IOTA – Integrable Optics Test Accelerator at Fermilab

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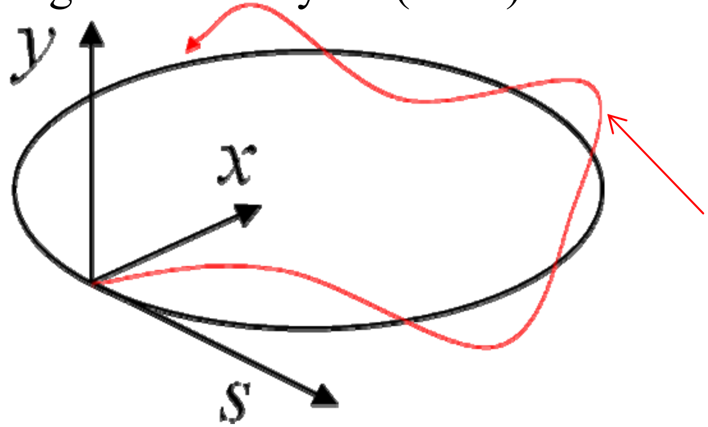
RuPAC 2012, St. Petersburg

Fermilab

Strong Focusing – our standard method since 1952

Christofilos (1949); Courant, Livingston and Snyder (1952)

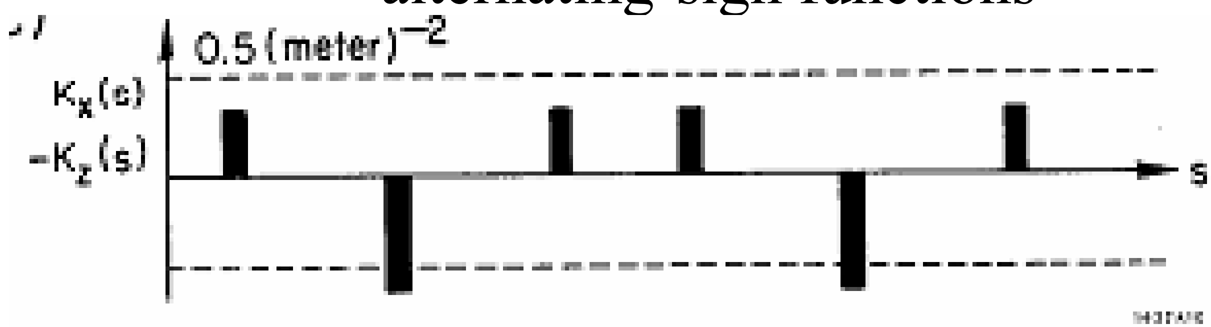


Particle undergoes betatron oscillations

$$\begin{cases} x'' + K_x(s)x = 0 \\ y'' + K_y(s)y = 0 \end{cases}$$

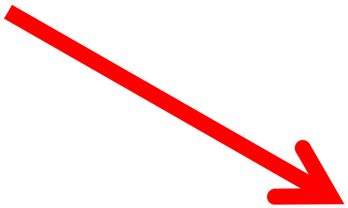
$$K_{x,y}(s + C) = K_{x,y}(s)$$ -- piecewise constant alternating-sign functions

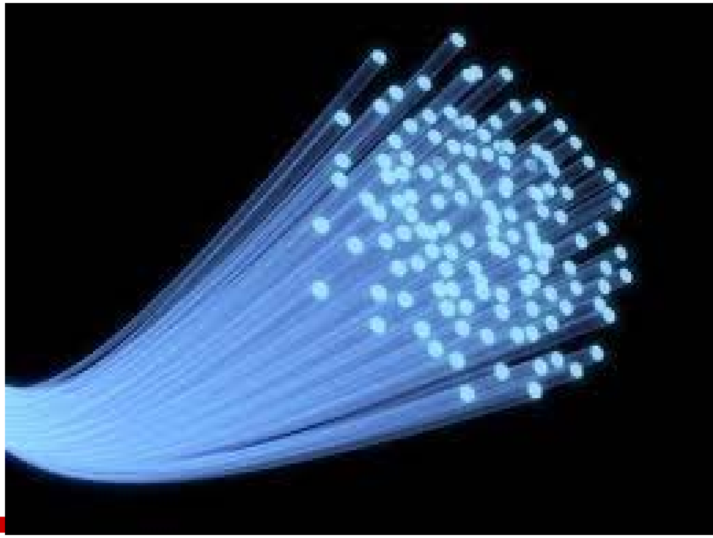
s is "time"



--Magnet lattice and focussing functions in the normal cells of a particular guide field.

Next generation beam focusing





Strong focusing

Specifics of accelerator focusing:

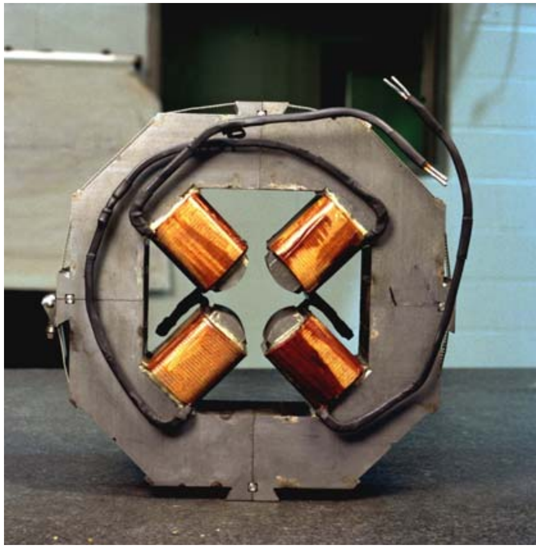
- Focusing fields must satisfy Maxwell equations in vacuum

$$\Delta\varphi(x, y, z) = 0$$

- For stationary fields: focusing in one plane while defocusing in another
  - quadrupole:

$$\varphi(x, y) \propto x^2 - y^2$$

  - However, alternating quadrupoles results in effective focusing in both planes



Courant-Snyder invariant

Equation of motion for betatron oscillations

$$z'' + K(s)z = 0,$$

$$z = x \text{ or } y$$

$$I = \frac{1}{2\beta(s)}\left(z^2 + \left(\frac{\beta'(s)}{2}z - \beta(s)z'\right)^2\right)$$

Invariant (integral) of motion, a conserved qty.

where $\left(\sqrt{\beta}\right)'' + K(s)\sqrt{\beta} = \frac{1}{\sqrt{\beta^3}}$

Linear oscillations

- Normalized variables

$$z_N = \frac{z}{\sqrt{\beta(s)}},$$

$$\psi' = \frac{1}{\beta(s)} \quad \text{-- new time variable}$$

$$p_N = p\sqrt{\beta(s)} - \frac{\beta'(s)z}{2\sqrt{\beta(s)}},$$

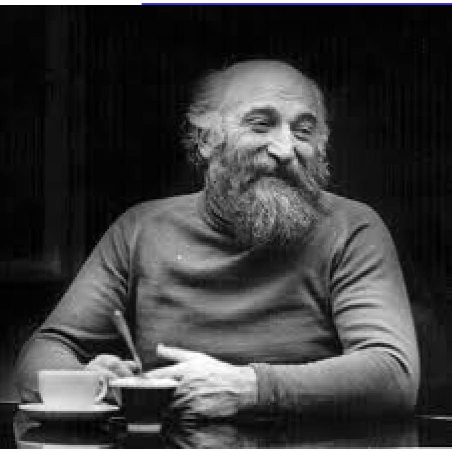
- In this variables the motion is a linear oscillator

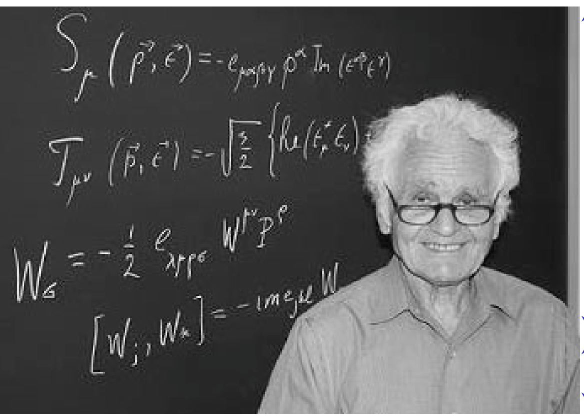
$$\frac{d^2 z_n}{d\psi^2} + \omega^2 z_n = 0$$

$$I = \frac{1}{2\pi}\oint p_n dz_n$$

- Thus, betatron oscillations are linear; all particles oscillate with the same frequency!

$$H = \omega_x I_x + \omega_y I_y$$





- 1954: ITEP (Moscow) decides to build a strong-focusing 7-GeV proton synchrotron.
  - Yuri Orlov recalls: "In 1954 G. Budker gave several seminars there. At these seminars he predicted that the combination of a high betatron frequency with even a small nonlinearity would result in stochasticity of betatron oscillations. "
  - "… I analyzed all reasonable linear and nonlinear resonances with tune-shifting nonlinearities and obtained well-defined areas of stability between and below resonances and the corresponding tolerances."
  - Work published in 1955.
  - Early example of Kolmogorov-Arnold-Moser (KAM) theorem applied to accelerators

COLLIDING BEAMS: PRESENT STATUS; AND THE SLAC PROJECT*

B. Richter

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Report at HEAC 1971

The discovery in the early '60's at the Princeton-Stanford ring of what was thought to be the resistive wall instability brought the realization that circular accelerators are fundamentally unstable devices because of the interaction of the beam with its environment. Stability is achieved only through Landau damping and/or some external damping system.

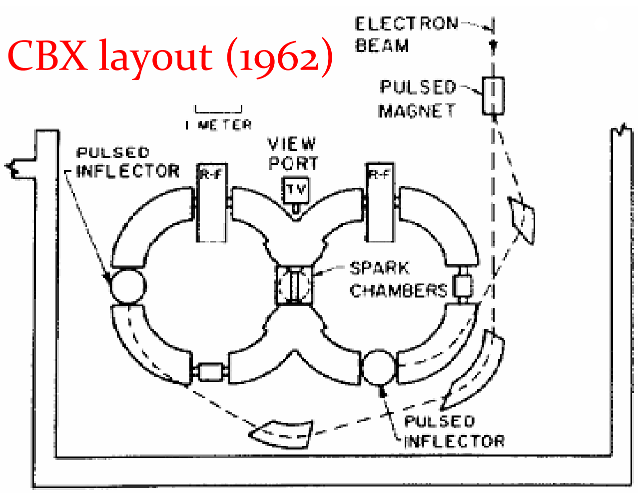
1965, Priceton-Stanford CBX: First mention of an 8-pole magnet

- Observed vertical resistive wall instability
- With octupoles, increased beam current from ~5 to 500 mA

CERN PS: In 1959 had 10 octupoles; not used until 1968

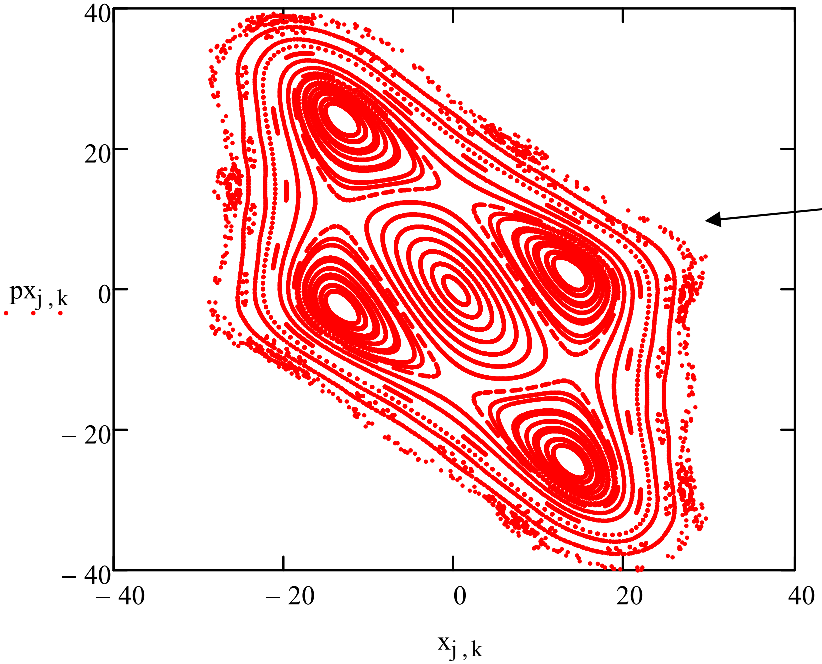
- At 10^{12} protons/pulse observed (1st time) head-tail instability. Octupoles helped.
- Once understood, chromaticity jump at transition was developed using sextupoles.
- More instabilities were discovered; helped by octupoles and by feedback.

CBX layout (1962)



Octupoles and tune spread

In all machines there is a trade-off between Landau damping and dynamic aperture (resonances).



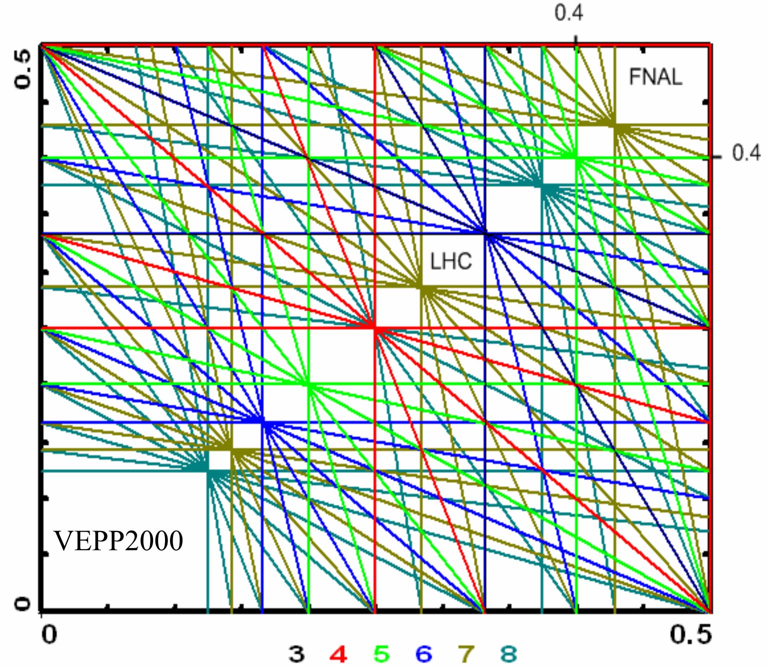
Typical phase space portrait:

1. Regular orbits at small amplitudes
2. Resonant islands + chaos at larger amplitudes;

Linear vs nonlinear

- Accelerators are linear systems by design (freq. is independent of amplitude).
- In accelerators, nonlinearities are unavoidable (SC, beam-beam) and some are useful (Landau damping).
- All nonlinearities (in present rings) lead to resonances and dynamic aperture limits.
- Are there "magic" nonlinearities that create large spread and zero resonance strength?
- The answer is – yes

(we call them "integrable")



$$k\nu_x + l\nu_y = m$$

3D: $H = F(J_1, J_2, J_3)$

Our goal

- Our goal is to create a practical nonlinear integrable focusing system with a large frequency spread and stable particle motion.

- Benefits:
  - Increased Landau damping
  - Improved stability to perturbations
  - Resonance detuning

First nonlinear focusing proposal

- In a series of reports 1962-65 Yuri Orlov has proposed to use non-linear focusing as an alternative to strong (linear) focusing.
  - Final report (1965):

FUNDAMENTAL PROPERTIES OF NON-LINEAR FOCUSING*

V. V. VECHESLAVOV and YU. F. ORLOV

(*Received* 23 *July* 1965)

Abstract—An analysis has been made of the fundamental properties of non-linear focusing taking the simple example of non-linear focusing in a symmetric magnetic field of the fifth degree. The dimensions of the first stability region with regard to small non-linear z-oscillations are determined. The influence of r–z-resonances was studied and also the maintenance of stability when allowing for adiabatic damping with the help of external or mutual r- and z-phase stabilization. It was found that mutual phase stabilization arises in the region of a r–z-resonance.

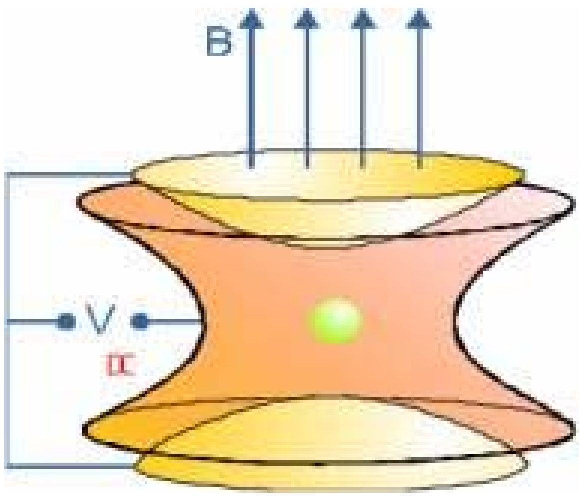
A numerical and partly analytical study of these effects has been made.

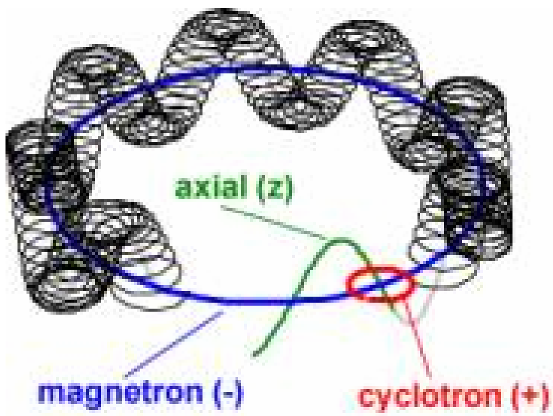
B. Chirikov has understood that this focusing is not integrable and would lead to chaotic particle motion

Example of integrable system: Ideal Penning trap

- The ideal Penning trap is a LINEAR integrable and system
  - It is a linear 3-d oscillator

$$H = \omega_1 J_1 + \omega_2 J_2 + \omega_3 J_3$$





Kepler problem – nonlinear integrable system

- Kepler problem: $$V = -\frac{k}{r}$$

- In spherical coordinates: $$H = -\frac{mk^2}{2\left(J_r + J_\theta + J_\phi\right)^2}$$

- Example of this system: the Solar system

A rectangular billiard

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right), \;\; 0 < x < a, \;\; 0 < y < b$$

$$H\left(J_1, J_2\right) = \frac{\pi^2}{2m}\left(\frac{J_1^2}{a^2} + \frac{J_2^2}{b^2}\right)$$

- This is a NONLINEAR integrable system

Nonlinear systems can be more stable!

- 1D systems: non-linear oscillations can remain stable under the influence of periodic external force perturbation. Example:

$$\ddot{z} + \omega_0^2 \sin(z) = a \sin(\omega_0 t)$$

- 2D: The resonant conditions

$$k\omega_1(J_1, J_2) + l\omega_2(J_1, J_2) = m$$

are valid only for certain amplitudes.

Nekhoroshev's condition guaranties detuning from resonance and, thus, stability.

Russian Math. Surveys 32:6 (1977), 1–65
From Uspekhi Mat. Nauk 32:6 (1977), 5–66

N. N. Nekhoroshev

AN EXPONENTIAL ESTIMATE OF THE TIME OF STABILITY OF NEARLY-INTEGRABLE HAMILTONIAN SYSTEMS

Example of a "good" nonlinear system

- Suppose that

$$\frac{d^2 z_n}{d\psi^2} + \omega^2 z_n + \alpha z_n^3 = 0, \quad \text{where } z_n \text{ is } x_n \text{ or } y_n$$

- This would be a nonlinear equivalent to strong focusing

- We do NOT know how realize this particular example in practice!

On the way to integrability

1) Colliding beams:

   a) Round beam – angular momentum conservation- 1D motion in *r* (Novosibirsk, 80's, realized at VEPP2000, tune shift around 0.15 achieved);

   b) Crab waist - decoupling x and y motion (P. Raimondi (2006), tune shift 0.1 achieved at DAΦNE).

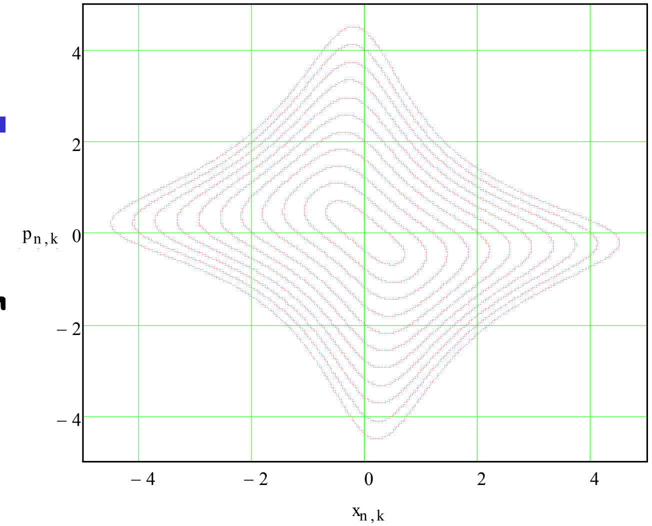
   c) Working point close to integer

2) Numerical methods to eliminate resonances (e.g. J. Cary and colleagues; D. Robin, W. Wan and colleagues);

3) Exact solutions for realization– our goal. The list is presented in next slides

   Major limiting factor: fields must satisfy Maxwell eqtns.

1-D nonlinear optics



- In 1967 E. McMillan published a paper

SOME THOUGHTS ON STABILITY
IN NONLINEAR PERIODIC FOCUSING SYSTEMS

Edwin M. McMillan

September 5, 1967

- Final report in 1971. This is what later became known as the "McMillan mapping":

$$x_i = p_{i-1}$$

$$p_i = -x_{i-1} + f(x_i)$$

$$f(x) = -\frac{Bx^2 + Dx}{Ax^2 + Bx + C}$$

$$Ax^2p^2 + B\left(x^2p + xp^2\right) + C\left(x^2 + p^2\right) + Dxp = \text{const}$$

If $A = B = 0$ one obtains the Courant-Snyder invariant

- Generalizations (Danilov, Perevedentsev, 1992-1995)

2D case with realistic fields

1. The 1-D McMillan mapping was extended to 2-D round thin lens by Danilov, Perevedentsev (1995)

Round lenses can be realized only with charge distributions:

a) 1 or 2 thin lenses with radial kicks $f_1(f_2)(r) = \dfrac{ar}{br^2 + c_1(c_2)}$

b) Time dependent potential $\dfrac{1}{\beta}U(\dfrac{r}{\sqrt{\beta}})$.

2. Approximate cases – J. Cary & colleagues - decoupling of x-y motion and use of 1D solutions;

3. Stable integrable motion without space charge in Laplace fields – the only known exact case is IOTA case (Danilov, Nagaitsev, *PRSTAB* 2010);

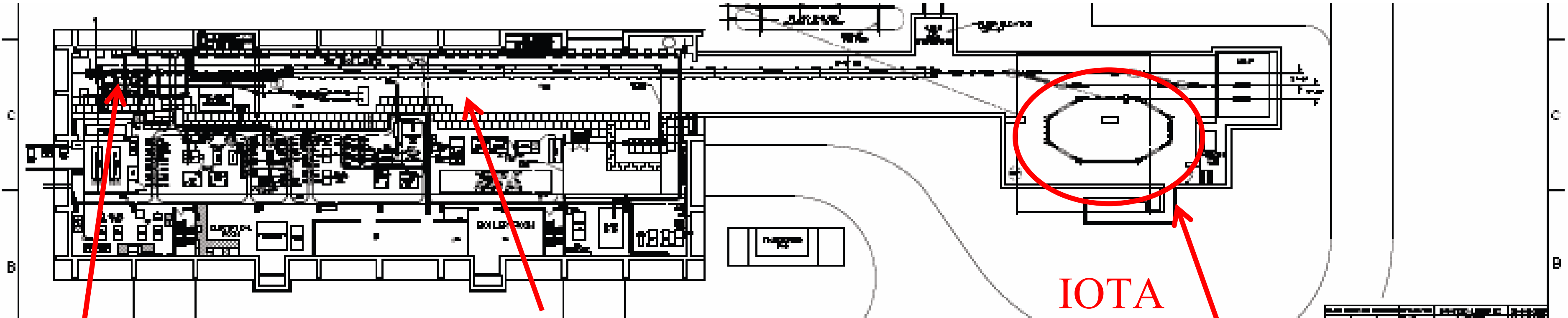
Choice of nonlinear elements

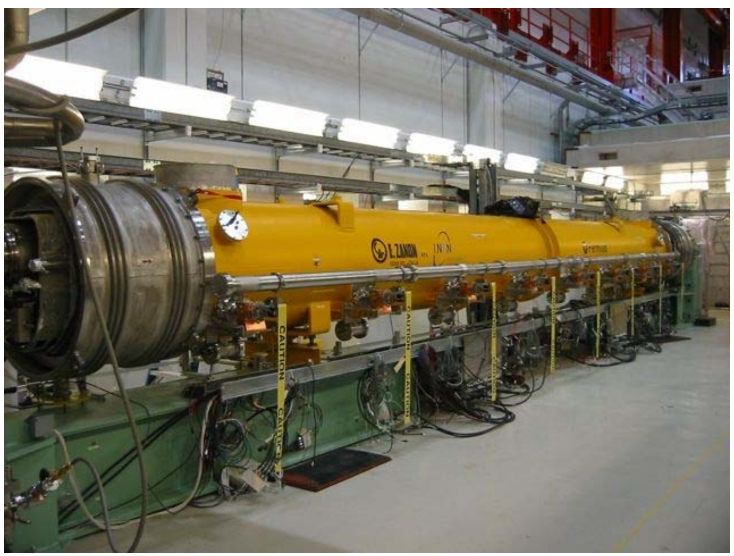
1) For large beam size accumulators and boosters – external fields can produce large frequency spread at beam size amplitudes;

2) Small beam size colliders need nonlinearities on a beam-size scale. The ideal choice is colliding beam fields (like e-lens in Tevatron)

- The IOTA ring can test both variants of nonlinearities.

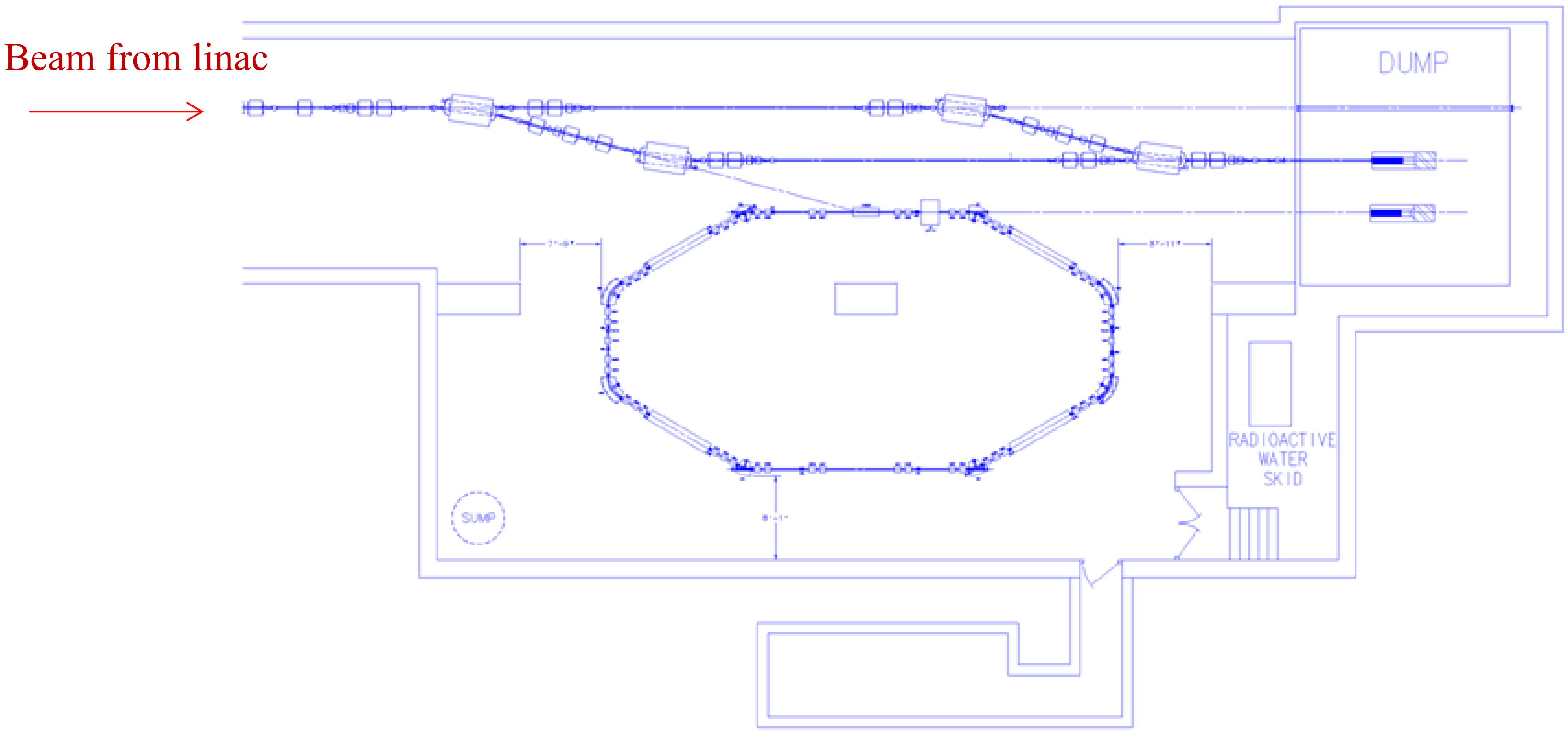
Advanced Superconductive Test Accelerator at Fermilab



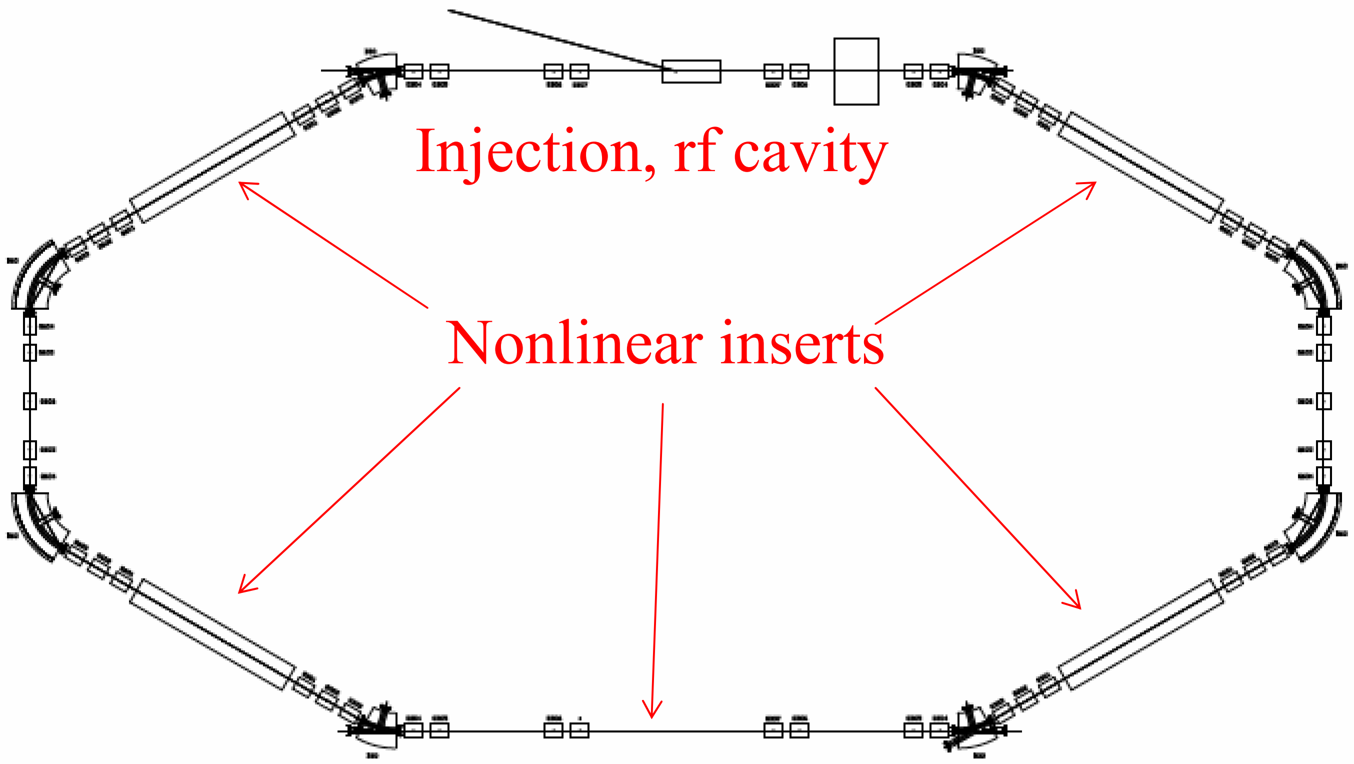


Layout

Beam from linac



IOTA schematic



- pc = 150 MeV, electrons (single bunch, 10^9)
- ~36 m circumference
- 50 quadrupoles, 8 dipoles, 50-mm diam vac chamber
- hor and vert kickers, 16 BPMs

Why electrons?

- Small size (~50 um), pencil beam
- Reasonable damping time (~1 sec)
- No space charge

- In all experiments the electron bunch is kicked transversely to "sample" nonlinearities. We intend to measure the turn-by-turn BPM positions as well as synch light to obtain information about phase space trajectories.

Proposed experiments

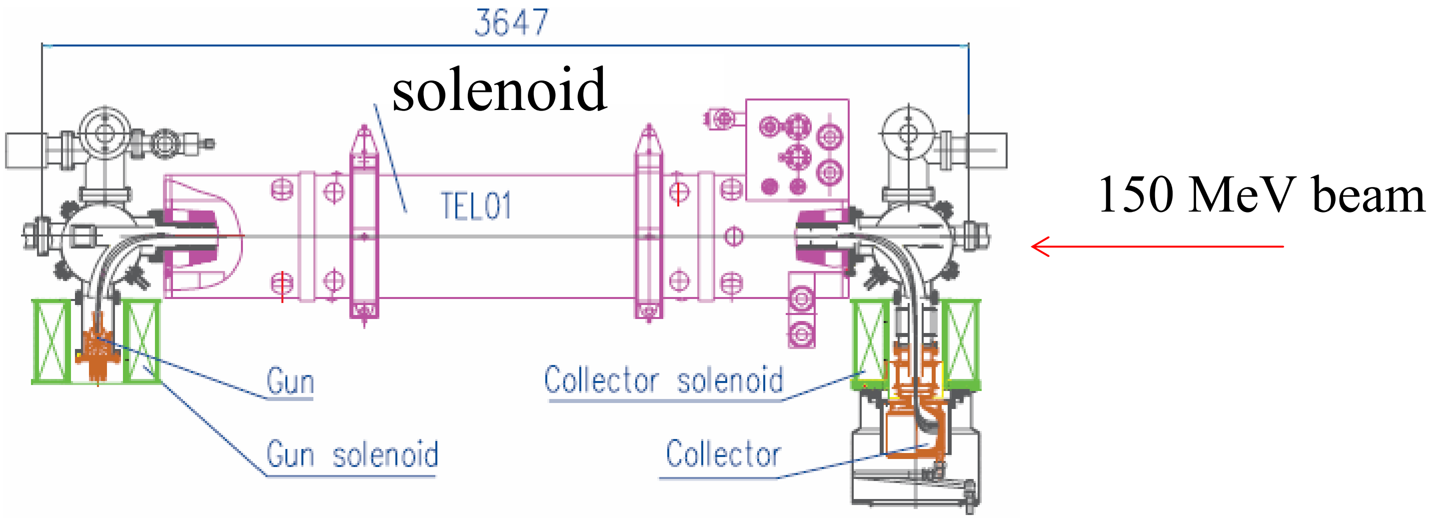
- We are proposing several experiments with nonlinear lenses
  - Based on the electron (charge column) lens

$$\nabla^2 U \neq 0$$

  - Based on electromagnets

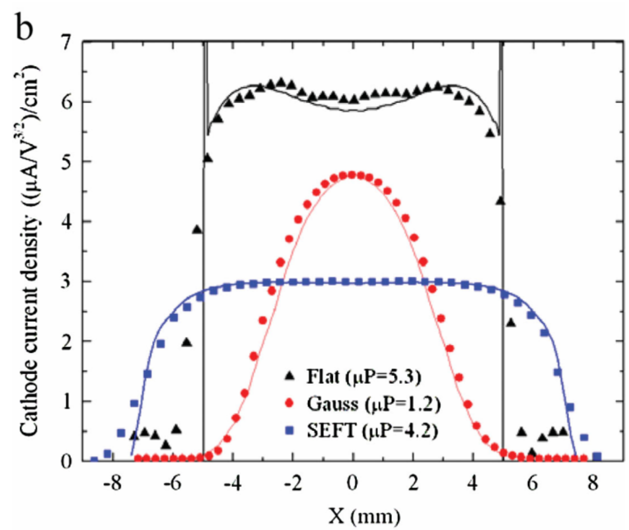
$$\nabla^2 U = 0$$

Experiments with electron lens



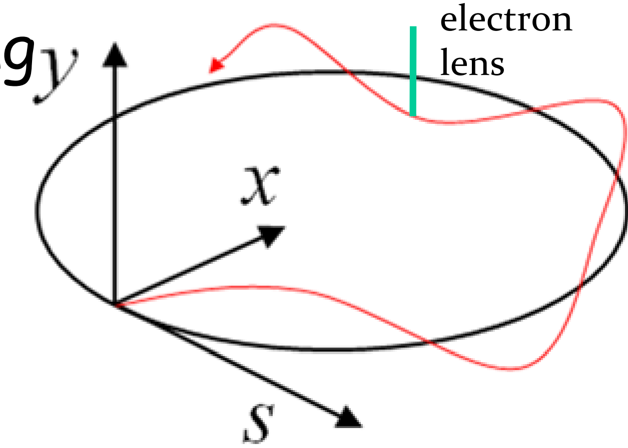
Example: Tevatron electron lens

- For IOTA ring, we would use a 5-kG, ~1-m long solenoid
- Electron beam: ~0.5 A, ~5 keV, ~1 mm radius



Experiment with a thin electron lens

- The system consists of a thin (L < β) nonlinear lens (electron beam) and a linear focusing ring



- Axially-symmetric thin McMillan lens:

$$\theta(r) = \frac{kr}{ar^2 + 1}$$

  - Electron lens with a special density profile

- The ring has the following transfer matrix

$$\begin{pmatrix} cI & sI \\ -sI & cI \end{pmatrix} \begin{pmatrix} 0 & \beta & 0 & 0 \\ -\frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\frac{1}{\beta} & 0 \end{pmatrix}$$

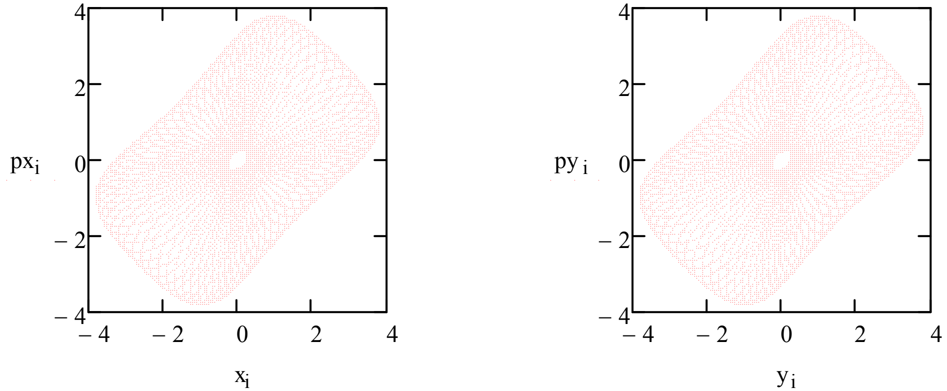
$$c = \cos(\phi)$$

$$s = \sin(\phi)$$

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Electron lens (McMillan – type)

- The system is integrable. Two integrals of motion (transverse):
  - Angular momentum: $xp_y - yp_x = const$
  - McMillan-type integral, quadratic in momentum



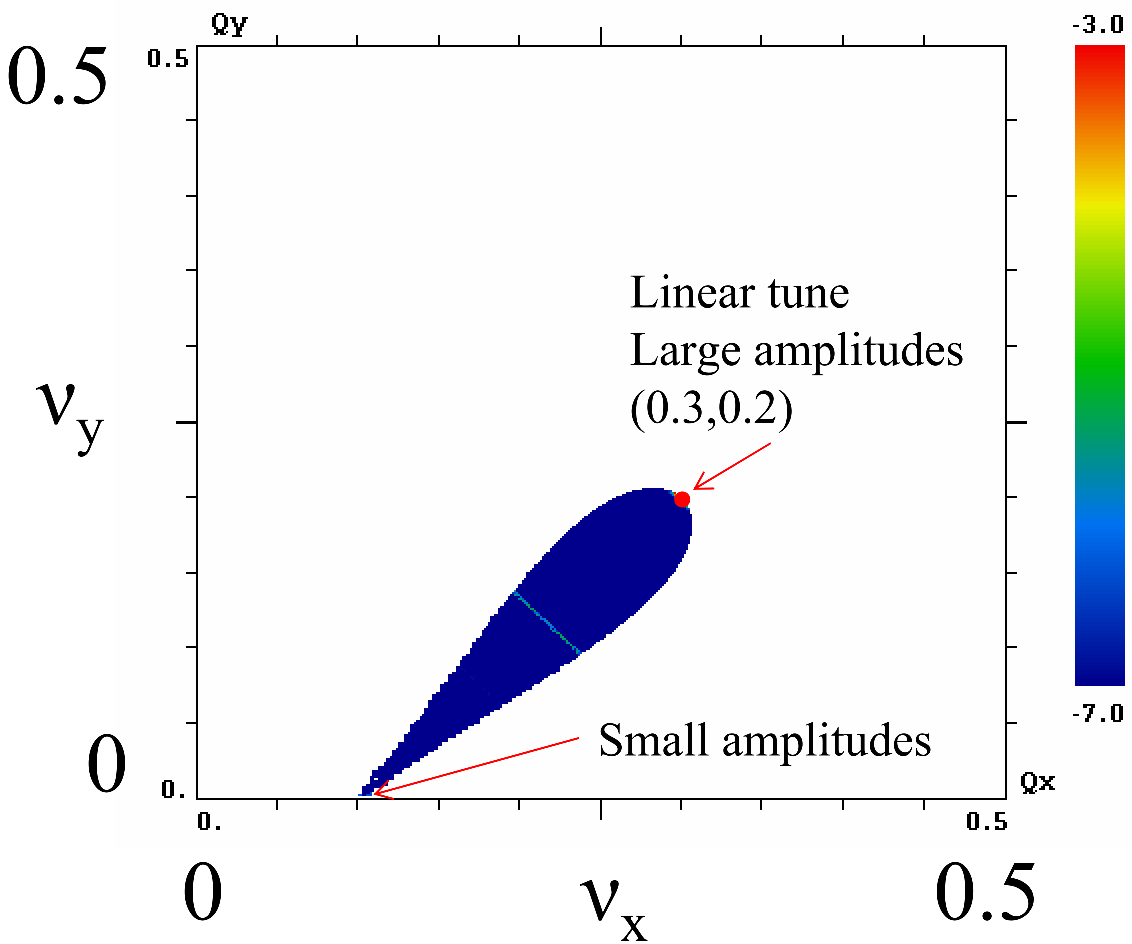
Electron lens current density:

$$n(r) \propto \frac{I}{\left(ar^2+1\right)^2}$$

- For large amplitudes, the fractional tune is 0.25
- For small amplitude, the electron (defocusing) lens can give a tune shift of ~-0.3
- Potentially, can cross an integer resonance

Ideal McMillan round lens

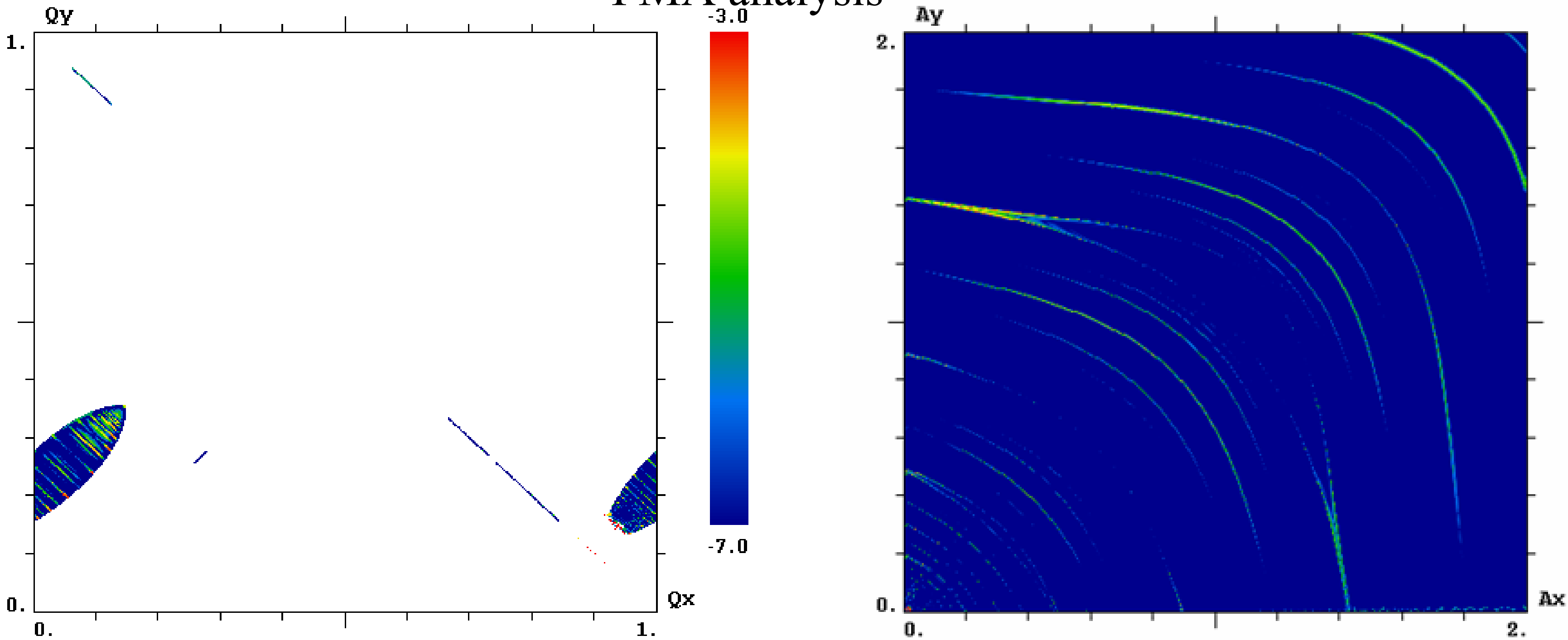
FMA fractional tunes



Practical McMillan round lens

e-lens (1 m long) is represented by 50 thin slises. Electron beam radius is 1 mm. The total lens strength (tune shift) is 0.3

FMA analysis



All excited resonances have the form $k \cdot (\nu_x + \nu_y) = m$

They do not cross each other, so there are no stochastic layers and diffusion

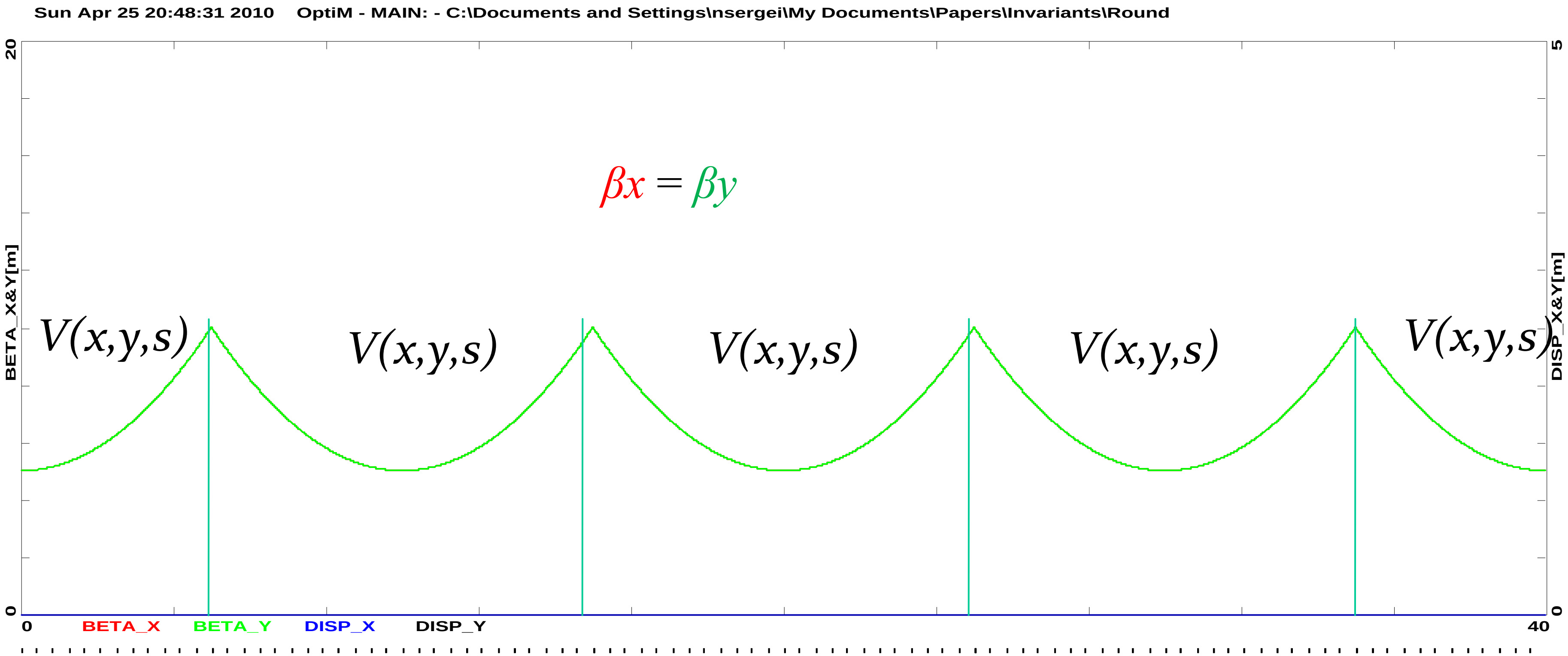
Experiments with nonlinear magnets

See: Phys. Rev. ST Accel. Beams 13, 084002 (2010)

Start with a round axially-symmetric LINEAR focusing lattice (FOFO)

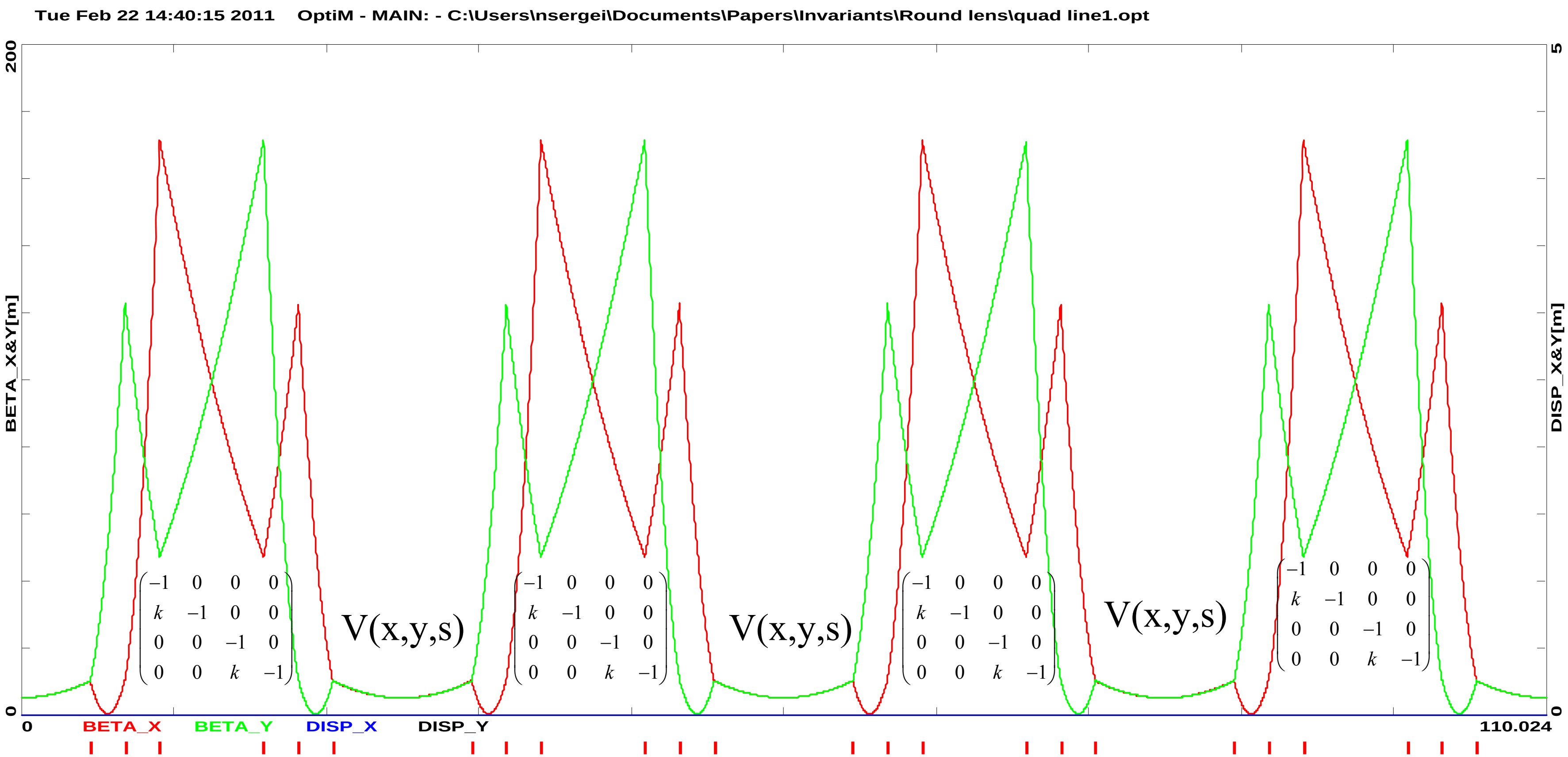
Add special non-linear potential $V(x,y,s)$ such that

$$\Delta V(x,y,s) \approx \Delta V(x,y) = 0$$



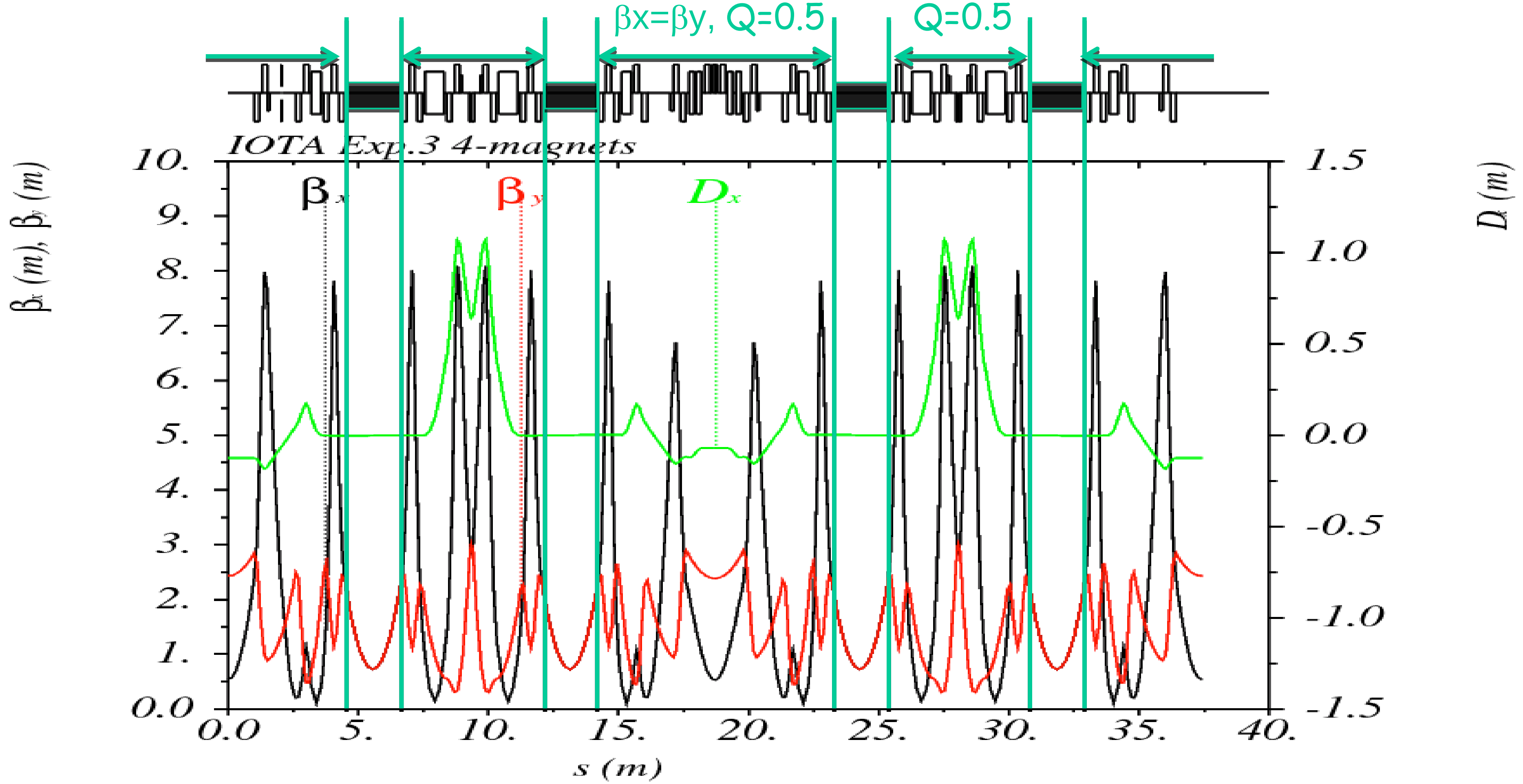
Fake thin lens inserts

Example only!



4-Magnet Lattice (Exp. 3-4)

- Equal beta-functions, $Qx=5.0+0.3\times4$, $Qy=4.0+0.3\times4$
- Dispersion=0 in the Nonlinear Magnet
- Maximum Vertical amplitude in the NM=11 mm
- $\alpha=0.015$



Main ideas

1. Start with a time-dependent Hamiltonian:

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + \beta(\psi) V\left(x_N \sqrt{\beta(\psi)}, y_N \sqrt{\beta(\psi)}, s(\psi)\right)$$

2. Chose the potential to be time-independent in new variables

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N)$$

3. Find potentials *U(x, y)* with the second integral of motion and such that $\Delta U(x, y) = 0$

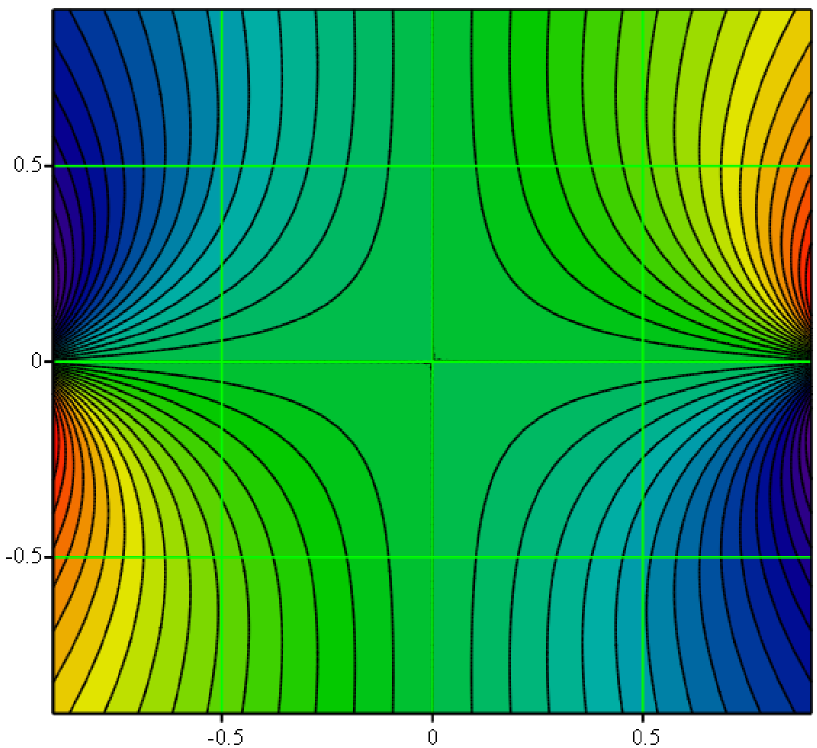
Nonlinear integrable lens

$$H = \frac{p_x^2 + p_y^2}{2} + \frac{x^2 + y^2}{2} + U(x, y)$$

This potential has two adjustable parameters: t – strength and c – location of singularities

Multipole expansion :

For $|z| < c$ $\quad U(x,y) \approx \frac{t}{c^2}\,\mathrm{Im}\left((x+iy)^2 + \frac{2}{3c^2}(x+iy)^4 + \frac{8}{15c^4}(x+iy)^6 + \frac{16}{35c^6}(x+iy)^8 + \ldots \right)$



For $c = 1$
$|t| < 0.5$ to provide linear stability for small amplitudes

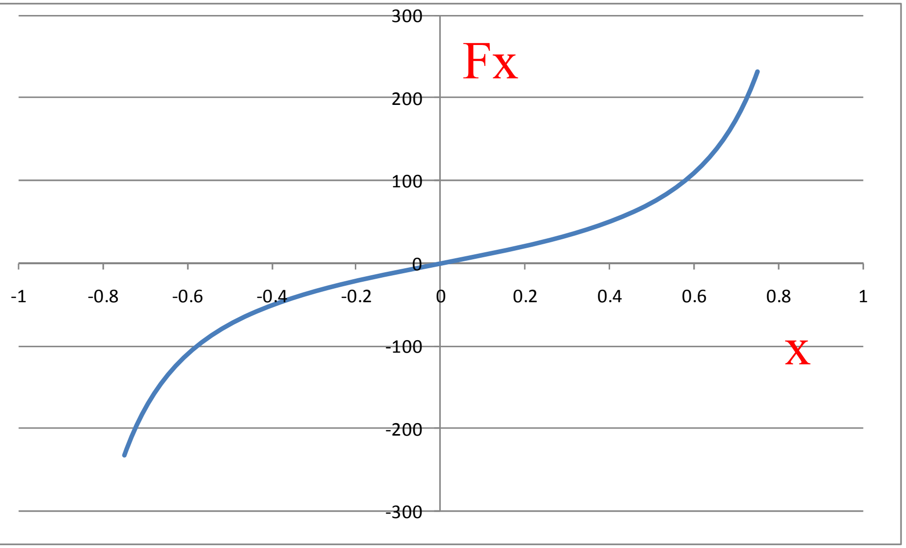
For $t > 0$ adds focusing in x

Small-amplitude tune s:

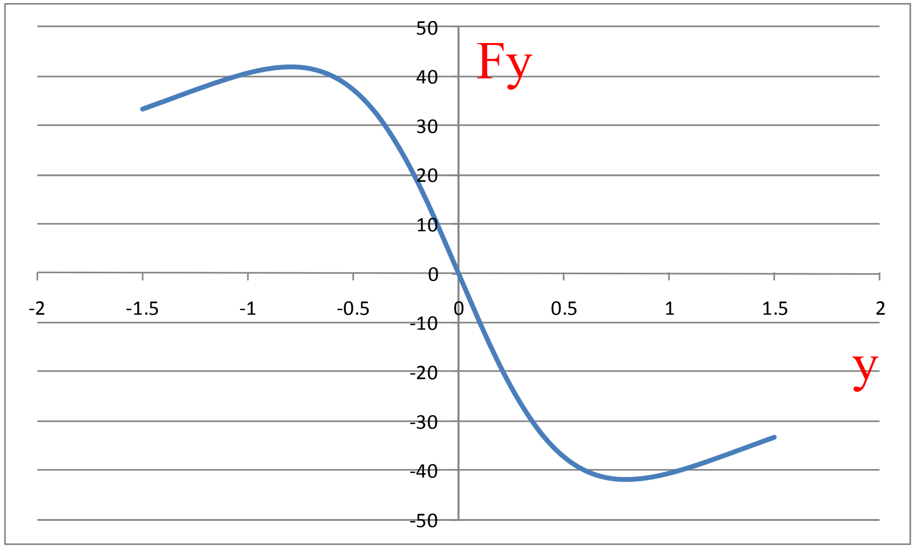
$$\nu_1 = \sqrt{1 + 2t}$$
$$\nu_2 = \sqrt{1 - 2t}$$

Transverse forces

Focusing in x

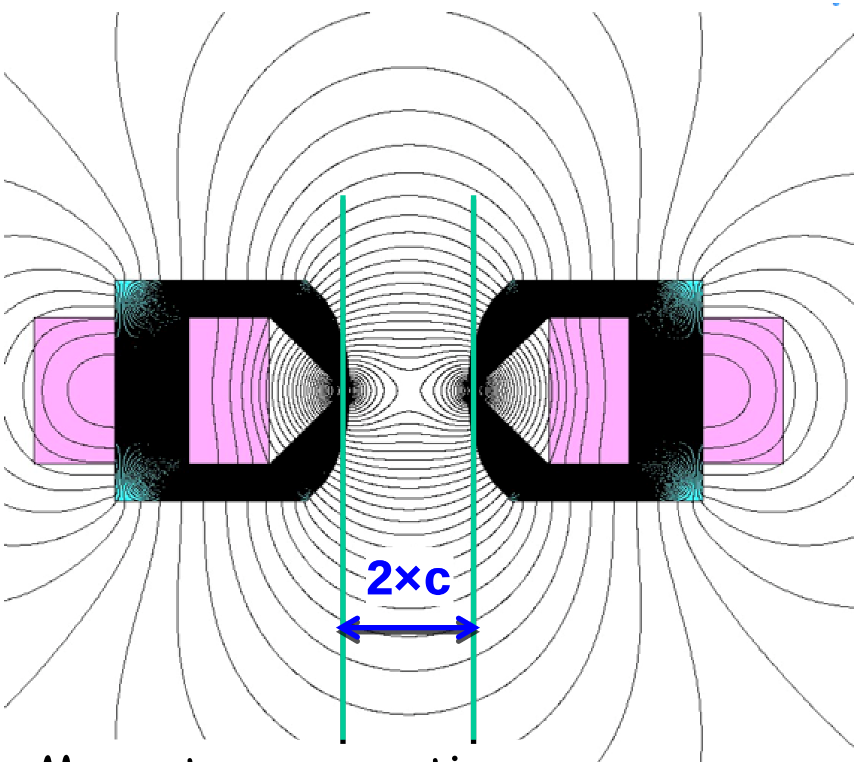


Defocusing in y



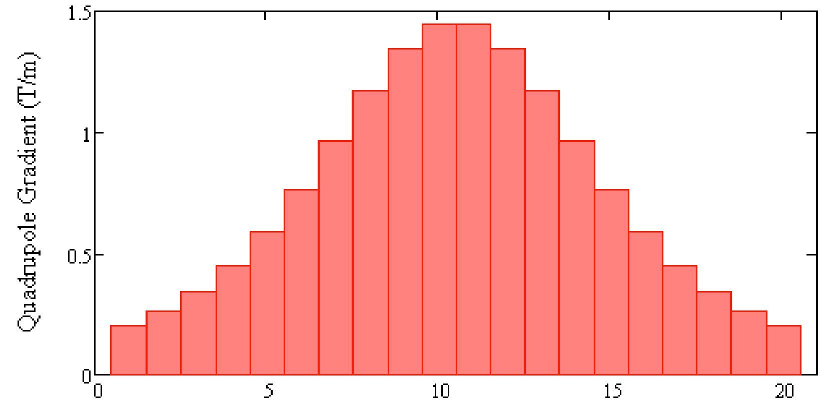
Nonlinear Magnet

- Practical design – approximate continuously-varying potential with constant cross-section short magnets

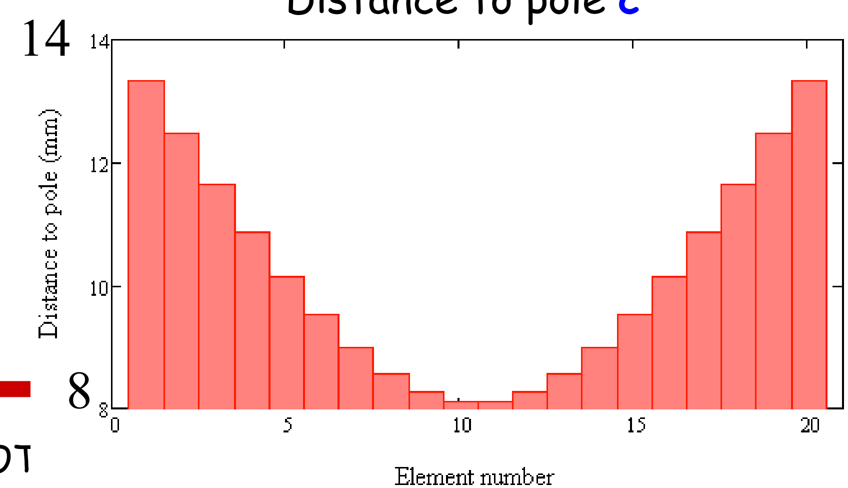


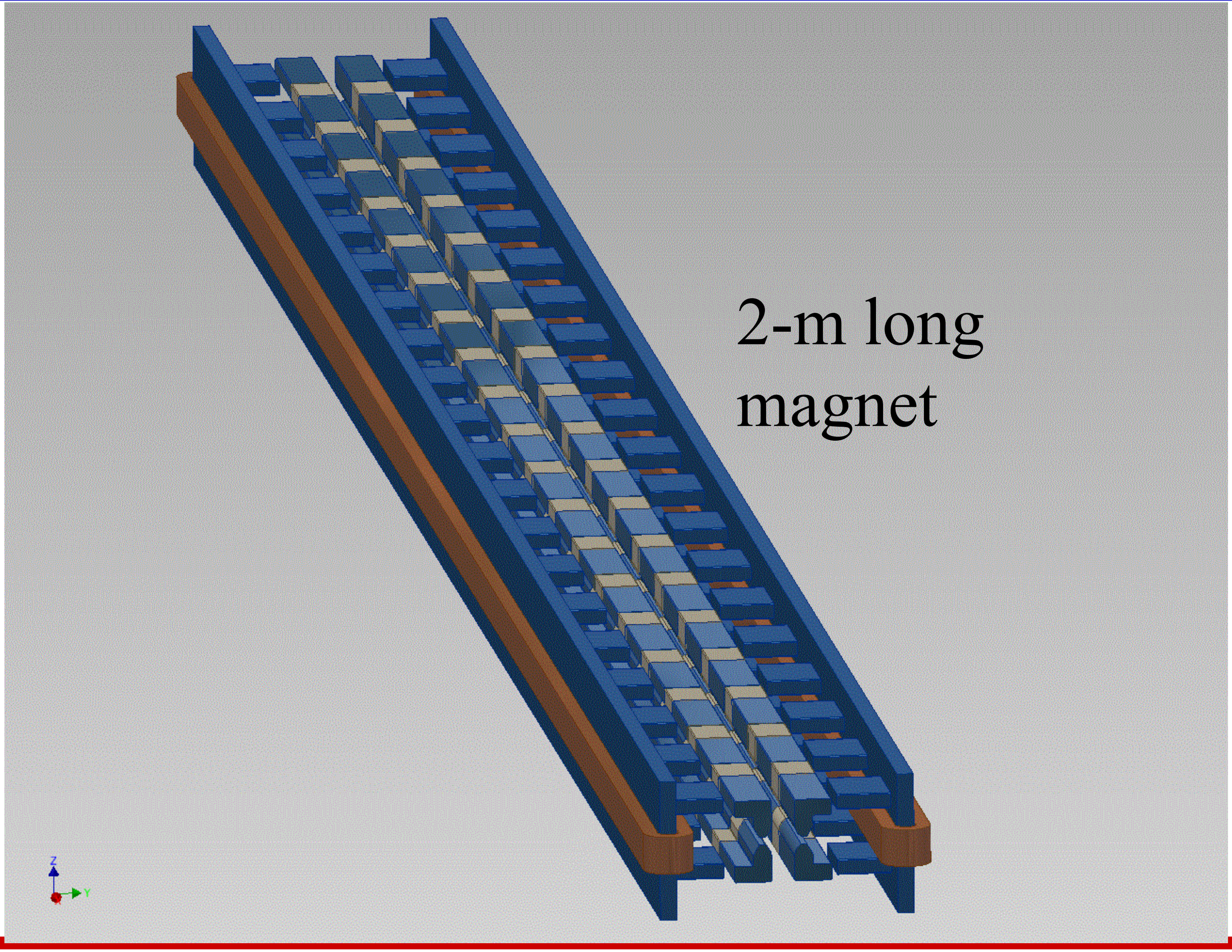
Magnet cross section
V.Kashikhin

Quadrupole component of nonlinear field

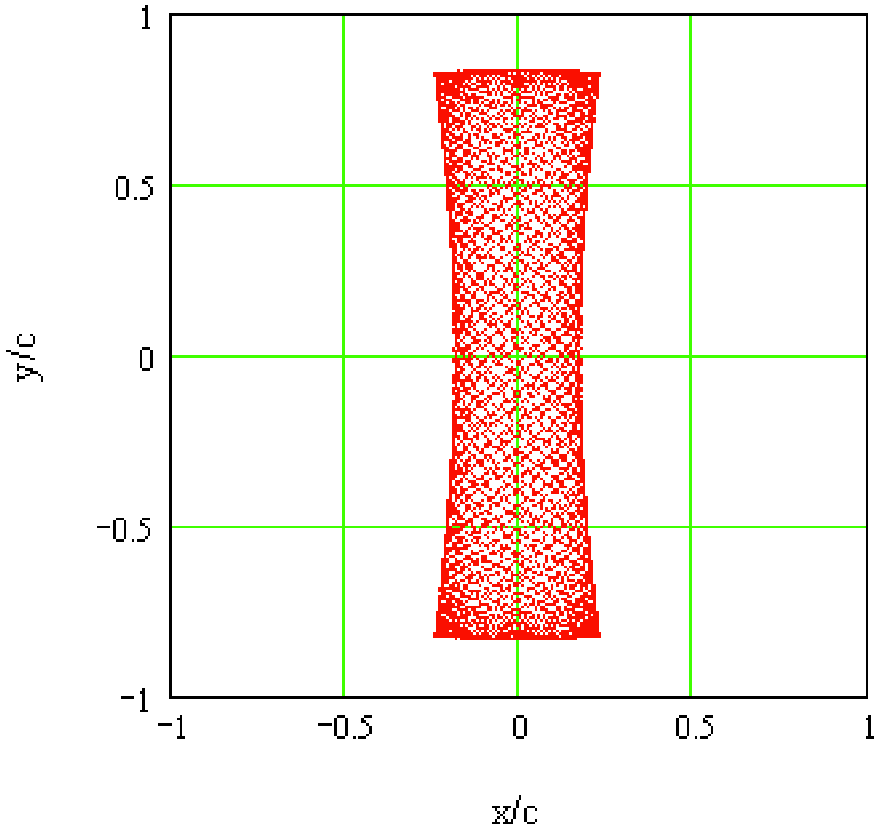


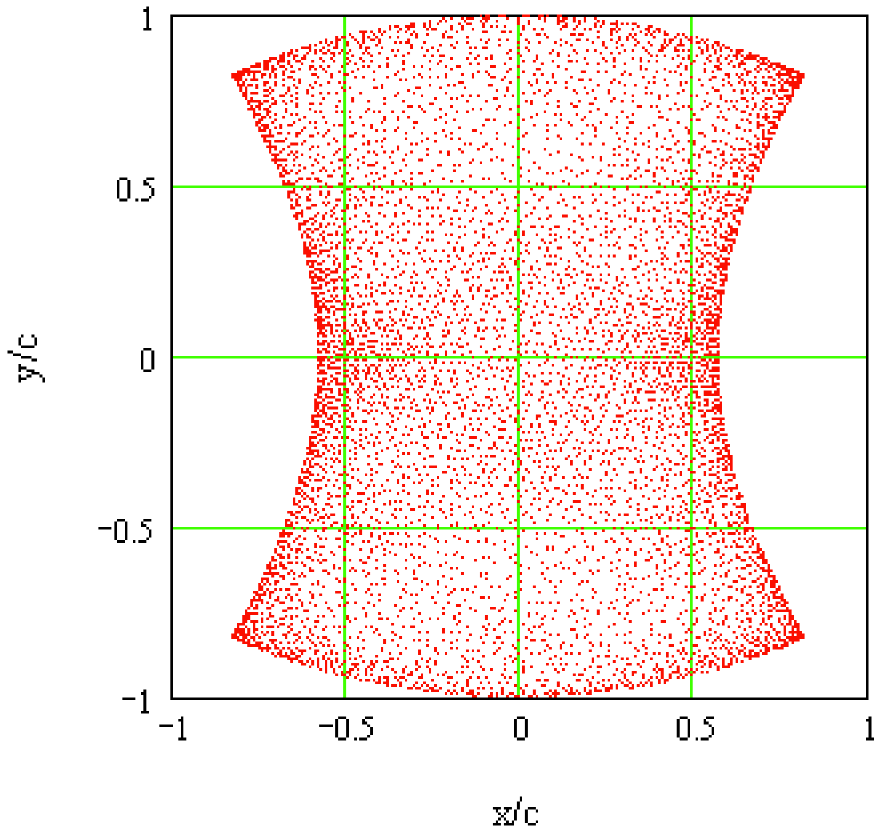
Distance to pole c




2-m long
magnet

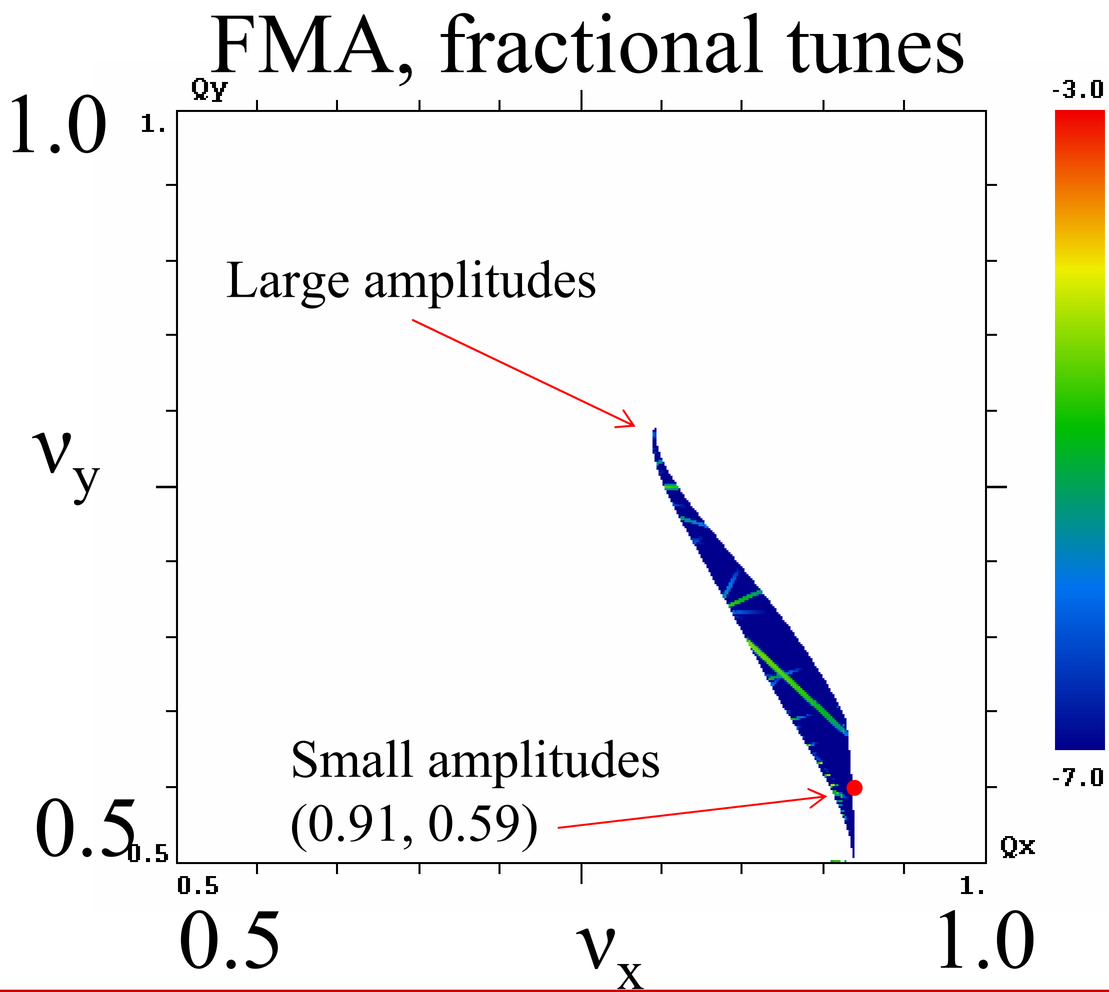
Examples of trajectories





Ideal nonlinear lens

- *A single 2-m long nonlinear lens creates a tune spread of ~0.25.*



Experimental goals with nonlinear lenses

- ***Overall goal is to demonstrate the possibility of implementing nonlinear integrable optics in a realistic accelerator design***

- Demonstrate a large tune shift of ~1 (with 4 lenses) without degradation of dynamic aperture
  - minimum 0.25

- Quantify effects of a non-ideal lens

- Develop a practical lens design.

Summary

- We have found first (practical) examples of completely integrable non-linear optics.

- The Integrable Optics Test Accelerator (IOTA) ring is now under construction. Completion expected in 2014.
  - Dipoles, vacuum chambers ordered
  - Quadrupoles exist (JINR)
  - Power supplies – reuse from Tevatron complex

- The ring can also accommodate other Advanced Accelerator R&D experiments and/or users
  - Current design accommodates Optical Stochastic Cooling

Collaboration

- Fermilab
- SNS
- Budker INP
- JINR (Dubna)
- BNL
- JAI (Oxford)
- Tech X

Extra slides

Demonstration of halo suppression

- In 2D, with space charge, mismatch can create a halo
  - this leads to 'breathing mode' or mismatch oscillations
  - particle-core halo model → halo forms [Wangler, Gluckstern, Fedotov, others]
  - adding a low-density "pre-halo" with correct matching → rapid formation

D.L. Bruhwiler, "Lowest-order phase space structure of a simplified beam halo Hamiltonian," AIP Conf. Proc. **377**, p. 219 (1996).

- PyORBIT sim's confirm rapid halo growth for linear lattice
  - predicted mismatch threshold $m = abs(r_{mismatched}/r_{matched} - 1)$ $m_{critical} \approx 0.25$
  - PyORBIT indicates $m_{critical} \approx 0.23$

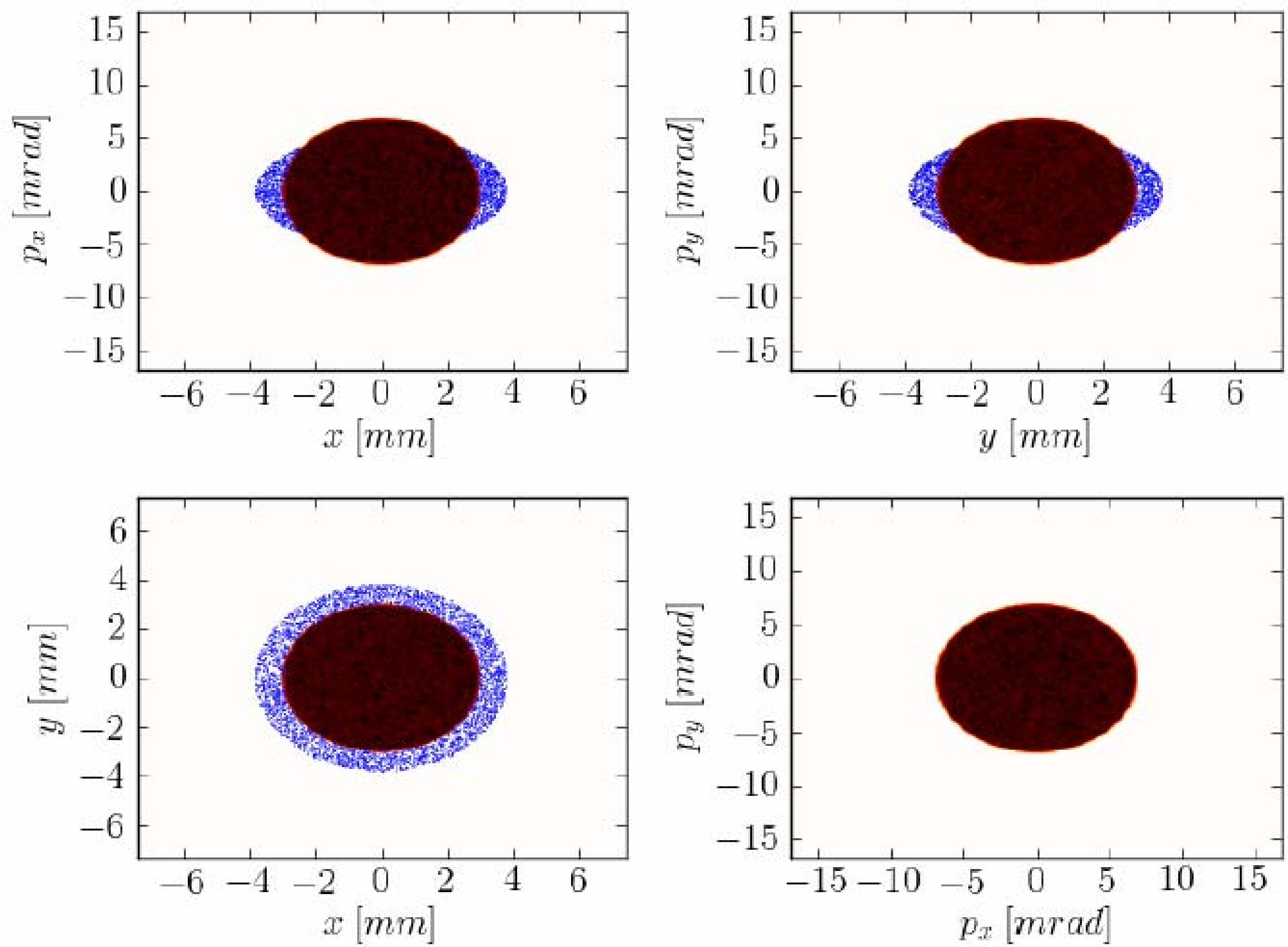
- Nonlinear lattices show suppression of halo
  - ➢ amplitude-dependent tune spread yields "nonlinear decoherence"
  - ➢ emittance growth is seen as edges of beam are smeared
  - ➢ final density is no-longer uniform in x-y space nonlinear space charge

System: linear FOFO; 100 A; linear KV w/ mismatch

Result: quickly drives test-particles into the halo

\# passes

0

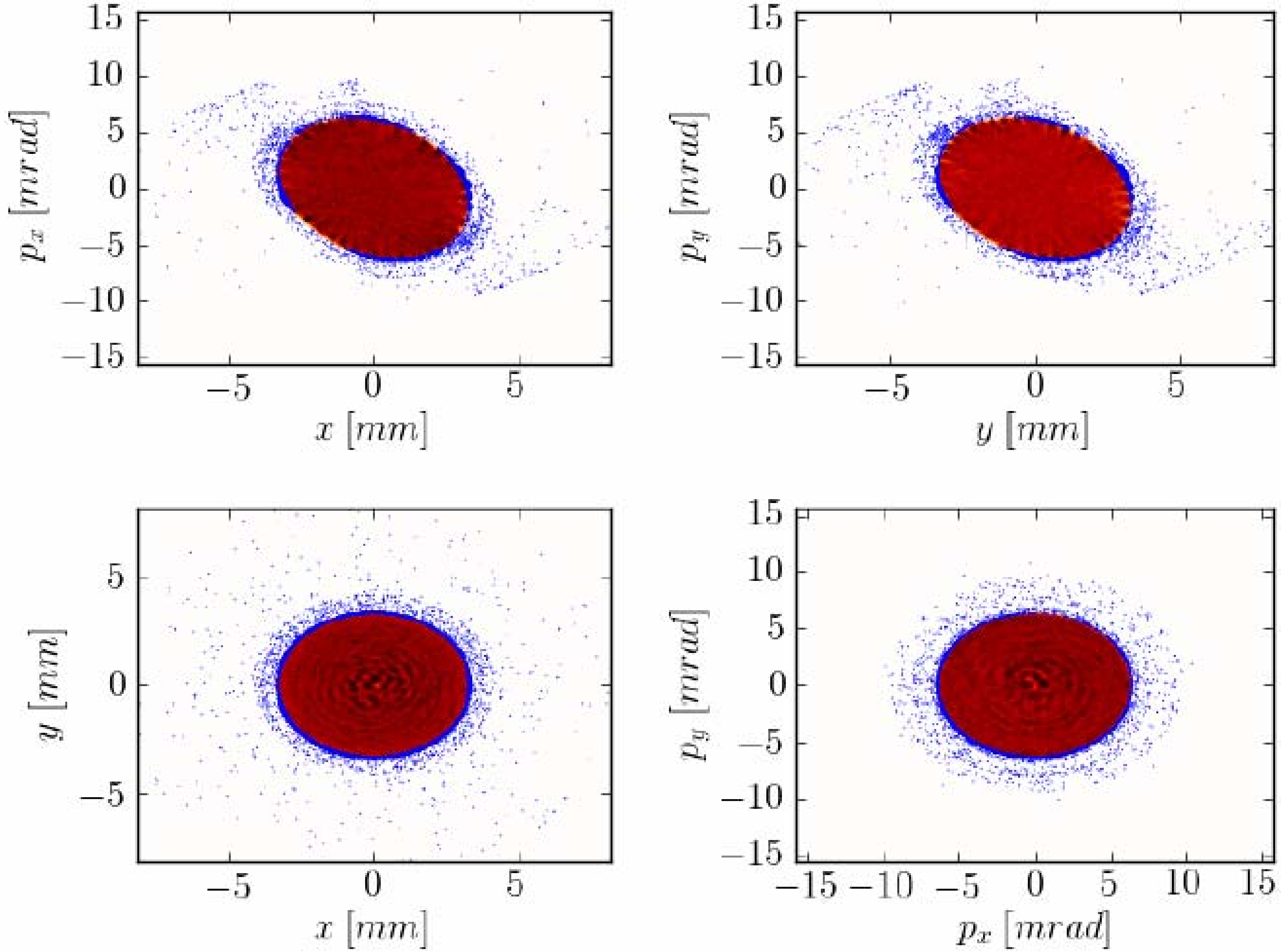


The beam core (red contours) is mismatched; the beam halo (blue dots) has 100x lower density.

System: linear FOFO; 100 A; linear KV w/ mismatch

Result: quickly drives test-particles into the halo

passes
500



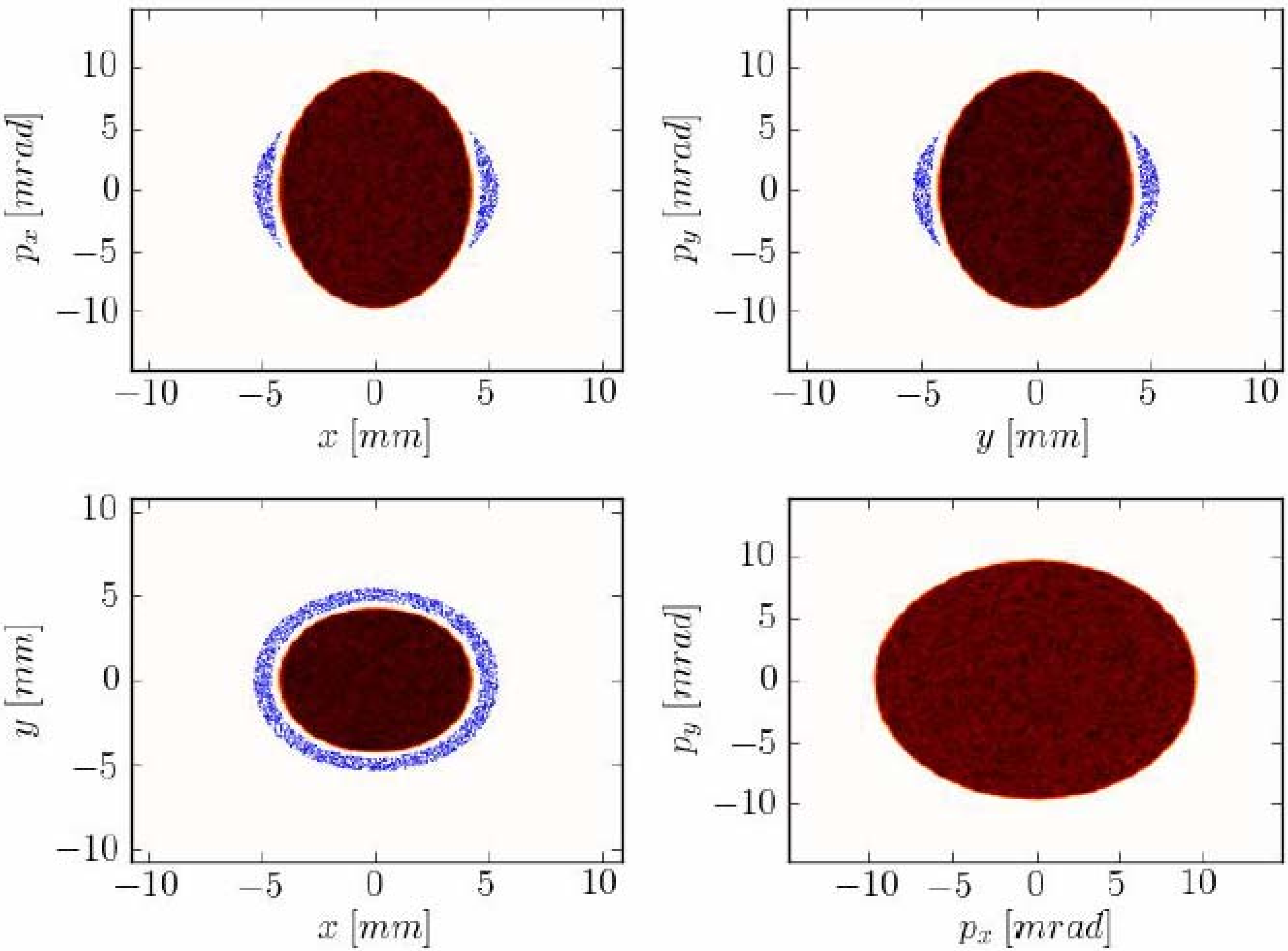
The beam core (red contours) is mismatched; the beam halo (blue dots) has 100x lower density.

System: octupoles; 100 A; generalized KV w/ mismatch

Result: nonlinear decoherence suppresses halo

passes

0



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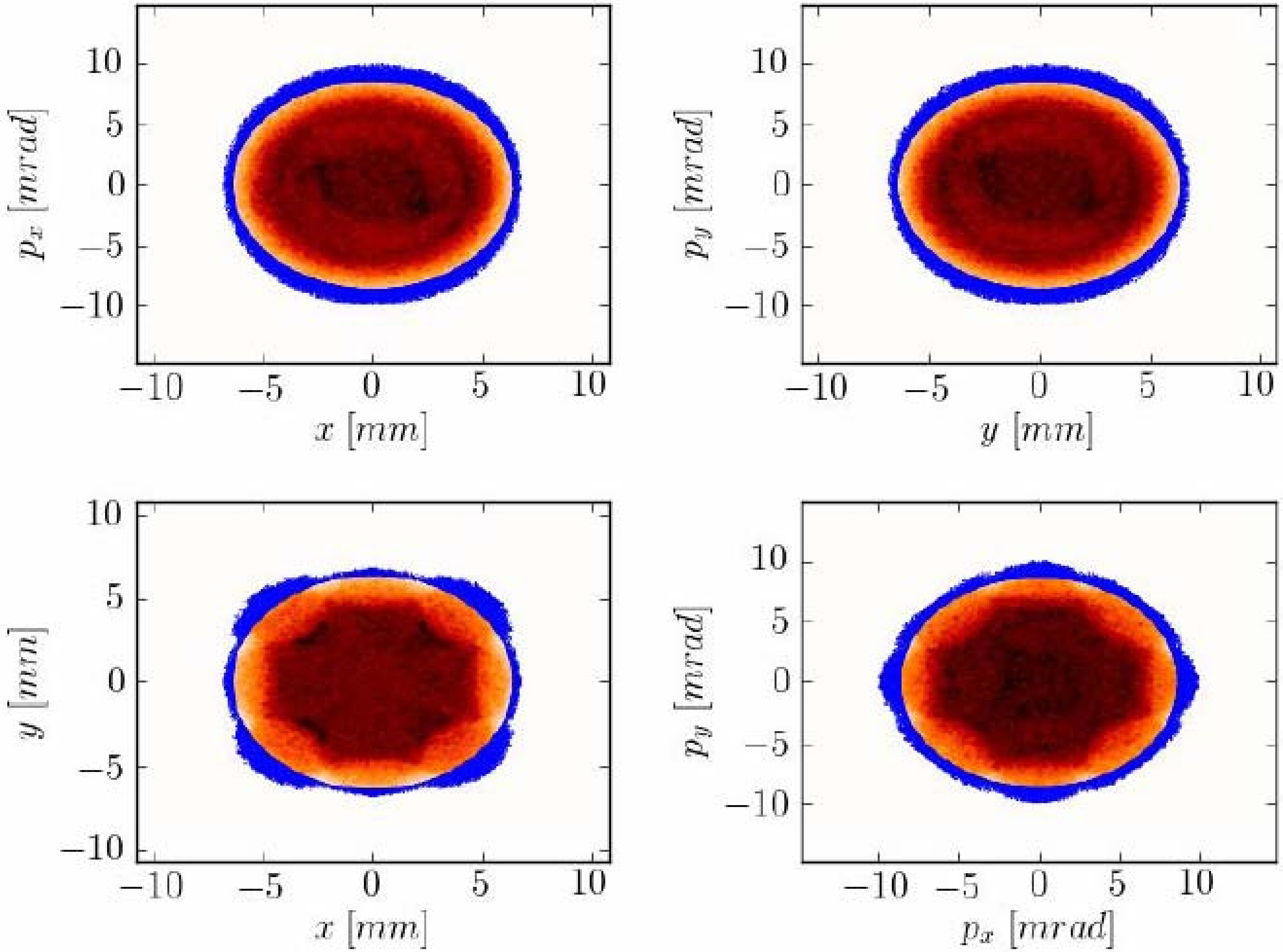
System: octupoles; 100 A; generalized KV w/ mismatch

Result: nonlinear decoherence suppresses halo

passes
500



The beam core (red contours) is mismatched; the beam halo (blue dots) has 100x lower density.