DIELECTRIC CHART AS A TOOL FOR DIAGNOSIS OF DIELECTRIC MATERIALS

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Abstract

One of the most informative diagnostic methods dielectric materials is the analysis of the complex permittivity depending on the frequency of the electric field [1]. Dielectric chart is the dependence of the imaginary part of the complex permittivity of its real part. Thus, difference between the real dielectric chart from the reference or change it during the operation can be a means of diagnostics of dielectric materials. Dielectric chart in the classical theory of Debye is a semicircle with its center lying on the real axis. For solid dielectric the dielectric chart deviation from the semicircle can be quite large, but it still remains a circular arc. This deviation is characterized by parameter α (in the case of the Debye $\alpha = 0$). To clarify the physical meaning of the deviations of the experimental data on the Debye theory, expressed in the value of α, several possible causes have been considered: the effect hindered reorientation of dipoles, the effect of the non-sphericity of the molecules, the complex nature of viscosity. However, the main cause of deviations, in our opinion, is the availability of the distribution of relaxation times around a central relaxation time, in particular, due to defects in the sample. Gaussian distribution width increases rapidly with increasing α. In this paper we propose an algorithm for calculating α, allowing you to quickly determine the condition of the sample on a single parameter.

INTRODUCTION

One of the most informative diagnostic methods for dielectric materials is the analysis of the complex permittivity ε^* depending on the frequency of the electric field [1, 2]. But the presentation in the form of frequency dependency does not allow to easily analyze data and assess the significance of the deviation from the expected relationship. A more appropriate presentation is a dielectric diagram (Argand diagram) in the complex plane when built dependence of the imaginary part of the complex permittivity ε'' of the real part of it ε', and each point is characterized by individual frequency (Fig. 1). Difference between the real dielectric chart from the reference or change it during the operation can be a means of diagnostics of dielectric materials.

DIELECTRIC DIAGRAM

Dielectric diagram according to the classical equations of Debye is a semicircle with its center lying on the real axis (ε'), and crosses the real axis at the points ε_0 and ε_∞ (see Fig. 1). We introduce the notation

$$\begin{aligned} u &= \varepsilon^* - \varepsilon_\infty, \\ v &= (\varepsilon^* - \varepsilon_\infty) i\omega\tau_0, \\ u + v &= \varepsilon_0 - \varepsilon_\infty, \end{aligned}$$

where τ_0 — relaxation time, ε_0 — the value of the real part of the dielectric permittivity at a frequency $\omega = 0$, ε_∞ — the value of the real part of the dielectric permittivity at a frequency $\omega \to \infty$, the difference between ε_0 and ε_∞ attributed to dipole.

The values v and u may be considered as vectors in the complex plane, and in Debye case they are perpendicular, and their sum is constant and equal to the real value of $\varepsilon_0 - \varepsilon_\infty$.

The deviation from the semicircle can be very large for solid dielectrics. Nevertheless, depending on ε' ε'' still represent circular arcs.

In equivalent circuit for the experimental dependence the impedance is $Z = \tau_0 (i\omega\tau_0)^{-\alpha}/(\varepsilon_0 - \varepsilon_\infty)$, and the phase angle between the active and reactive components does not depend on the frequency and is equal to $\alpha\pi/2$. Since the angle between the axis ε' and the radius vector to the point ε_∞ on diagram arc circle in the complex plane is also equal to $\alpha\pi/2$, it is reasonable to assume that the properties of the dielectric are determined by the value α. Angle $(1-\alpha)\pi/2$ (between the vectors v and u in the complex plane) does not depend on the frequency and is equal to half of the arc angle. Consequently,

$$u + v = u[1 + f(\omega) e^{i(1-\alpha)\pi/2}] = \varepsilon_0 - \varepsilon_\infty,$$

where $f(\omega)$ — real function of frequency and other parameters. Since $e^{i(1-\alpha)\pi/2} = i^{(1-\alpha)}$, then

$$\varepsilon^* - \varepsilon_\infty = (\varepsilon_0 - \varepsilon_\infty)/[1 + i^{(1-\alpha)} f(\omega)].$$

From general considerations, it can be assumed that this relationship will look $\omega^{(1-\alpha)}$, when the complex form ε^* is the result of the initial hypothesis that the applied field is given by $E = E_0 e^{i\omega t}$. If ω is the result of linear operations over the exponent, the dependence on ω is identical to depending on the imaginary unit i, so that

$$\varepsilon^* - \varepsilon_\infty = (\varepsilon_0 - \varepsilon_\infty)/[1 + (i\omega\tau_0)^{1-\alpha}].$$

for $0 < \alpha < 1$.

The dependence of $\ln|u/v|$ on $\ln\omega$:

$$\ln|u/v| = (1-\alpha)\ln\omega\tau_0 = (1-\alpha)\ln\omega + (1-\alpha)\ln\tau_0,$$

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ISBN 978-3-95450-181-6

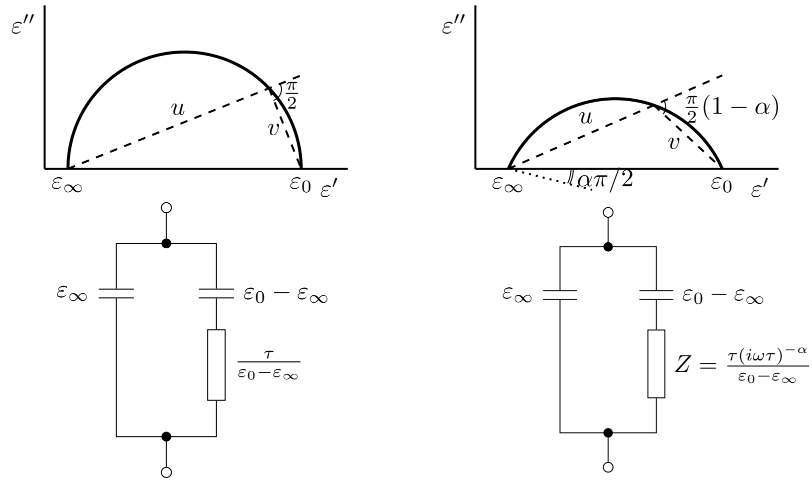

Figure 1: Dielectric diagrams and respective equivalent circuits.

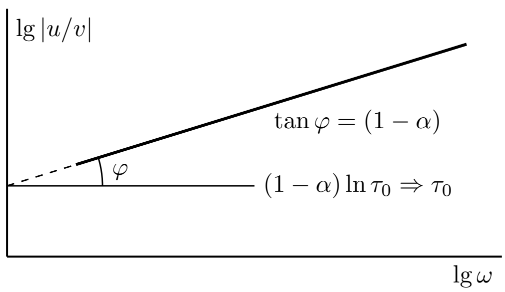

Figure 2: The dependence of $\ln|u/v|$ on $\ln\omega$.

is a linear function of which can be determined α and τ_0 (see Fig. 2).

Then

$$\begin{aligned}
&\varepsilon' - \varepsilon_0 = \\
&\frac{\varepsilon_0 - \varepsilon_\infty\left[1 + (\omega\tau_0)^{1-\alpha}\sin\frac{1}{2}\alpha\pi\right]}{1 + 2(\omega\tau_0)^{(1-\alpha)}\sin\frac{1}{2}\alpha\pi + (\omega\tau_0)^{2(1-\alpha)}} = \\
&\frac{1}{2}(\varepsilon_0 - \varepsilon_\infty)\left[1 - \frac{\sinh(1-\alpha)x}{\cosh(1-\alpha)x + \cos\frac{1}{2}\alpha\pi}\right], \\
&\varepsilon'' = \frac{(\varepsilon_0 - \varepsilon_\infty)(\omega\tau_0)^{1-\alpha}\cos\frac{1}{2}\alpha\pi}{1 + 2(\omega\tau_0)^{1-\alpha}\sin\frac{1}{2}\alpha\pi + (\omega\tau_0)^{2(1-\alpha)}} = \\
&\frac{1}{2}(\varepsilon_0 - \varepsilon_\infty)\frac{\cos\frac{1}{2}\alpha\pi}{\cosh(1-\alpha)x + \sin\frac{1}{2}\alpha\pi},
\end{aligned} \tag{1}$$

where $x = \ln\omega\tau_0$. These expressions are reduced to the case of Debye when $\alpha = 0$. If $\alpha > 0$ $(0 \leqslant \alpha \leqslant 1)$ the range of variation of ε^* is expanding, and the maximum of ε'' (at $\omega = 1/\tau_0$) decreases.

Expressions (1) provide the following maximum value ε'' at the point $\omega = 1/\tau_0$:

$$\varepsilon''_{max} = \frac{1}{2}(\varepsilon_0 - \varepsilon_\infty)\tan[(1-\alpha)\frac{\pi}{4}].$$

At very low frequencies (see (1))

$$\varepsilon = (\varepsilon_0 - \varepsilon_\infty)(\omega\tau_0)^{2-\alpha}\cos\frac{1}{2}\alpha\pi.$$

If $\omega \ll 1/\tau_0$, then admittance $\sigma \sim \omega^\gamma$ $(1 \leqslant \gamma \leqslant 2)$, corresponding $0 \leqslant \alpha \leqslant 1$.

EVALUATION OF DEFECTS IN THE DIELECTRIC MATERIAL

To clarify the physical meaning of the deviations of the experimental data from Debye theory, expressed in the value of α, several possible causes have been considered: the effect of dipoles hindered reorientation, the effect of the non-sphericity of the molecules, the complex nature of viscosity. However, the main cause of deviations, in our opinion, is the availability of the distribution of relaxation times around a central τ_0, in particular, due to defects in the sample. Here we are not talking about different phenomena of polarization or non-homogeneity of the material. The distribution of the relaxation times, proposed by Wagner [3], is given by the logarithmic Gaussian type distribution

$$F(s)ds = (b/\sqrt{\pi})\exp(-b^2 s^2)ds,$$

where $s = \ln(\tau/\tau_0)$, parameter b determines the width of the distribution. If we use the expression (1) for ε'', we obtain a different distribution function $F(s)$ for relaxators:

$$F(s)ds = \frac{1}{2\pi}\frac{\sin\alpha\pi}{\cosh(1-\alpha)s - \cos\alpha\pi}ds.$$

If we define the ε' and ε'' using numerical integration of this expression it is possible to find b, which most closely matches a given α. For example, if $\alpha = 0.23$, found by the above procedure b is equal to 0.6. Obviously, the Gaussian distribution requires too high a concentration of the relaxation times nearby τ_0, and distribution using α much wider (see Fig. 3). Distribution width increases rapidly with increasing α, and Gaussian distribution is getting worse approximation. For $\alpha = 0.5$ 95% of relaxation times are in the range $0.001 < \tau/\tau_0 < 1000$ whereas only $\alpha = 0.75$ only 72% of relaxation times are in the specified range.

ISBN 978-3-95450-181-6

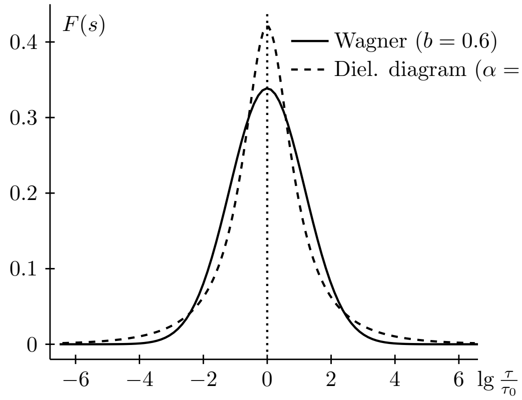

Figure 3: Comparing the distributions of relaxation times, obtained by Wagner and dielectric diagrams.

CONCLUSION

The paper deals with the application of the dielectric diagram as a tool for processing of dielectric information. It has been shown that its study allows to characterize the material under test by parameter α, associated with the emergence of the distribution of relaxation times around the classic Debye relaxation time τ_0. It should be noted that the proposed method is limited to processing of dielectric diagrams representing symmetrical arcs. Also the difficulties of the physical explanation of relaxation time distribution $F(s)$ remain. In addition, the physical sense of the real and imaginary parts of the impedance of the equivalent electrical circuit $Z = \tau(i\omega\tau)^{-\alpha}/(\varepsilon_0 - \varepsilon_\infty)$ are unclear.

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ISBN 978-3-95450-181-6