USING GENETIC ALGORITHMS FOR ELECTRODE SHAPE OPTIMIZATION IN ACCELERATORS WITH RF FOCUSING

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Abstract
The drift tubes shape choice which provides the necessary distribution of the spatial RF field harmonic amplitudes is an important problem in the design of RF focusing accelerators. It is necessary to have various relationships of the main (accelerating) and the first (as main focusing) harmonics of RF field for different types of accelerators. High order harmonics should be negligible for accelerators with an external focusing, and this ratio should be $E_1/E_0 = 3-5$ for the efficient operation of the axially symmetric RF focusing accelerator. Thus, the distribution and harmonic amplitude's ratios at the accelerator axis which provides stable beam dynamics are always known. The drift tubes shape study problem cannot be solved directly by ordinary methods because of unknown boundary conditions belongs to a class of incorrectly defined problem. At present, this problem can be solved by using genetic algorithms (GA). For this purpose, we will define electrode's shape, and then solve the Laplace equation with boundary conditions of Dirichlet and Neumann. The necessary electrodes shape can be quickly and easily simulated using the adaptive search.

INTRODUCTION
The acceleration of high intensity low beta ion beams is one of the priority tasks of applied accelerating technology. But conventional RF low energy linacs are needed to use any external focusing elements as solenoids or quadrupole lenses to provide the beam transverse focusing. Any type of radiofrequency focusing is an alternative. It’s necessary to control the spatial RF field harmonics spectrum to provide focusing condition. As an example, in axi-symmetrical RF focusing linac (ARF) [1] the base (zero order) spatial harmonic is the accelerating and high order harmonics are used for focusing. The ratio of focusing and accelerating harmonics should be equal $E_1/E_0 = 3-5$ for effective beam focusing. In the other hand in RF linear undulator accelerator (UNDULAC, [2]) the acceleration is realized without synchronism with anyone of RF harmonics (beat-wave acceleration) and this ration should be equal $E_1/E_0 = 0.25$-$0.3$ for the stable beam motion. Note than in the second case the channel period can be simplest in contrast to ARF linac.

The problem of drift tubes geometry definition providing the necessary spatial RF field harmonics spectrum is not an easy task because it is an incorrectly defined problem. An other way to define the necessary electrodes geometry is to solve the analysis-syntheses problem with numerical optimization.

SIMULATION MODEL
Let we consider the structure with the drift tubes (Figure 1).

![Figure 1: Layout of periodic structure.](image_url)

It is necessary to solve electromagnetic problems for accelerating facilities designing. In this case, it is possible to use a quasi-static approach and necessary to solve the Poisson equation in a system with a complex geometry. As stated above, the problem of electrode shapes definition for a specified field on the axis is an incorrectly defined problem. In the other hand, the problem of the field distribution simulation over the electrodes is simple, and there are many numerical methods for this simulation. For the specified field distribution we must choose the correct shape and dimensions of the electrodes, which in some cases can be a difficult problem, since changing several parameters may take a long time to find the optimal one. Therefore to solve this problem, we propose to use genetic algorithms (GA), and using one is discussed in this article.

tubeOpt console code was developed to solve this problem, which allows optimizing the accelerating structure geometry and carrying out the field calculation. tubeOpt code consists of two parts: the first electrostatic solver to find the field distribution in the cavity by numerical simulation the Laplace equation with boundary conditions of Dirichlet and Neumann, the second - a genetic algorithm which optimizes the drift tubes geometric parameters. The chosen accelerating structure has axially symmetric geometry and we can consider a quarter one (Figure 2). In this case, the following parameters will be optimized: the bore radius of the drift tube $R_{in}$, the external radius of the drift tube $R_{out}$, the bending radii $R_{b1}$ and $R_{b2}$ and the half-length of the tube $L_{tube}$ (see Fig. 2).
There are two main methods of Laplace and Poisson equations simulation. In case when the structure has a strong symmetry or consists of several simple geometry elements (sphere, cylinder, box for example), an axis or plane symmetry coincide, the simulation area can be divided into smaller areas, where the equation can be easily solved analytically or using any simple numerical methods. The stitching of solutions can be done in case when the solutions in each area are founded and the field distribution definition in whole area is difficult. This method is called the finite volumes method. Another method is called the finite differences method. In this case, a spatial grid is imposed on the field of simulation and the solution of the electrodynamics problem can be provided on the grid. Generally a rectangular or triangular grid and their combinations are used. Obviously, the solution of differential equations on the grid is replaced by the solution of the system of finite-difference equations. This method is used in the tubeOpt code.

Genetic algorithm is a search heuristic that resembles the process of natural evolution. This heuristic can be used to generate useful solutions to optimization and search problems. GA generates solutions to optimization problems using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover. The error between the desired and the obtained potential distribution along the accelerator axis can be selected as the fitness function and calculation carried out to minimize the error.

The tubeOpt code works using the following algorithm. At first the genetic algorithm generates a population of a few individual solutions (further individuals), i.e. combinations of the optimizing parameters that represents number of possible solutions of this problem. Each individual is evaluated to a measure of its "fitness", i.e. correspondence to the required solutions. The elite individuals have the opportunity to "reproduce" the population by crossover with other individuals from the population or undergo mutation. This leads to the appearance of new individuals, which combines some of the characteristics that are inherited from their parents. Selection of elite individuals from the previous generation replicates a new population and gets a lot of new individuals. This new generation contains more tailored individuals. The crossover of the elite individuals leads to exploration of the most promising area of the optimization space. Eventually, the population will converge to the optimal solution.

Evolution cycle continues until one of the three conditions is true: the number of generations has reached the limit, the best individual solution in the population does not change over a given number of generations, individual solutions in the population became to similar. The fitness function takes the first five harmonics variations on the axial potential. The potential on the axis is expanded into a Fourier series and compared to the ratio of the first and third harmonics, as well as minimized the variations of the second, fourth and fifth harmonics:

\[
error = \sqrt{\left( \frac{U_1}{U_0} \right)^2 + U_2^2 + U_4^2 + U_5^2 - \eta}.
\]

We performed several numerical experiments to determine the optimal geometric parameters of the drift tubes. The results of the simulation are shown in Table 1. The optimal solution was found pending 78 generations (168 minutes). In this case, the population contains of 20 individuals, each with 5 genes (according to the optimizing parameters), the reproduction of the population set to 90%.

<table>
<thead>
<tr>
<th>Population</th>
<th>Reproduction, %</th>
<th>Generations</th>
<th>Best solution accuracy</th>
<th>Runtime, min</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10</td>
<td>74</td>
<td>4.17·10^{-3}</td>
<td>58</td>
</tr>
<tr>
<td>20</td>
<td>100</td>
<td>100</td>
<td>1.7·10^{-2}</td>
<td>214</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>29</td>
<td>4.24·10^{-3}</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>90</td>
<td>78</td>
<td>3.79·10^{-3}</td>
<td>168</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>12</td>
<td>2.54·10^{-3}</td>
<td>15</td>
</tr>
</tbody>
</table>

The optimal variant of the accelerating structure geometric parameters is presented in Table 2. Parameters are presented relative to the half-period L.

<table>
<thead>
<tr>
<th>L</th>
<th>L_{tube}</th>
<th>R_{in}</th>
<th>R_{out}</th>
<th>R_{b2}</th>
<th>R_{b2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.210</td>
<td>0.226</td>
<td>0.557</td>
<td>0.060</td>
<td>0.054</td>
</tr>
</tbody>
</table>

The potential distribution was built according to the optimized geometric of the accelerating structure (see Fig. 3). A drift tube section is shown by gray color.
The simulated potential distribution along the accelerating structure axis was compared to the ideal distribution (Fig. 4). The figure shows the ideal distribution of the potential (red color), and the calculated potential (blue color). It can be seen that the calculated distribution is almost identical to the desired.

We did the potential distribution comparison on the accelerating structure axis. The distribution obtained by using tubeOpt electrostatic solver (blue curve) was compared with the results of a standard electrodynamics code (red curve). It can be seen that the curves are sufficiently similar (see Fig. 7).

Figure 3: Equipotential surfaces obtained by tubeOpt code.

Figure 4: Calculated (blue curve) and ideal axial potential distribution (red curve).

Figure 5 shows the change of the best individual solution during evolution process. It can be seen that the optimal solution obtained sufficiently fast.

Figure 5: Optimization progress.

Figure 6: Sample equipotential surfaces.

Figure 7: Comparison of the sample (red curve) and a tubeOpt (blue dotted curve) potential calculation.

CONCLUSION

It is shown that use of genetic algorithms allows to quickly and accurately optimize the accelerating structure parameters. Verification of the parameters definition by specially designed tubeOpt code shows a good agreement with the simulation by well-known codes.

REFERENCES