Generation of High-Energy Photons with Large ORBITAL Angular Momentum

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4.b. General case

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This report is based mainly on recent papers:

[1] U.D. Jentschura, V.G. Serbo "Generation of High–Energy Photons with Large Orbital Angular Momentum by Compton Backscattering", Phys. Rev. Lett. 106 (2011) 013001

[2] U.D. Jentschura, V.G. Serbo "Compton Upconversion of Twisted Photons: Backscattering of Particles with Non-Planar Wave Functions", Eur. Phys. Journ. C 71 (2011) 1571

[3] I.P. Ivanov, V.G. Serbo "Scattering of twisted particles: extension to wave packets and orbital helicity", Phys. Rev. A 84 (2011) 033804 and arXive:1105.5575v2 [hep-ph]

1. Introduction

An interesting research direction in modern optics is related to experiments with so-called **"twisted photons".**

These are states of the laser beam whose photons have a defined value $\hbar m$ **of the ORBITAL angular momentum projection** on the beam propagation axis where m is a (large) integer

L. Allen et al., Phys. Rev. A 45, 8185 (1992);

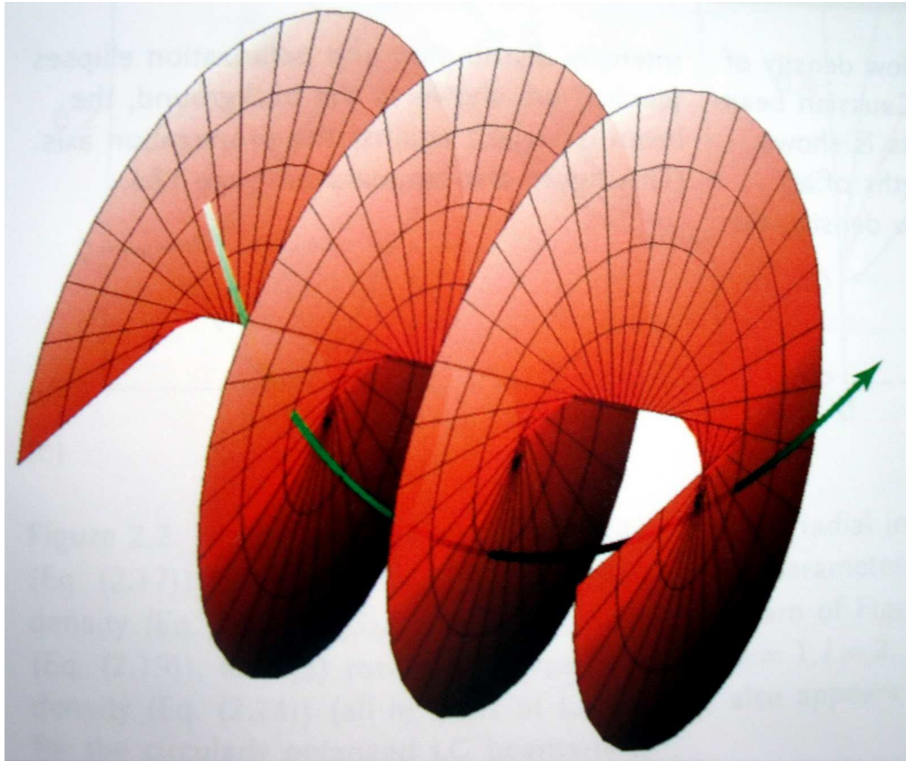
S. Franke-Arnold, L. Allen, M. Padgett, Laser and Photonics Reviews 2, 299 (2008);

A.M. Yao, M.J. Padgett, Advances in Optics and Photonics 3, 161 (2011).

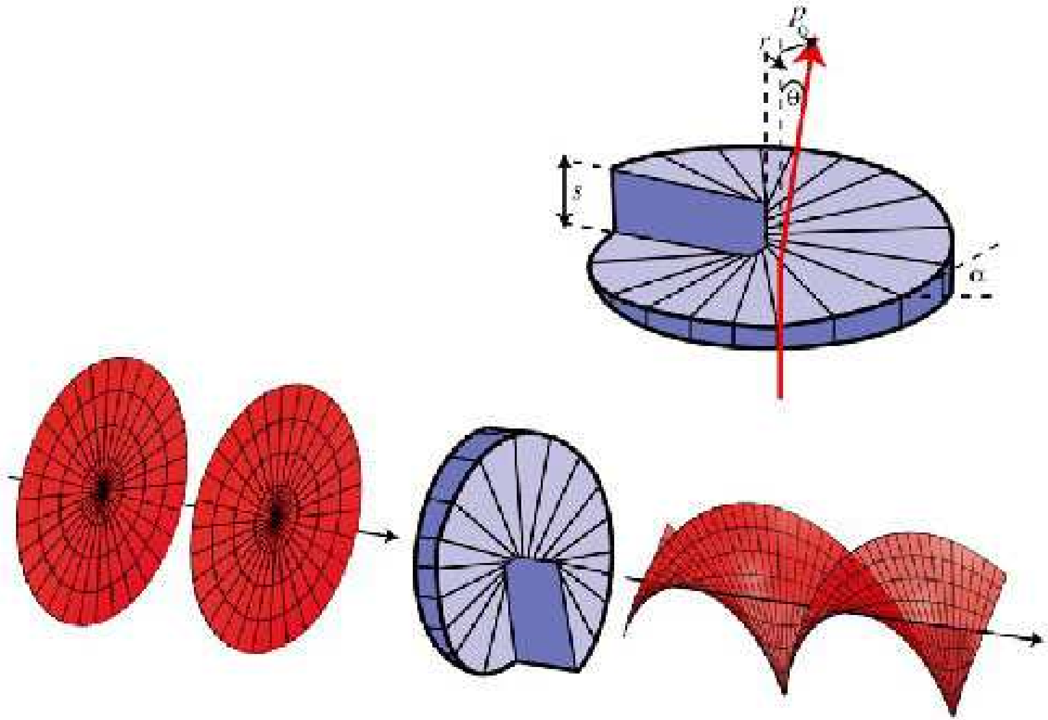
An experimental realization exists for states with projections as large as $m = 200$

J. E. Curtis, B. A. Koss, and D. G. Gries, Opt. Commun. **207**, 169 (2002).

The wavefront of such states **rotates around the propagation axis**, and their Poynting vector looks like **a corkscrew**:

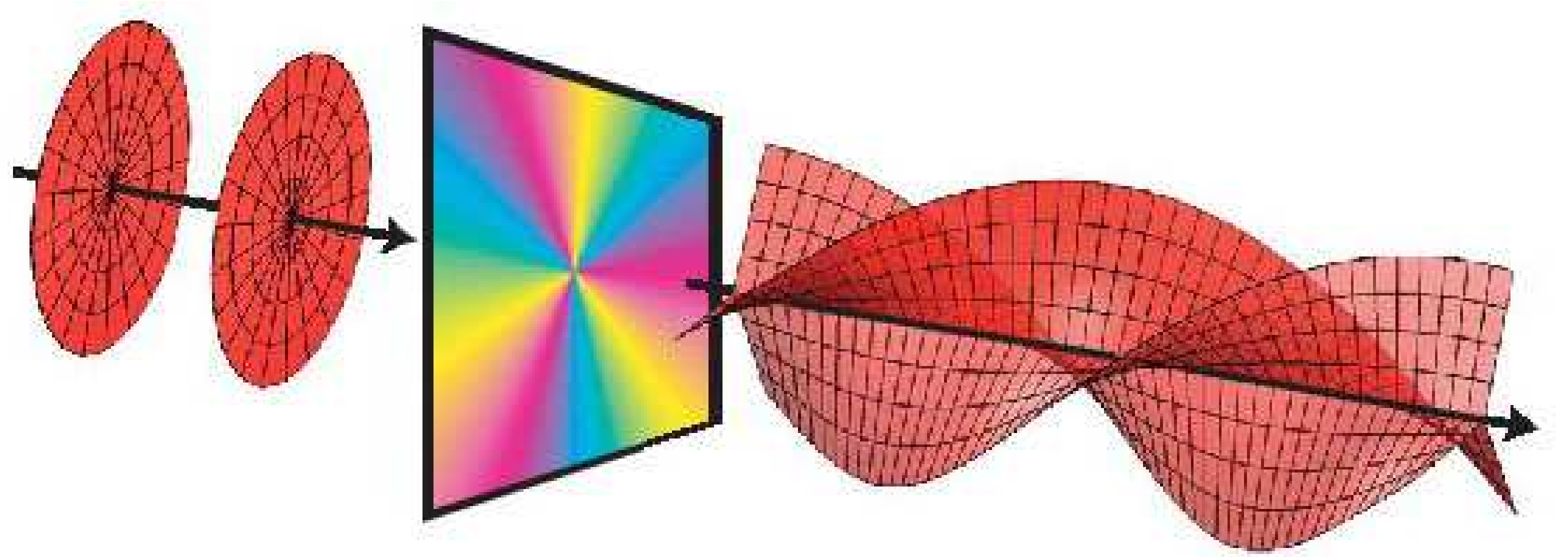


Such photons can be created, for example, from usual laser beams using **spiral phase plate:**



A spiral phase plate can generate a helically phased beam from a Gaussian. In this case $\ell = 0 \to \ell = 2$.

or a **numerically computed hologram:**



A helical phase profile $\exp(i\ell\phi)$ converts a Gaussian laser beam into a helical mode whose wave fronts resemble an ℓ-fold corkscrew. In this case $\ell = 3$.

Some examples of applications of such photons:

See quite a new book

Twisted photons

(Applications of light with orbital angular momentum)

edd. by J. P. Torres and L. Torner

(Wiley-VCH Weinheim, Germany 2011)

[1.] Micro-machines — it was demonstrated that micron-sized Teflon and calcite "particles" start to rotate after absorbing twisted photons

[2.] Astrophysics — the observation of orbital angular momentum of light scattered by rotating black holes could be very instructive

[3.] Rotating atoms with light — rotating Bose-Einstein condensates

[4.] Spiral phase contrast microscopy

[5.] Optical torques in liquid crystals

[6.] Quantum information — quantum features in high-dimensional Hilbert spaces

Very recently several groups have reported successful creation of **twisted electrons**, first using phase plates

M. Uchida and A. Tonomura, Nature **464**, 737 (2010)

and then with computer-generated holograms

J. Verbeeck, H. Tian, P. Schlattschneider, Nature **467**, 301 (2010); B. J. McMorran et al, Science **331**, 192 (2011)

Such electrons carried the energy as high as 300 keV and the orbital quantum number up to $m = 75$.

It is very conceivable that when these electrons are injected into a linear electron accelerator, their energy can be boosted into the multi-MeV and even GeV region.

Such vortex beams can be manipulated and focused just as the usual electron beams, and very recently remarkable focusing of a vortex electron beam to the focal spot of **less than 0.12 nm in diameter** was achieved in the paper:

"Atomic scale electron vortices for nanoresearch"

Verbeeck, Schattschneider, Lazar, Stöger-Pollach, Löffler, Steiger-Thirsfeld, Van Tendeloo, Appl. Phys. Lett. **99**, 203109 (2011)

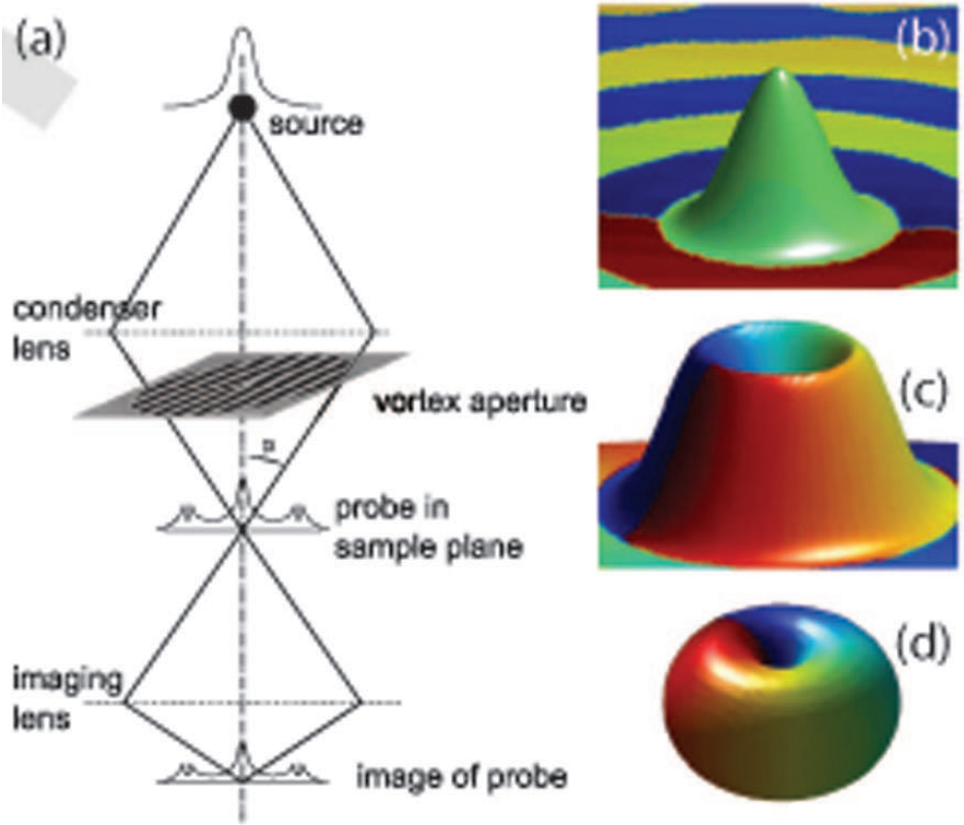


FIG. 1. (Color online) (a) Sketch of the setup to create focused vortex probes in a transmission electron microscope. The probe is formed in the sample plane and can be used to perform atomic resolution experiments in that plane. The probe is magnified for observation by the imaging system. The convergence angle α can be adjusted which allows to tune the size of the vortex. (b), (c) Artist impression of the intensity distribution for a conventional airy disc and a vortex beam with the same opening angle. (d) Sketch of the surface of a $2p_1$ orbital in nitrogen containing 80% of the electron density. The image is approximately to scale with (b), (c) for our experimental setting of $\alpha = 21.4$ mrad. Color coding indicates the phase distribution from 0 (blue) to 2π (red). Note the big similarity in both phase

In the papers [1–3] we had shown that it is possible to convert twisted photons from an energy range of about 1 eV to a higher energies of up to a hundred GeV using Compton backscattering off ultra-relativistic electrons.

In principle, Compton backscattering is **an established method** for the creation of high-energy photons and is used successfully in various application areas:

For example, in Novosibirsk Budker INP there is ROKK-2M device with the electron energy 5 GeV which produces **final photons with the energy up to 0.5 GeV** for the study of **photo-nuclear reactions** and for NQED experiments including of **the photon splitting** and Delbrúck scattering.

Another example — **projects of photon colliders** where high energy photon beams should be obtained by Compton backscattering of the intense laser beam on the bunches of high-energy electrons of linear e^+e^- collider.

However, the central question is how to treat Compton backscattering of twisted photons, whose field configuration is manifestly different from plane waves.

2. Twisted photons

Twisted scalar particle

Usual plane-wave state of a scalar particle with mass equal to zero has a defined 3-momentum $\boldsymbol{k}$, energy $\omega = |\boldsymbol{k}|$ and

$$\Psi_{\boldsymbol{k}}(t, \boldsymbol{r}) = \frac{\mathrm{e}^{-\mathrm{i}(\omega t - \boldsymbol{k}\boldsymbol{r})}}{\sqrt{2\omega}}, \tag{1}$$

(here and below $\hbar = 1$ and $c = 1$).

A twisted scalar particle has the following quantum numbers:
longitudinal momentum k_z,
absolute value of the transverse momentum $\varkappa$,
energy $\omega = |\boldsymbol{k}| = \sqrt{\varkappa^2 + k_z^2}$
and projection m of the **orbital angular momentum** onto the z axis:

$$\partial_\mu \partial^\mu \, \Psi_{\varkappa m k_z}(t, \boldsymbol{r}) = 0 \, , \tag{2}$$

$$\check{p}_z \, \Psi_{\varkappa m k_z} = k_z \, \Psi_{\varkappa m k_z} \, , \;\; \check{p}_z = -\mathrm{i} \frac{\partial}{\partial z}, \tag{3}$$

$$\check{L}_z \, \Psi_{\varkappa m k_z} = m \, \Psi_{\varkappa m k_z} \, , \quad \check{L}_z = -\mathrm{i} \frac{\partial}{\partial \varphi_r} \, . \tag{4}$$

Its evident form in cylindrical coordinates r, φ_r, z is

$$\Psi_{\varkappa m k_z}(r, \varphi_r, z, t) = \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_z z)}}{\sqrt{2\omega}}\, \psi_{\varkappa m}(r, \varphi_r)\,,$$

$$\psi_{\varkappa m}(r, \varphi_r) = \frac{\mathrm{e}^{\mathrm{i} m \varphi_r}}{\sqrt{2\pi}}\sqrt{\varkappa}\, J_m(\varkappa\, r)\,, \qquad (5)$$

where $J_m(x)$ is **the Bessel function**.

For small $r \ll 1/\varkappa$, the function $\psi_{\varkappa m}(r, \varphi_r)$ is of order of r^m, has a maximum at $r \sim m/\varkappa$ and then drops at large values $r \gg 1/\varkappa$

$$\psi_{\varkappa m}(r, \varphi_r) \approx \frac{\mathrm{e}^{\mathrm{i} m \varphi_r}}{\pi\sqrt{r}} \cos\left(\varkappa\, r - \frac{m\,\pi}{2} - \frac{\pi}{4}\right). \qquad (6)$$

The function $\psi_{\varkappa m}(r,\varphi)$ may be expressed as a **superposition of plane waves** in the xy plane,

$$\psi_{\varkappa m}(r,\varphi) = \int a_{\varkappa m}(\boldsymbol{k}_\perp)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp \boldsymbol{r}}\, \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\,, \tag{7}$$

where the Fourier amplitude $a_{\varkappa m}(\boldsymbol{k}_\perp)$ is concentrated on the circle with $k_\perp \equiv |\boldsymbol{k}_\perp| = \varkappa$,

$$a_{\varkappa m}(\boldsymbol{k}_\perp) = (-\mathrm{i})^m\, \mathrm{e}^{\mathrm{i}m\varphi_k}\, \sqrt{\frac{2\pi}{\varkappa}}\, \delta(k_\perp - \varkappa)\,. \tag{8}$$

Therefore, the function $\Psi_{\varkappa m k_z}(r,\varphi_r,z,t)$ can be regarded as a superposition of plane waves with defined longitudinal momentum k_z, absolute value of transverse momentum $\varkappa$, energy $\omega = \sqrt{\varkappa^2 + k_z^2}$ and different directions of the vector $\boldsymbol{k}_\perp$ given by the angle φ_k.

Twisted photons

The wave function of a twisted photon (vector particle) can be constructed as a generalization of the scalar wave function. We start from the plane-wave photon state with **a defined 4-momentum** $k = (\omega, \boldsymbol{k})$ and **helicity** $\Lambda = \pm 1$,

$$A^{\mu}_{k\Lambda}(t, \boldsymbol{r}) = \sqrt{4\pi}\, e^{\mu}_{k\Lambda}\, \frac{\mathrm{e}^{-\mathrm{i}(\omega\, t - \boldsymbol{k}\boldsymbol{r})}}{\sqrt{2\omega}}\,, \tag{9a}$$

$$e_{k\Lambda} \cdot k = 0\,, \quad e^{*}_{k\Lambda} \cdot e_{k\Lambda'} = -\delta_{\Lambda\Lambda'}\,, \tag{9b}$$

where $e^{\mu}_{k\Lambda}$ is the polarization four-vector of the photon.

The twisted photon vector potential

$$\mathcal{A}^{\mu}_{\varkappa m k_z \Lambda}(r, \varphi_r, z, t) = \int a_{\varkappa m}(\boldsymbol{k}_\perp)\, A^{\mu}_{k\Lambda}(t, \boldsymbol{r})\, \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \tag{10}$$

$$= (-\mathrm{i})^m \sqrt{2\pi\varkappa} \int\limits_0^{2\pi} \mathrm{d}\varphi_k \int\limits_0^{\infty} \mathrm{d}k_\perp\, \delta(k_\perp - \varkappa)\, \frac{e^{\mathrm{i}m\varphi_k}}{(2\pi)^2}\, A^{\mu}_{k\Lambda}(t, \boldsymbol{r})$$

is given as a two-fold integral over the perpendicular components $\boldsymbol{k}_\perp = (k_x, k_y, 0)$ of the wave vector $\boldsymbol{k} = (k_x,\, k_y,\, k_z)$.

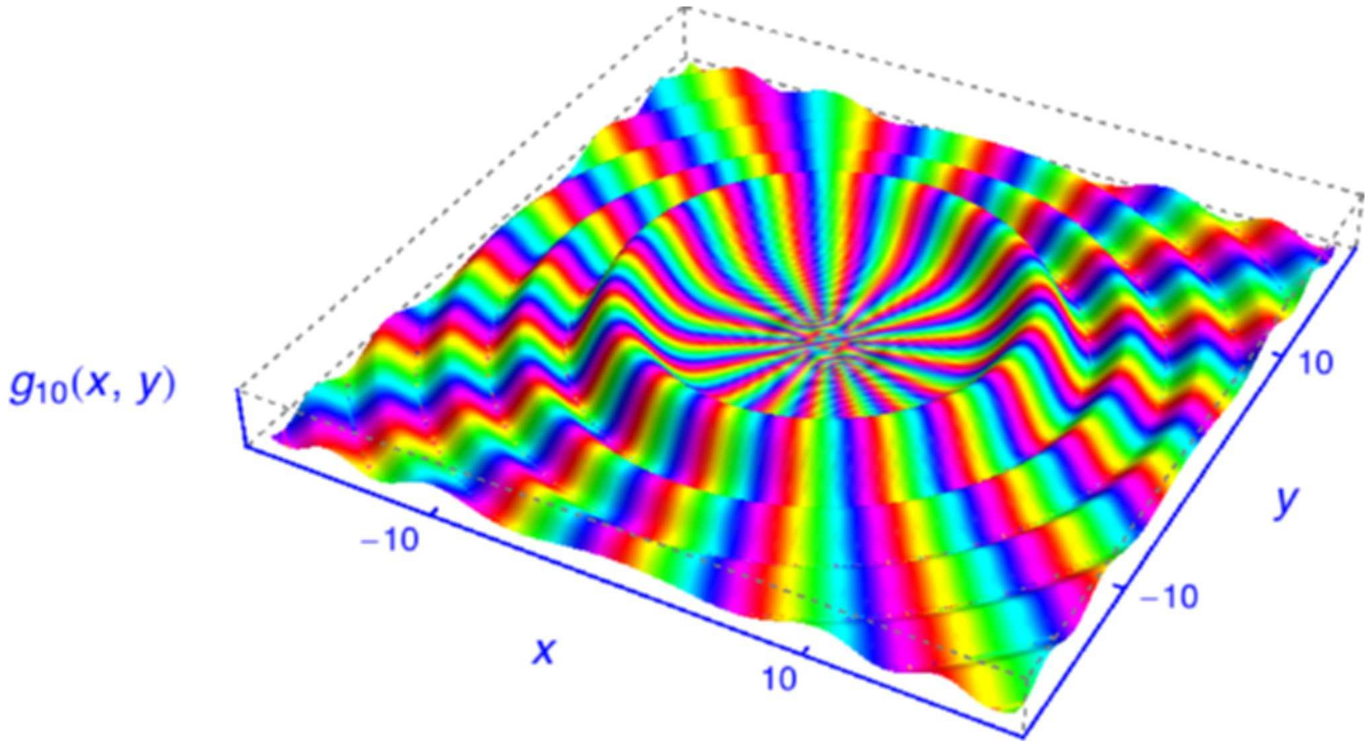


Fig. 2B Vector potential $\mathcal{A}^{\mu}_{\varkappa m k_z \Lambda}$ of a twisted photon: $g_m(x,y) = |\mathcal{A}^{\mu}_{\varkappa m k_z \Lambda}(0,x,y,0)|^2$ as a function of x and y; upper plot $m = 5$, lower plot $m = 10$. The parameters are $\mu = 1$ (x component), $\varkappa = 1$.

3. Compton scattering for twisted photons in the initial state

1. Compton scattering of plane-wave photons

The S-matrix element for plane waves is well known

$$S_{fi}^{(\mathrm{PW})} = \mathrm{i}\,(2\pi)^4\,\delta^{(4)}(p+k-p'-k')\,\frac{M_{fi}}{4\sqrt{E\,E'\,\omega\,\omega'}}\,, \qquad (11)$$

where M_{fi} is the amplitude.

For **a head-on collision of a plane-wave photon and electron**, **the photon propagates almost along the direction of momentum of the initial electron**.

2. Compton scattering with the twisted photon in the initial state

Since a twisted photon is a superposition of plane-wave photons, for the case where the **initial photon is twisted** m-photon, but the **outgoing one is a plane-wave** photon, we have

$$S_{fi}^{(\mathrm{m})} \equiv \langle k', \Lambda'; p', \lambda' | S | \varkappa, m, k_z, \Lambda; p, \lambda \rangle = \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} S_{fi}^{(\mathrm{PW})} \, a_{\varkappa m}(\boldsymbol{k}_\perp) \,. \tag{12}$$

A detailed consideration shows that the corresponding cross section is given by the previous expression with the only replacement

$$x = \frac{4\omega E}{m_e^2} \to \frac{4\omega E}{m_e^2} \cdot \cos^2(\alpha_0/2) \,, \tag{13}$$

where α_0 is the conical angle of the initial photon $\tan \alpha_0 = k_\perp / |k_z|$.

It is important since it proves that the cross section for

twisted initial photons has no additional smallness

as compare with the ordinary Compton scattering.

This result looks very natural since the initial photon state is nothing else but **a superposition of plane waves with the same absolute value of their transverse momentums**.

4. Compton scattering for twisted photons in the initial and final states

4.a. Strict BACKWARD scattering and PRINCIPAL conclusion

For strict backward Compton scattering the transverse momentum of the **final electron** $\mathbf{p}'_\perp = 0$.

Therefore, the **final twisted** m' photon is a superposition of plane waves with
large energy $\omega' \sim 4\gamma^2\omega$
and **small transverse momentum** $\boldsymbol{k}'_\perp = \boldsymbol{k}_\perp$:

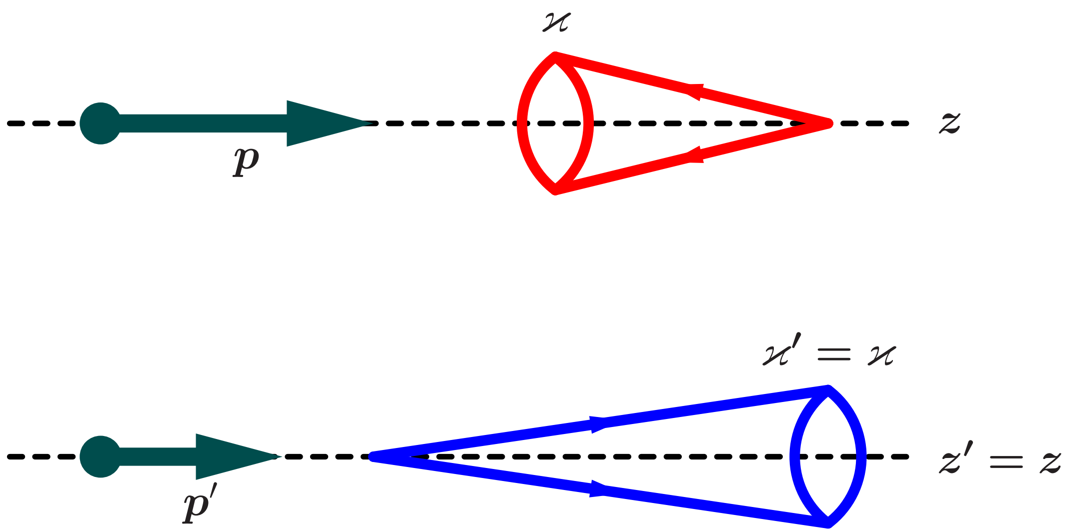


Fig. 4. Initial (above) and final (below) states for the head-on Compton **strick-backscattering** geometry of a twisted photons

The S matrix element $S_{fi}^{(\mathrm{TW})}$ for the scattering of a twisted (TW) photon $|\varkappa, m, k_z, \Lambda\rangle$ into the state $|\varkappa', m', k_z', \Lambda'\rangle$ is given as a convolution:

$$S_{fi}^{(\mathrm{TW})} = \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 k'_\perp}{(2\pi)^2} a^*_{\varkappa' m'}(\boldsymbol{k}'_\perp)\, S_{fi}^{(\mathrm{PW})}\, a_{\varkappa m}(\boldsymbol{k}_\perp)\,. \qquad (14)$$

Therefore, transition from PW to TW **in this case** means:

$$M_{fi}(\boldsymbol{k}'_\perp = \boldsymbol{k}_\perp = 0) \to M_{fi}(\boldsymbol{k}'_\perp = \boldsymbol{k}_\perp \neq 0), \qquad (15)$$

$(2\pi)^2 \delta^{(2)}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp - \boldsymbol{p}'_\perp - \boldsymbol{k}'_\perp) \to$ **the master integral**

$$\mathcal{I}_{mm'} = \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 k'_\perp}{(2\pi)^2} a^*_{\varkappa' m'}(\boldsymbol{k}'_\perp) a_{\varkappa m}(\boldsymbol{k}_\perp) (2\pi)^2 \delta^{(2)}(\boldsymbol{p}_\perp + \boldsymbol{k}_\perp - \boldsymbol{p}'_\perp - \boldsymbol{k}'_\perp).$$

For strict backscattering, we thus calculate **the new amplitude M_{fi}** and new **the master integral**.

As a result, for strict backscattering, the S matrix element reads

$$S_{fi}^{(\mathsf{TW})} = \mathsf{i}\,(2\pi)^2\,\delta_{mm'}\,\delta(\varkappa-\varkappa')\,\delta(E+\omega-E'-\omega') \times\ \delta(p_z+k_z-p_z'-k_z')\frac{M_{fi}}{4\sqrt{EE'\omega\omega'}}\,. \qquad (16)$$

This result states that **for strict backscattering**, the angular momentum projection $m' = m$ and the conical momentum spread $\varkappa' = \varkappa$ of the twisted photons **are conserved**, but the energy of the final twisted photon **is increased** dramatically: $\omega'/\omega \sim 4\gamma^2 \gg 1$.

It means that there is a principal possibility to create high-energy photons with large orbital angular momenta projections.

4.b. General case
(orbital helicity; wave pockets for twisted states)

What happens for not strict backward scattering of twisted photons?

This problem is studied in detail in the paper

[3] I.P. Ivanov, V.G. Serbo "Scattering of twisted particles: extension to wave packets and orbital helicity", Phys. Rev. A 84 (2011) 033804 and arXive:1105.5575v2 [hep-ph]

In general case the final electron momentum $\boldsymbol{p}' \nparallel \boldsymbol{p}$ and it is natural to consider the final twisted photon as propagating along the axis z' which is the axis of **the average propagation direction** of the final photon. **The corresponding projection of OAM we call ORBITAL HELICITY**

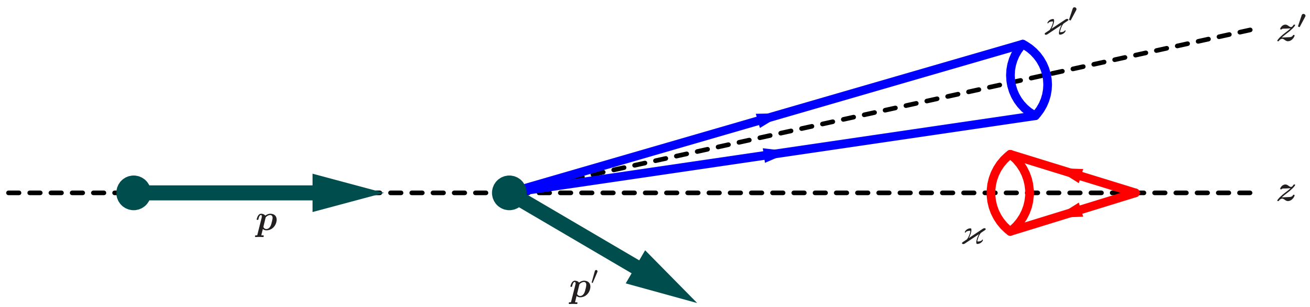


Fig. 6. Initial (red) and final (blue) states of twisted photons for the head-on Compton **not strick-backscattering** geometry.

We denote the angle between axes z and z' as $\langle\theta_\gamma\rangle$

For the **pure Bessel** states we again obtained that the distribution over m' is **rather wide.**

Then we consider **more realistic case** when the pure initial twisted state $\psi_{\varkappa m}(r, \varphi)$ is replaced by **a wave packet** with a defined m:

$$\psi_{\varkappa m}(r, \varphi) \to \psi_m(r, \varphi) = \int_0^\infty f(\varkappa) \psi_{\varkappa m}(r, \varphi) \, d\varkappa$$

with a narrow weight function of the gaussian type

$$f(\varkappa) = N \exp\left[-\frac{(\varkappa - \varkappa_0)^2}{2\sigma^2}\right], \qquad (17)$$

peaked at $\varkappa = \varkappa_0$ and having a width σ.

We assume the similar smearing for the **final** twisted state $\psi_{\varkappa' m'}(r, \varphi)$ with the weight function $g(\varkappa')$.

A further consideration shows that the distribution over the final projection m' is now **concentrated near the initial value** m:

$$|m' - m| \lesssim 4\langle\theta_\gamma\rangle \, \frac{\omega}{\sigma} \, .$$

(Note that the right hand side of this inequality is **small** for small enough angles $\langle\theta_\gamma\rangle$).

As a result, we proved that if the initial and final states are wave packets, then

final orbital helicity m' stays close to m

the final $\varkappa'$ stays close to $\varkappa$

5. Conclusion

1. We have investigated the scattering of a twisted photon by an incoming ultra-relativistic electron, in the Compton backscattering geometry.

2. We found out that the OAM projection m' and the conical momentum spread $\varkappa'$ of the final twisted photon are close to those of the initial twisted photon, but the energy ω' of the final twisted photon is increased dramatically:

$$\omega' \sim 4\gamma^2\omega \gg \omega \,.$$

3. As a result, we prove the principal possibility to create high-energy photons with large orbital angular momentum projections.

4. Such photons may be useful for a number of experimental studies regarding the excitation of atoms into circular Rydberg states, for studying the photo-effect, the pair production off nuclei and so on in previously unexplored regimes.

THANK YOU FOR YOUR ATTENTION!