EFFECT OF THE VERTICAL VELOCITY COMPONENT ON PROPERTIES OF SYNCHROTRON RADIATION

O.E. Shishanin, Moscow State Industrial University, Moscow, Russia

Abstract

This subject determines more precisely characteristics of synchrotron radiation when the charge particle moves on the spiral in physical devices and a space. For this purpose the Bessel functions of a high order are approximated to within the second approach. It is discussed that the vertical component of velocity in alternating magnetic fields of accelerators significantly changes the spectral and angular distributions of the radiation intensity.

Theory of synchrotron radiation when the electron has a spiral trajectory considered by many authors (see, for example, [1]). In this paper we find a more accurate synchrotron radiation formulas for the spiral and circular motiom of electron in a constant and homogeneous magnetic field. For this purpose, first of all we define the asymptotic representation of the Bessel functions. Previously, several asymptotic expressions for the Bessel functions with large index were obtained [1] - [3]. Based on these methods we have extended the calculations up to the second order of accuracy [4].

However, we can take the integral representation of the Bessel functions

$$J_\nu(y) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i\nu\varphi - iy\sin\varphi}d\varphi.$$

When the circular motion, it will $y = \nu\beta\sin\theta$, where $\beta = v/c$ (v is the electron velocity) and θ is the spherical angle of radiation. Following Schwinger [5], we assume φ as a small parameter because the radiation is removed from the small part of the orbit in a certain direction.

To study the problem with a spiral trajectory there is a need to replace β by

$$\beta_0 = \sqrt{(\beta^2 - \beta_3^2)/(1-\beta_3^2)}$$

and $\sin\theta$ by

$$\sin\theta_0 = (\sqrt{1-\beta_3^2}\sin\theta)/(1-\beta_3\cos\theta);$$

$c\beta_3$ here is the velocity component along the magnetic field. Thus, we have

$$J_\nu(\nu\beta_0\sin\theta_0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i\nu(\varphi - \beta_0\sin\theta_0\sin\varphi)}d\varphi.$$

Expanding the right-hand side in terms of φ and introducing a new variable as $\varphi = pt$, where

$$p = \sqrt[3]{6/(\nu\beta_0\sin\theta_0)}\cdot t,$$

we get

$$J_\nu(\nu\beta_0\sin\theta_0) = \frac{p}{2\pi}\int_{-\infty}^{\infty} dt e^{i(xt+t^3)}[1 - i\frac{\nu}{120}(\frac{6}{\nu})^{5/3}t^5]$$

with $x = p(1-\beta_0\sin\theta_0)\nu$. Here the integral limits were extended to infinity because φ is small.

Then we use the following expressions:

$$\int_0^\infty \cos(t^3+xt)dt = \frac{\sqrt{x}}{3}K_{1/3}(x_1),$$

$$\int_0^\infty t^5\sin(t^3+xt)dt = \frac{x^3}{27\sqrt{3}}K_{2/3}(x_1) - \frac{4}{27}x^{3/2}K_{1/3}(x_1),$$

where $x_1 = 2(x/3)^{3/2}$. In the first equality we took into account the terms of order

$$\varepsilon = 1 - \beta_0^2\sin^2\theta_0.$$

Finally for asymptotics of the Bessel function and its derivative, we obtain

$$J_\nu(\nu\beta_0\sin\theta_0) \approx \frac{\sqrt{\varepsilon}}{\pi\sqrt{3}}[K_{1/3} + \frac{1}{10}\varepsilon(K_{1/3} - 2\mu K_{2/3})], \tag{1}$$

$$J'_\nu(\nu\beta_0\sin\theta_0) \approx \frac{\varepsilon}{\pi\sqrt{3}}[K_{2/3} + \frac{1}{5}\varepsilon(2K_{2/3} - (\frac{1}{\mu}+\mu)K_{1/3}], \tag{2}$$

where $\mu = \nu\varepsilon^{3/2}$ and $\mu/3$ is an argument of functions K_i. The neglected terms are of the order ε with respect to the main term.

Formulas for spectral and angular distributions in the case of the spiral motion we can get a direct calculation or by the Lorentz transformations. Then for the components of the linear polarization of the radiation intensity(in the orbital plane and perpendicular to it, respectively) we get

$$dW_\sigma(\nu,\theta_0) = W_1\beta_0^2 J_\nu'^2(\beta_0\nu\sin\theta_0)\sin\theta_0 d\theta_0, \tag{3}$$

$$dW_\pi(\nu,\theta_0) = W_1\cot^2\theta_0 J_\nu^2(\beta_0\nu\sin\theta_0)\sin\theta_0 d\theta_0, \tag{4}$$

where

$$W_1 = \frac{3}{2}W_0\nu^2\varepsilon_0(1+\beta_3\cos\theta_0), W_0 = \frac{2}{3}\frac{e_0^4H^2}{m_0^2c^3}, \varepsilon_0 = 1-\beta_0^2.$$

Radiation frequency ω will be

$$\frac{e_0H}{mc}\cdot\frac{\nu}{1-\beta_3\cos\theta_0},$$

where H is the magnetic field strength. Using asymptotics, right-hand sides of (3) and (4) can be written as

$$\frac{1}{3\pi^2}W_1\beta_0^2\varepsilon^2K_{2/3}^2[1+\frac{2}{5}\varepsilon(2-(\frac{1}{\mu}+\mu)K_{1/3}/K_{2/3})]\sin\theta_0 d\theta_0, \tag{5}$$

ISBN 978-3-95450-170-0

$$\frac{1}{3\pi^2}W_1\cot^2\theta_0\varepsilon K_{1/3}^2[1+\frac{1}{5}\varepsilon(1-2\mu K_{1/3}/K_{2/3})]\sin\theta_0 d\theta_0. \quad (6)$$

We note that at a fixed frequency in (4) and (6) π - component is equal to zero when $\theta = \pi/2$. At this angle σ - component has a maximum. Integrating these expressions over angle θ_0 we obtain spectral formulas

$$W_\sigma(\nu) = W_2[\int_y^\infty K_{5/3}(x)dx+K_{2/3}(y)-\frac{1}{20}\varepsilon_0(7K_{2/3}(y)$$
$$+(16\mu_0+\frac{11}{\mu_0})K_{1/3}(y)+5\int_y^\infty K_{1/3}(x)dx)],$$
$$W_\pi(\nu) = W_2[\int_y^\infty K_{5/3}(x)dx-K_{2/3}(y)+\frac{1}{20}\varepsilon_0(7K_{2/3}(y)$$
$$+\frac{3}{\mu_0}K_{1/3}(y)-15\int_y^\infty K_{1/3}(x)dx)],$$

where

$$W_2 = \frac{\sqrt{3}}{4\pi}W_0\nu\varepsilon_0^2, \quad y = \frac{2}{3}\mu_0, \quad \mu_0 = \nu\varepsilon_0^{3/2}.$$

It is clear that small corrections to the known formulas are proportional ε_0.

Then we take into account that at high ν the change of radiation frequency almost continuously. It gives us the opportunity to perform integration in the last formulas. Thus, we find the radiation intensities for components of linear polarization

$$W_\sigma = \frac{7}{8}W_0\frac{1}{\varepsilon_0}(1-\frac{8}{7}\varepsilon_0), \quad W_\pi = \frac{1}{8}W_0\frac{1}{\varepsilon_0}. \quad (7)$$

Summing (7) we obtain total intensity

$$W = \frac{2}{3}\frac{e_0^2H^2}{m_0^2c^3}\frac{\beta^2-\beta_3^2}{1-\beta^2}. \quad (8)$$

Found formulas apply also for the electron motion in a circle with a relativistic velocity if instead of β_0 and $\sin\theta_0$ take β and $\sin\theta$, respectively; in addition, it is necessary to put $\beta_3 = 0$. In this case, the important spectral formula applicable for accelerators becomes

$$W(\nu) = \frac{\sqrt{3}}{2\pi}W_0\frac{\nu}{\gamma^4}\{\int_{y_1}^\infty K_{5/3}(x)dx-$$
$$\frac{1}{\gamma^2}[\frac{1}{5}(\frac{2\nu}{\gamma^3}+\frac{\gamma^3}{\nu})K_{1/3}(y_1)-\frac{1}{2}\int_{y_1}^\infty K_{1/3}(x)dx]\},$$

where lorentz-factor $\gamma = 1/\sqrt{1-\beta^2}$, $y_1 = (2\nu)/(3\gamma^3)$.

In the case of electron motion in a spiral requires special consideration of the use of the asymptotics (1) and (2). First of all it is necessary that $\beta_3 \ll \beta$; if β_3 is close to β, it will be produced only basic tone and the integration over a quasi-continuous spectrum is meaningless. On the other hand, the asymptotics with corrections for the spiral motion can be used in the case when $\beta_3 > \varepsilon_0$; for $\beta_3 < \varepsilon_0$ terms in the spectral and angular distributions, which are proportional to $\varepsilon_0^2\beta_3^2$, will be less than the rejected terms proportional to ε_0^4. For $\beta_3 < \varepsilon_0$ is necessary to restrict the asymptotics for circular motion.

We now turn to an alternating magnetic fields of accelerators and storage rings. For example, in the case of an axially symmetric magnetic field we have a simple solution for the vertical oscillations of the form

$$z = B\cos(\sqrt{n}\omega_0 t+\delta),$$

where B is the amplitude of oscillations, δ is the initial phase, n is the field gradient, frequency $\omega_0 = (e_0H)/(mc)$. Study the problem of radiation has led to the fact that along with parameter $\cos\theta$ should be considered a derivative of the vertical movement, which defines the tangent of sloping angle. We must bear in mind new value

$$v_z/c|_{t=0} = \alpha\cdot\cos\delta, \qquad \alpha = \sqrt{n}B/R.$$

In the corresponding formulas it is necessary to carry out averaging over phase δ.

By this we take into account the scatter of the particle beam during the injection and difference of electron amplitudes in the cross section. Vertical betatron oscillations change the behavior of spectral and angular curves [6]. Intensity of π - component in the orbital plane is not equal to zero; maximum of σ - component decreases.

Quantity α varies for different machines. For storage rings, as is known,

$$z = \sqrt{\frac{\beta_z A_z}{\pi}}\cos(\int\frac{ds}{\beta_z}+\delta),$$

where A_z is the emittance, β_z is the betatron function. In this case we have [7]

$$\alpha = \sqrt{\frac{A_z}{\pi}}[\frac{1}{\sqrt{\beta_z}}\sqrt{1+(\frac{1}{2}\frac{d\beta_z}{ds})^2}]_{\varphi=0}.$$

Here the azimuth angle is fixed at the point where the radiation is emitted; the derivative is found as the ratio of the legs of the triangle.

REFERENCES

[1] A.A.Sokolov, I.M.Ternov, Radiation from Relativistic Electrons, AIP, New York, 1986.

[2] G.N.Watson, Theory of Bessel Functios, Cambridge University Press, 1944.

[3] V.A.Fock, Doklady AN SSSR 1(1934)96 (in Russian).

[4] O.E.Shishanin, Vestnik of Moscow University, physics, astronomy, 4(1969)103 (in Russian).

[5] J.Schwinger, Phys.Rev. 75(1949)1912.

[6] V.Ch.Zhukovsky, O.E.Shishanin, JETP 34(1972)729.

[7] O.E.Shishanin, JETP 90(2000)725.

ISBN 978-3-95450-170-0