

APPROXIMATE METHOD FOR CALCULATION OF FIELD OF CHARGED PARTICLE MOVING THROUGH DIELECTRIC OBJECT*

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Abstract

Cherenkov radiation is widely used for particle detection. As well, it is prospective for particle bunch diagnostics. Therefore, it is actual to elaborate methods for calculation of the fields of bunches moving in the presence of different dielectric objects. We offer the approximate method based on calculation of the field in unbounded medium and accounting of boundary influence with help of geometrical optics. First, we consider the problem concerning the field of charge crossing a dielectric plate. This problem has an exact solution. It is used as a "test" problem for estimation of precision of the approximate method. Computation of the field is performed using both methods and the results have a good agreement. Further, we analyze the cases of more complex objects, in particular, a dielectric cone. Note that the offered method allows to obtain wave fields using neither complex analytical transformations nor laborious numerical calculations.

INTRODUCTION

Problems of radiation of charged particles in the presence of dielectric objects are of interests for some important applications in the accelerator and beam physics. It can be mentioned for example a new method of bunch diagnostics offered recently [1]. For realization of this method, it is necessary to calculate the field of radiation outside a dielectric object. As a rule the form of object in such problems does not allow obtaining an exact analytical solution. Computer simulation of electromagnetic field is also very cumbersome. Therefore development of approximate analytical methods for analyses of radiation in such problems is an actual task. One of such methods will be offered and developed in this paper.

BASIS OF THE METHOD

The method offered here concerns problems which are characterized by some large geometric parameters. Let a charged particle bunch move in some dielectric or magnetic object. It is possible as well that the charge moves in a vacuum channel in the object, and radius of the channel can be arbitrary. In addition, the case of charge moving along one of borders of object can be considered, and in such case the distance from this border to the charge trajectory can be arbitrary. Anyway we

assume that the sizes of the object are much more than wave lengths under consideration. Therefore, the Cherenkov radiation (CR) excited by the bunch runs inside the object some distance which is much more than wave lengths.

Under such conditions we can apply the following approach. At first, we calculate analytically the field of the charge in the infinite medium without "external" border. It is important that we can take into account such peculiarities of the problem as a vacuum channel (if the charge moves into the object) or finite distance from trajectory to the object's border (if the charge moves along the object). We underline that a lot of such problems have been solved in the literature.

The second step is approximate calculation of radiation going out of the object (sometimes it is named "Cherenkov-transition radiation" (CTR) [2]). The idea of this calculation is related to Fok's method of analysis of reflection of waves from arbitrary surfaces [3] but we deal with transmission instead of reflection. The incident field is multiplied by the Fresnel transmission coefficient, and then we should take into account decrease of the radiation because of spreading of a ray tube in the external medium. Thus we obtain the first of refracted rays of CTR. Probably, this will be enough for the majority of applied problems. If it will be necessary, multiple reflections and refractions on the object borders can be taken into account.

TESTING OF THE TECHNIQUE FOR DIELECTRIC LAYER

For testing the method, we use the problem about the field of point charge flying through the dielectric layer with permittivity ε placed at $0 < z < d$. The charge density is set in the form $\rho = q\delta(x, y, z - Vt)$. Such a problem has exact solution [4] which has been proved by us independently. We compare computations performed with use of exact formulae and approximate ones. Some results concerning the magnetic strength Fourier component $H_{\phi\omega}$ are given in Fig. 1. Note that approximate curves have a break on the boundary of "the light bar". Naturally, the exact solution is continuous everywhere (excepting the layer boundaries).

One can see that some agreement takes place even for $d \sim \lambda_2 = 2\pi c / (\omega\sqrt{\varepsilon})$ if the distance from the plate is $\sim \lambda_2$ as well. In the case when $d \sim 10\lambda_2$ we have very good agreement for the most part of "the light bar". This result is very encouraging, and it stimulates applying the

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method under consideration for more complex object where the exact solution cannot be obtained.

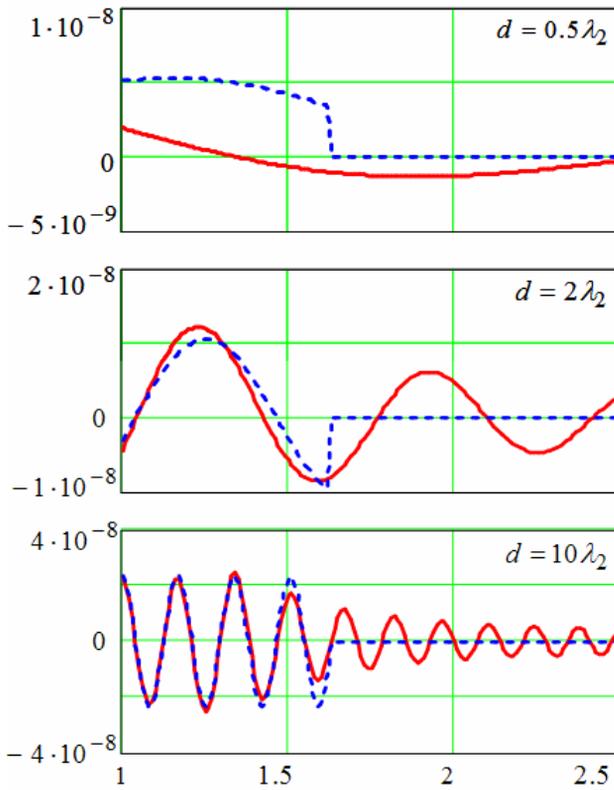


Figure 1: The magnetic strength Fourier component $H_{\phi\omega}$ (A·s/m) depending on distance z (cm) for charged particle crossing the plate: computations using exact (red solid curve) and approximate (blue dashed curve) formulae; $q = -1nC$, $\varepsilon = 1.5$, $\rho = 0.6$ cm, $\beta = 0.99$

THE CASE OF CONE WITH CHANNEL

We apply the method under consideration to the case of a dielectric cone with a cylindrical vacuum channel (Fig. 2). The external boundary of the cone is determined by the equation $\rho = (z_0 - z)\tan\alpha$ where $0 < \alpha < \pi/2$. The channel radius a can be both less and more than the typical wave lengths. A point charge moves along the channel axis (z -axis). It is assumed that the wave of CR in the dielectric passes a distance much more than wave lengths.

The first step of our consideration is solving the problem with infinite medium with vacuum channel (the solution of this problem is known [5]). At the second step, we firstly determine the point M_* of incidence of wave of CR on the cone boundary. The coordinates ρ_*, z_* of this point is a function of the coordinates of the observation point. Further, one can show that spreading of the ray tube outside the cone is the same as inside it. As result we obtain the expression for field outside the cone in the form

$$H_{\phi\omega} \approx H_{\phi\omega}^* \sqrt{\frac{\rho_*}{\rho}} T \exp(i\omega L/c),$$

where $H_{\phi\omega}^*$ is an incident field at M_* , T is Fresnel transmission coefficient, L is a ray path in vacuum.

Figures 3 - 6 illustrate some properties of the solution obtained by the method under consideration. All graphics

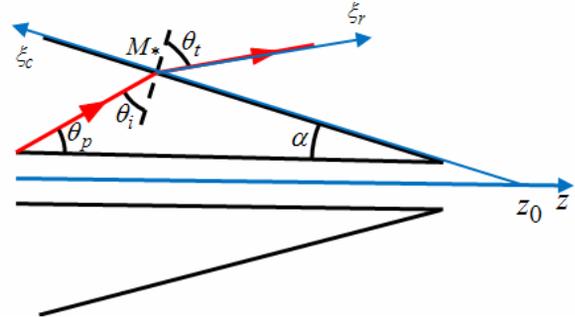


Figure 2: Cross-section of the cone.

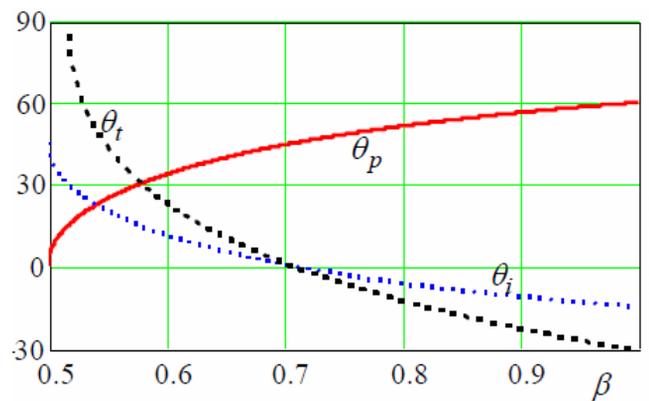


Figure 3: Angle of CR, angle of incidence and angle of refraction (in degrees) depending on the charge velocity for $\varepsilon = 4$.

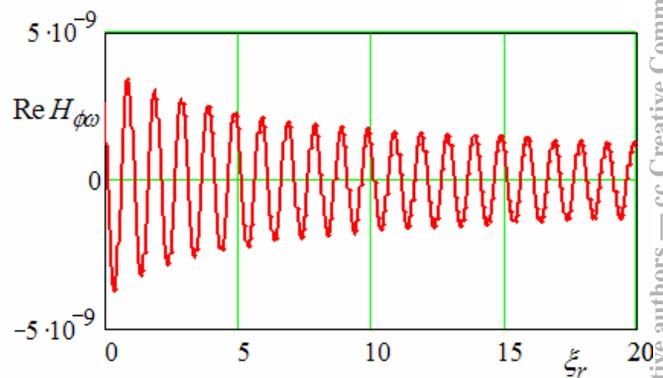


Figure 4: The magnetic field Fourier transformation (A·s/m) depending on distance ξ_r (cm) from the cone surface along the transmitted ray; $q = -1nC$, $\omega = 2\pi \cdot 3 \cdot 10^{10} s^{-1}$, $\varepsilon = 4$, $a = 2$ mm, $\alpha = 45^\circ$, $\beta = 0.99$

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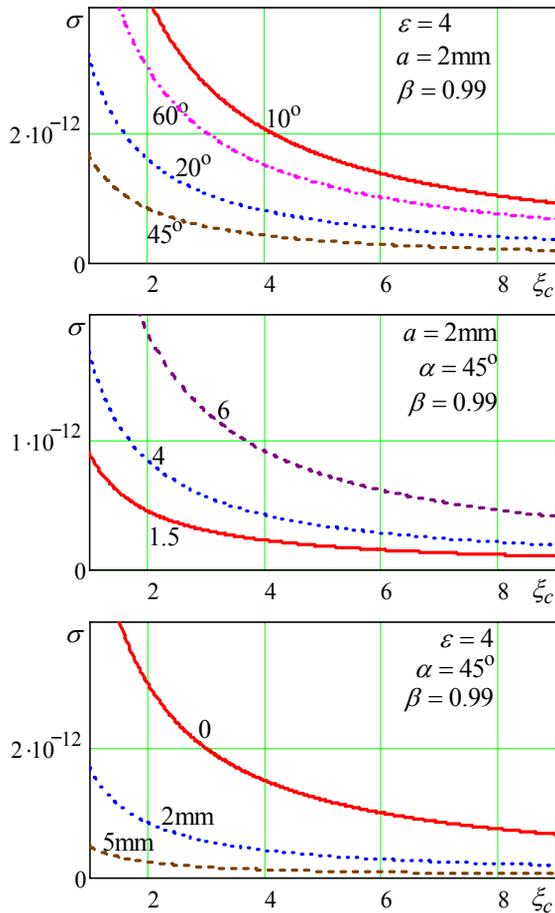


Figure 5: Dependency of the spectral density of the radiation energy σ ($\text{J}\cdot\text{s}/\text{m}^2$) on the distance ξ_c (cm) along the cone surface for different angles α (top), different permittivity values ε (middle), and different channel radii a (bottom).

All graphics have been obtained for $q = -\ln C$ and $\omega = 2\pi \cdot 3 \cdot 10^{10} \text{ s}^{-1}$. Figure 3 shows the angle of CR, the incidence angle and the refraction one (see Fig. 2) depending on the charge velocity. Note that positive values of θ_i and θ_r correspond to the case shown in Fig. 2, these angles are negative at other positional relationship of the rays and the normal to the cone boundary.

Figure 4 illustrates typical dependency of the magnetic field Fourier transformation on the distance ξ_r from the cone surface along the direction of the transmitted wave propagation. Naturally, decrease of amplitude is explained by cylindrical divergence of radiation.

Figure 5 shows the spectral density of the radiation energy $\sigma \sim |H_{\phi\omega}|^2$ depending on the distance ξ_c along the cone surface (see Fig. 2). This value determines the total energy Σ passing through a unit square:

$$\Sigma = \int_0^{\infty} \sigma d\omega = \int_{-\infty}^{\infty} S dt, \text{ where } S \text{ is a Poynting vector.}$$

Dependencies of σ on the charge velocity and the cone angle at some fixed point on the cone surface are shown in Fig. 6. It is interesting, for example, that dependence of σ on the cone angle is not monotonous. Note that for some value of α this magnitude tends to infinity that is connected with approach of θ_i to the limit angles of total reflection.

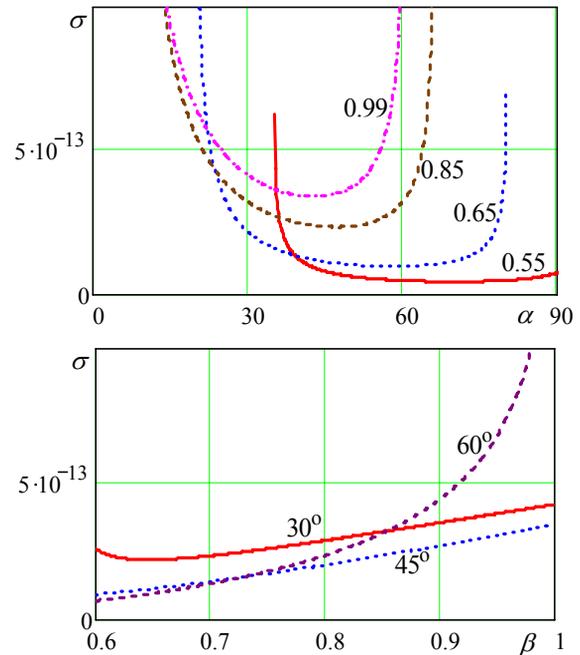


Figure 6: The spectral density of the radiation energy σ ($\text{J}\cdot\text{s}/\text{m}^2$) on the cone surface at $\xi_c = 5 \text{ cm}$ depending on α for different β (top) and on β for different α (bottom); $\varepsilon = 4$, $a = 2 \text{ mm}$.

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