THE ELECTRON TRAJECTORIES CONSTRUCTION IN THE SYSTEM WITH A "REAL" GEOMETRY OF THE FIELD CATHODE

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Abstract

The problems of the trajectories constructing for field electrons and emission images simulation are considered. As an approximation of the emitter shape an explicitly and implicitly defined surfaces are selected. Hyperboloidal, ellipsoidal and paraboloidal models are studied. Also an equipotential surface of the charged sphere-on-orthogonal-cone system is used. A simple solution of the field distribution problem is allowed to formulate the Cauchy problem for the motion equations. The shape of the anode (the projector microscope screen) can be selected in any desired form. Achieving a screen by electrons is performed with dense output technique in numerical approach to the solution. The distribution of the work function on the cathode surface is obtained. The trajectories for the projection of the field emission activity as the image are used.

INTRODUCTION

The phenomenon of field electron emission is a recognized tool for the analysis of electron sources. These sources are necessary for accelerating technology and for high-precision microscopes. Surface shape, distribution of the force field in the interelectrode space and the cathode material affect emission main characteristics. Advanced emitters search will be more relevant for a long time [1].

In this paper the number of the tasks required to build emission images are considered. Emission images show the current density distribution on the surface of the cathode. The source itself and the work function maps modeling, and the construction of electron trajectories in diode systems with elements of a statistical approach are performed.

PHYSICAL AND MATHEMATICAL MODELS

To initiate a field electron emission at low voltages V, sources are often made in the form of tips (figure 1).

Figure 1: Shapes of the real cathodes ([2], [3] and [4]).

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As the approximation of the emitter surface the part of the two-sheeted hyperboloid, prolate ellipsoid of revolution, paraboloid of revolution and sphere-on-orthogonal-cone system are utilized.

As emission control parameters of the system the radius of curvature r_0 at the top of the cathode, the distance d from the top of the cathode to the anode (screen), the voltage V between the electrodes are considered. As the example the following values were used: $r_0 = 1.00$ μm, $d = 2.00$ cm, $V = 20.0$ kV.

RESULTS AND DISCUSSION

The potential distribution is found by solving the Laplace equation. It exists in an analytical form for all of the systems in question [3], [5]. It leads us to the determination of the force field structure in the gap between the electrodes. For the example the picture of the field in the system with the paraboloidal tip is presented (figure 2).

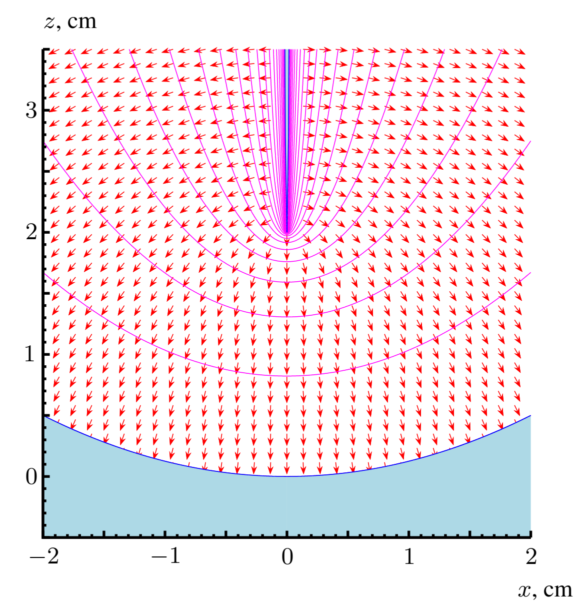

Figure 2: Equipotentials and force field.

For a single crystal cathode it is important to have an idea of the work function Φ distribution of on its surface. In the metal samples case the semi-empirical model proposed in [6] is well established:

$$\Phi = A + B\Delta t.$$

Here A and B must be found by linear regression method with the experimental data. Parameter Δt depends on the type of crystal lattice and Miller indices. The model has

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successfully passed extensive statistical testing for materials such as W, Mo, Pt, Ir, Ta, Cu, Nb, Ni, Re, Ru [7], [8].

The figure 3 shows the distributions of the work function for Ni. The work function varies within regression model from 4.3 eV to 5.3 eV. Small values of Φ are colored in green, average values — in yellow, high values — in red. An algorithm for work function map constructing is published in [8].

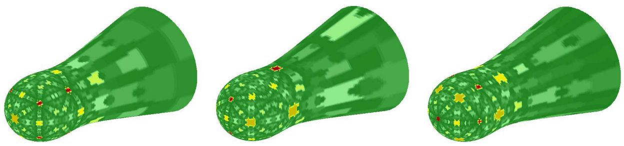

Figure 3: Maps of Φ: faces (001), (011), (111).

The basic law that reflects the dependence of the emission current density j on the electric field strength E and the work function Φ, is the Fowler—Nordheim formula [9]:

$$j = \frac{e^3}{8\pi h \Phi t^2(y_0)} E^2 \exp\left[-\frac{4\sqrt{2m}}{3\hbar e E}\Phi^{\frac{3}{2}} v(y_0)\right],$$

$$y_0 = \frac{e}{\Phi}\sqrt{\frac{eE}{4\pi\varepsilon_0}}.$$

Here e is the elementary charge, h is the Planck constant, m is the electron mass, $t(y)$ and $v(y)$ are the Nordheim's elliptic functions. One of the most successful approximations of the Nordheim functions presented in [10].

In fact, j determines the probability density of the electron escape from a specific point on the cathode surface. This value can be calculated with more accurate way to solving the Schrödinger equation and using the temperature dependence of the electron energy distribution [11]. Runge—Kutta—Nyström method with Dormand—Prince coefficients of 12-th order with embedded method of 10-th order is used [12]. Integration of the current density is performed with Simpson and Bode rules [13].

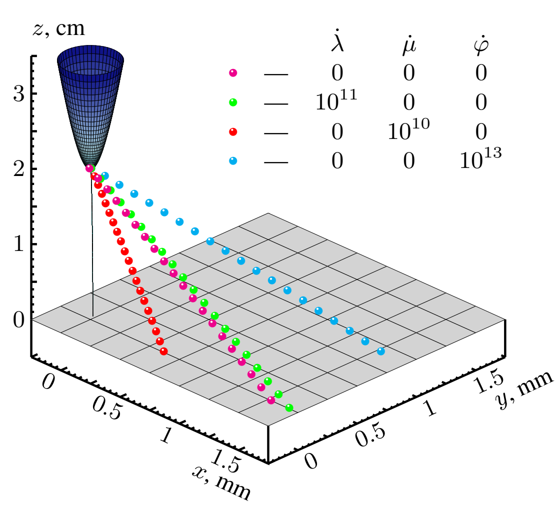

Figure 4: Electron trajectories.

The next step is the projection of the current density distribution on the screen. This procedure can be accomplished through field lines or by constructing the trajectories of emitted particles [14]. Figure 4 demonstrates the calculated trajectories for the hyperboloidal tip. Here λ, μ and φ are prolate spheroidal coordinates. Generalized velocities corresponds to the thermal values. Integration of the motion equations is carried out by Runge—Kutta method with Dormand—Prince coefficients of 8th order with two embedded methods of 5th and 3rd order [12]. Screen surface achievement is monitored by dense output based on method's continuous expansion of 7th order.

The velocity distribution of electrons required to the formulation of the Cauchy problem for the motion equations is obtained on the basis of the total and normal energy distributions (figure 5). Here $E = 5.0 \cdot 10^9$ V/m, $T = 300$ K, $\Phi = 4.5$ eV, Fermi energy $\mathscr{E}_F = 5.78$ eV (last two values corresponds to tungsten) [13].

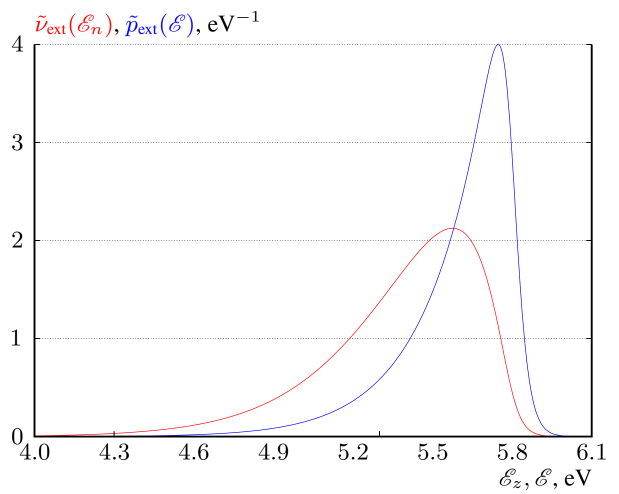

Figure 5: Normalized electron energy distributions.

Figure 6 illustrates emission images of tungsten obtained in the natural experiment.

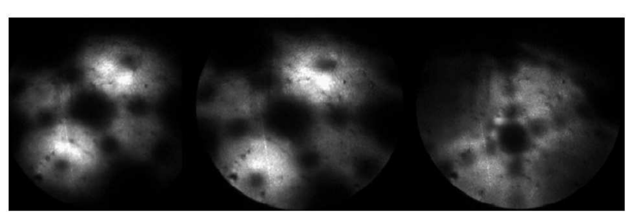

Figure 6: Tungsten emission images [15].

Figure 7 demonstrates the results of the statistical modeling for the tungsten emission image. On the top of the tip the face with Miller indices (001) is located. We can say that the picture qualitatively faithfully describes the situation (it should be noted that the real surfaces has unavoidable defects and inhomogeneities).

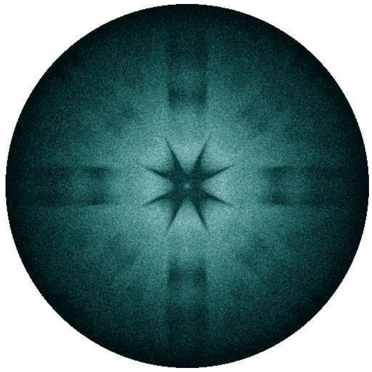

Figure 7: Emission image modeling.

CONCLUSION

In the paper the field emission images simulation of single crystal cathode of tip form is performed. The software package implements surface modeling and work function distribution algorithms. On the basis of the electric field and the current density evaluations the electron energy distributions and coordinates of escape are constructed. The projection of the emission image on the screen is carried out with the motion equations solution.

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