# VERTICAL SIZE OF AN ELECTRON BEAM AT SIBERIA-2

A.G.Valentinov, V.N.Korchuganov, Yu.V.Krylov, Yu.L.Yupinov, NRC Kurchatov Institute, Moscow, Russia

*Abstract*

Brightness of the synchrotron radiation light sources is defined by electron beam sizes at radiation point. Horizontal size depends mainly from designed magnetic structure. Vertical size is defined by two processes: first, betatron coupling between vertical and horizontal motions and second, presence of vertical dispersion function at bending magnets. Vertical dispersion creates non-zero vertical emittance even without coupling.

The report is dedicated to methods of vertical beam size decreasing at SIBERIA-2 storage ring. There are two families of skew-quadrupoles on the ring, one lens of every family in each of 6 cells of the magnetic structure. After analyzing of betatron coupling coefficient equation we stayed only two lenses in each family. As a result power supplies' currents for coupling compensation became much lower.

In order to decrease vertical dispersion a special algorithm was developed and tested. Vertical dispersion on beam position monitors (BPM) azimuths was corrected by vertical displacements of chromaticity compensating sextupoles. Maximal value of the dispersion became four times lower. It leaded to prominent vertical beam size decreasing.

## INTRODUCTION

The vertical size of the electron beam in storage ring SIBERIA-2 is mainly determined by two factors. Firstly, there is a vertical emittance $\varepsilon_z$, which is generated by the vertical dispersion function $\eta_z$ in the places, where the electrons radiate energy - similar to the way that the horizontal dispersion function leads to the appearance of the horizontal emittance. Vertical dispersion occurs in the presence of any horizontal fields on the orbit, such as fields of vertical corrective magnets, fields due to deviation of the beam from the centers of quadrupoles and sextupoles, any parasitic fields. In addition, there are the inevitable errors in the position of the magnetic elements, which may lead to the distortion of vertical orbit. Secondly, there is a coupling of vertical and horizontal betatron. It appears in the presence of skew-quadrupole fields on a closed orbit. These fields can be a consequence of the rotation of quadrupole lenses along their longitudinal axis, as well as errors in the upright position of the sextupole lenses. The betatron coupling leads to the periodic transfer of energy between the horizontal and vertical betatron oscillations.

Other mechanisms to increase the vertical size of the beam, such as multiple internal scattering of electrons in the bunch (the Toushek effect), and the interaction of a beam with the currents, induced on the walls of the vacuum chamber, give negligibly small contribution to the vertical size of the beam at SIBERIA-2 at 2.5 GeV.

To decrease the vertical size of the beam it is necessary to control the vertical dispersion function, as well as the coupling of betatron oscillations.

## VERTICAL DISPERSION FUNCTION CONTROL

In the case of small connection between transversal betatron oscillations an equation for the vertical dispersion is written as

$$\eta_z'' + K_1\eta_z = K_2 z_c \eta_x - K_{1s}\eta_x + K_1 z_c - \frac{1}{\rho_z} = F \qquad (1)$$

where $\eta_x$ - horizontal dispersion function, $K_1$, $K_2$, $K_{1s}$ - normalized power of quadrupoles, sextupoles and skew-quadrupoles respectively, $z_c$ - vertical deviation of a closed orbit, $\rho_z$ is the radius of rotation in vertical plane [1]. The solution to this equation has the same form like for a closed orbit:

$$\eta_z = \frac{\sqrt{\beta_z(s)}}{2\sin\pi\nu_z}\int_s^{s+C}\sqrt{\beta_z(y)}\cos\left(\phi_z(s) - \phi_z(y) + \pi\nu_z\right)F dy \qquad (2)$$

where $\nu_z$ - vertical betatron frequency, C – ring circumference, $\beta_z$ - vertical $\beta$-function.

For us the important thing here is that $\eta_z$ is described by exactly the same formulas as the vertical closed orbit. This means that $\eta_z$, as well as the orbit, can be corrected to obtain its acceptable view. As for correction of the closed orbit, the goal is to decrease the value of RMS $\eta_z$ deviations from zero: $\sigma_{\eta z} \to 0$. Magnetic elements contributing to the function F can be used as correctors. In fact, with already adjusted closed orbit, one can affect only the second term in the expression for F, because in this case the following conditions are satisfied: $z_c$ = const and $\rho_z$ = const. Thus, only the magnetic elements with skew-quadrupole component in the field can influence on the vertical dispersion without the distortion of the orbit. They must be located inside achromatic bend with a non-zero $\eta_x$. It is preferable to have skew-quadrupole lenses with independent power supplies inside each achromatic bend. There are no that kind of lens at SIBERIA-2, but we can use sextupole lenses for natural chromaticity correction as correctors for $\eta_z$. We can get skew-quadrupole field $K_{1s}$ from sextupole vertical displacement $\Delta z$: $K_{1s}$= $-K_2 \cdot \Delta z$. Impact on the closed orbit is small enough, because components of the field arising from sextupole moving $\Delta B_z$ and $\Delta B_x$ are proportional, respectively, to $z_c \cdot \Delta z$ and $x_c \cdot \Delta z$, where $z_c$ and $x_c$ - vertical and horizontal orbit distortions at sextupoles' azimuths. Each of the $z_c$, $x_c$ and $\Delta z$ does not exceed 1-2 mm, and the



ISBN 978-3-95450-125-0

angular deviation of the orbit generated by the field, does not exceed $2 \cdot 10^{-6}$. This is a very small value in comparison with the typical angular deviation $10^{-4}$, arising from typical 0.01 mm transverse quadrupole deviations. The maximum orbit distortion because of the shift of one sextupole will not exceed several tens of microns.

SIBERIA-2 sextupoles are not designed for this type of $\eta_z$ correction, so their vertical movements are quite limited. Vacuum chamber of the storage ring is an obstacle. About one-third of the 24 sextupoles may be displaced for no more than 0.2 - 0.3 mm; others allow a more notable movement up to 1.5 mm.

The same mathematical methods can be used for the correction of $\eta_z$ as for the correction of the closed orbit (LSQ, MICADO, SVD). A response matrix for ideal magnetic structure is calculated, linking the shift of the sextupoles and vertical dispersion changes at azimuths of beam position monitors (BPM). Vertical dispersion is measured through changes in the frequency of RF system (and hence the revolution frequency $f_{REV}$). Energy deviation $\Delta p/p$ is connected with change in revolution frequency $\Delta f_{REV}$ through momentum compaction factor $\alpha$ = 0.0103. So we can calculate vertical dispersion from

$$\eta_z = \frac{\Delta z}{\Delta p / p} \qquad \frac{\Delta p}{p} = -\frac{1}{\alpha}\frac{\Delta f_{REV}}{f_{REV}} \tag{3}$$

where $\Delta z$ – vertical orbit distortions from BPMs, $f_{REV}$ = 2.4151 MHz, $\Delta f_{REV}$ usually equals $\pm$ 20 Hz.

Solution of the $\eta_z$ correction problem is a set of shift values for sextupoles. Next you need to select the variant of correction, which can be physically implemented, that is there would be no obstacles for sextupoles' moving, and then make it. After this the cycle of correction can be repeated. Neighboring sextupoles, located on the edges of the quadrupole doublet, affect the $\eta_z$ practically by the same way (with accuracy up to the sign!), because of small betatron phase distance between them. Thus, there is 12 independent «correctors» to regulate $\eta_z$ at azimuths of 24 BPMs. Of course, it is not possible to reach full compensation of dispersion function, moreover, a behavior of $\eta_z$ remains unknown between BPMs.

The correction process led to a strong decrease in the $\sigma_{\eta z}$ value at BPM azimuths. Figure 1 shows the vertical dispersion function at SIBERIA-2 before and after correction. 8 sextupoles were moved, maximal shift was $\Delta z$ = 1.5 mm. As a result of $\sigma_{\eta z}$ decreased from 3.4 cm down to 0.9 cm, the maximum value of $\eta_z$ decreased from 6 cm down to 3 cm. Theoretically we can make the $\sigma_{\eta z}$ approximately 2 times less by using all of the 12 independent «correctors», however, sextupoles' shifts would be too big.

Reduction of dispersion led to a considerable reduction in the vertical size of the electron beam, visible even on the TV screen, which shows the profile of the beam in visible light. In the visible range there is quite a strong dispersion of light in an optical observation system, which leads to an increase in the visible vertical size. However, the effect of correction is well seen in this case (Fig. 2).

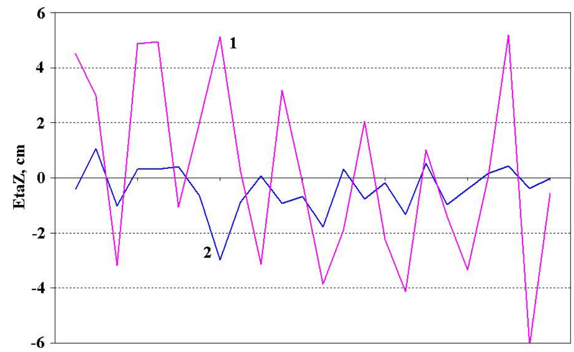


Figure 1: Vertical dispersion function at SIBERIA-2 BPMs before correction (1) and after it (2).

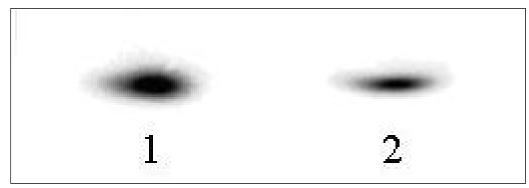


Figure 2: Electron beam image on TV screen (converted colors) before (1) and just after (2) $\eta_z$ correction.

## BETATRON COUPLING

The coupling coefficient can be described analytically (excluding the effect of solenoid fields) [2]:

$$C^- = \frac{1}{2\pi R}\int_0^{2\pi}\sqrt{\beta_x \beta_z}\cdot K(\theta)\cdot \exp^{i\Psi(s)}\, d\theta \tag{4}$$

$$\Psi(s) = (\mu_x - \mu_z) - (Q_x - Q_z - p)\theta$$

Here R is the average radius of the ring, $\beta_{x,z}$ - optical functions, $\mu_{x,z}$ - betatron phase, $Q_{x,z}$ – betatron tunes, $\theta$ = s/R, p - the difference between integer parts of the betatron tunes, and K is the normalized gradient of the skew-quadrupole lenses:

$$K(\theta) = \frac{R^2}{2\cdot B\rho}\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_z}{\partial z}\right) \tag{5}$$

$C^-$ is a complex value, its module $|C^-|$ corresponds to minimum distance between betatron tunes, which can be reached on the approach to the resonance $Q_x$ - $Q_z$ - p = 0. The values of the imaginary and real parts of $C^-$ depend on the starting point, while the value $|C^-|$ remains unchanged. The vertical emittance due to betatron coupling can be written as:



ISBN 978-3-95450-125-0

$$\frac{\varepsilon_z}{\varepsilon_x} = \frac{\left|C^-\right|^2}{\Delta^2 + \left|C^-\right|^2}, \qquad (6)$$

where $\varepsilon_{x,z}$ – horizontal and vertical emittances, $\Delta = Q_x - Q_z - p$, $Q_x = 7.77$, $Q_z = 6.70$ for SIBERIA-2 structure, $|C^-|$ value without skew-quadrupole lenses was equal approximately to 0.03 after compensation of closed orbit distortions.

In order to compensate betatron coupling the magnetic structure of SIBERIA-2 contains 2 skew-quadrupole families SQ1 and SQ2 - one lens of the family in each of the 6 machine superperiods. It turned out, however, that their forces are not enough to compensate the coupling: the requested currents were about 30 A with the highest possible level of 25 A. The fact is that the influence of each skew-quadrupole depends on the phase $\Psi(s)$ in the exponential factor of the expression (4). If the magnetic structure consists of N identical superperiods, the phase advance of $\psi(s)$ on one superperiod is equal to $2\pi \cdot p/N$. Therefore, the lenses of one family, located in the same manner within each superperiod, will be equally influence the imaginary and real parts of the $C^-$ only if p is equal to zero or a multiple number of superperiods N. In our case p = 1 and N = 6, so phase advance of $\psi(s)$ for one superperiod is equal to 60 degrees. In this case, the skew-quadrupoles, belonging to one family, but located in different superperiods, will compensate the influence of each other. They affect on betatron coupling only because there are differences in the betatron functions and phases between different superperiods because of imperfections of the magnetic structure.

For more efficient use of the skew-quadrupoles we changed commutation inside the SQ1 and SQ2 families. In principle, it was possible to leave on one lens in each family. In practice two lenses was left on opposite sites of the ring (in the 2nd and 5th superperiods), and the sign of skew-quadrupole gradient in 2nd superperiod was changed to the opposite. The phase advance between lenses inside one family became equal to 180˚, the phase difference between SQ1 and SQ2 lenses inside one superperiod is equal to ~60˚. This allowed the halving of the required power supply current for the coupling compensation. New type of commutation allows to achieve full compensation of the coupling with the currents SQ1 = -0.1 A, SQ2 = -2.8 A.

## CONCLUSIONS

As a result we have been able to reduce the vertical size of the electron beam considerably. The new commutation in the families of the skew-quadrupole lenses allowed us to reduce currents (and, hence, the field) needed to compensate the betatron coupling. Thus, vertical emittance due to coupling can be easily reduced to an amount not exceeding 0.01% from the horizontal one. The correction of the vertical dispersion function led to 3.5 times lower $\eta_z$ value at BPM azimuths. All these factors provide a substantial decrease of the electron beam vertical size in radiation points of the bending magnets.

## REFERENCES

[1] T.O.Raubenheimer, R.D.Ruth, "Analytic Estimates of Coupling in Damping Rings", PAC 1989, p.1435.
[2] G.Guignard, "The General Theory of All Sum and Difference Resonances in a Three-dimensional Magnetic Field in a Synchrotron", CERN 76-06 (1976).



ISBN 978-3-95450-125-0