega_1) > 0$).

By the property of the stability of the characteristic indices the system (21-23) is unstable also at sufficiently small $a \neq 0$, at least for $|a| < k \frac{\omega_m \omega_z}{\Omega_c \sqrt{2}}$ (the condition (15)), i.e. similar to the system (3-5).

CONCLUSION

Given the stability of characteristic indices of the linear systems under consideration with respect to small perturbations, magnetron motion is unstable for sufficiently small $a \neq 0$ (the condition (15)). This contradicts the conclusions of the article [1] that there is a possibility of the compression of magnetron motions in the case of $|a| \rightarrow 0$ ($|a| \ll k\sqrt{\omega_z \Omega_c}$). The contradiction may indicate that the simplified model of the dynamics and the approach used by the author [1] of the article do not accurately describe the dynamics of the process, because it leads to qualitative discrepancies in the behavior of the original system and simplified system. And, as shown above, at least for $|a| < k \frac{\omega_m \omega_z}{\Omega_c \sqrt{2}}$ magnetron motion is unstable and the compression is impossible.

It is also doubtful that a positron bunch compression may occure when the rotating electric field is applied on the all length of the storage area (see eq.(1)). It was observed in the experiment [4] that good compression can be achieved as long as the axial extent of the RW electrode is less than half of the plasma length. We also observed [5] the inability of accumulation region in the case where the rotating electric field length coincides with the length of the storage region.

It should be noted that the existence of compression found in experiment can not be explained with the analysis presented in [1,2]. There is other explanation [6] of the rotating electric field effect on the charged particles accumulation in traps at large and small particle densities [5,6].

REFERENCES

- [1] C.A. Isaac, Phys. Rev. A 87, 043415 (2013).
- [2] D.P. van der Werf, C.A. Isaac, C.J. Baker, T. Mortensen, S. J. Kerrigan and M. Charlton, New Journal Phys. 14, 075022 (2012).
- [3] N.A. Izobov, Lyapunov exponents and stability. Stability Oscillations and Optimization of Systems. Cambridge: Cambridge Scientific Publishers. 380 p. (2013).
- [4] J.R. Danielson and C.M. Surko, Phys. Plasmas, 13, 055706 (2006).
- [5] M.K. Eseev, A.G. Kobets, I.N. Meshkov, A.A. Sidorin, O.S. Orlov, JETP letter 102, 261 (2015)
- [6] M.K. Eseev, I N. Meshkov, Phys. Usp. 59, 304 (2016)