THE MULTI-TIP FIELD EMISSION CATHODE MATHEMATICAL MODELING

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Abstract

The multi-tip field cathode as the field emission cathode arrays for rectangular lattice is considered. The field emission cathodes are of interest for vacuum nano-scale electronic devices. The electrostatic potential distribution is presented for the periodic system of free-number thin tips on a plane substrate as a field emission cathode and a plane substrate as an anode. The tips shape may be various. The potential of the substrate and cathode is equal the zero, the anode's potential is equal a constant. The effect of space charge is neglected. The each tip is represented as a system of the point charges. The point charges are determined to the zero equipotential coincides with the cathode's shape. The potential distribution is found for whole region of the field emission cathode arrays. The exact three-dimensional solution to the Laplace/Poisson equation has been obtained in the Cartesian coordinate system. This solution has direct applications in three-dimensional calculations of electron trajectories in micron- and submicron-sized field-emitter arrays.

INTRODUCTION

Field emission is of great commercial interest in electronic devices. Over the last decade, carbon-based and several others nanomaterials, such as carbon nanotubes, nanotips, various zinc oxide nanostructures, have attracted considerable attention due to their unique physical, chemical, and mechanical properties [1–3]. These nanostructures as the field cathodes are applied for surface diagnostics, low-energy electron diffraction, Auger-spectroscopy, scanning tunneling microscopy and others potential applications in the areas of electron field emission [4–6].

In this work the multi-tip field cathode as the field emission cathode arrays for rectangular lattice is under investigation.

PROBLEM BACKGROUND

The solution of Laplace's equation for the electrostatic potential distribution is presented for the diode systems: the multi-tip emitter as a field emission cathode of the on a flat metal substrate (base) and a plane as an anode. Each thin tip is placed in the rectangular lattice point (see Fig.1). The tip's shape may be various. The effect of the space charge is neglected.

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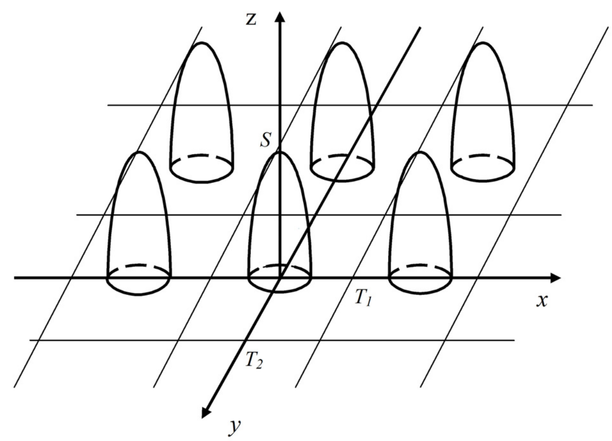

Figure 1: Illustration of a field emission cathode arrays for rectangular lattice.

To solve the electrostatic problem the variable separation method is employed. The cartesian coordinate system (x, y, z) is used.

The parameters of the problem are as follows:

$z = Z_{N+1}$ — the surface of anode;

$z = 0$ — the surface of substrate;

S — the length of tip;

$z_0(x, y)$ — the surface of tip;

T_1 — x-direction half-period;

T_2 — y-direction half-period;

$V(x, y, Z_{N+1}) = V_0$ — the boundary condition on anode;

$V(x, y, 0) = 0$ — the boundary condition on substrate.

The potential of the tip is assumed to be zero without the loss of general character of the problem.

Let us to interchange each tip influence for periodic lattice cell with a charge system q_i $(i = \overline{1, N})$ effect so that the tip surface is matched with the zero's equipotential as the virtual cathode (see Fig.2).

The parameters of the charge system are as follows:

N — the number of charges;

q_i — the values of charges;

$(0, 0, Z_i)$ — the coordinates of charges.

ISBN 978-3-95450-125-0

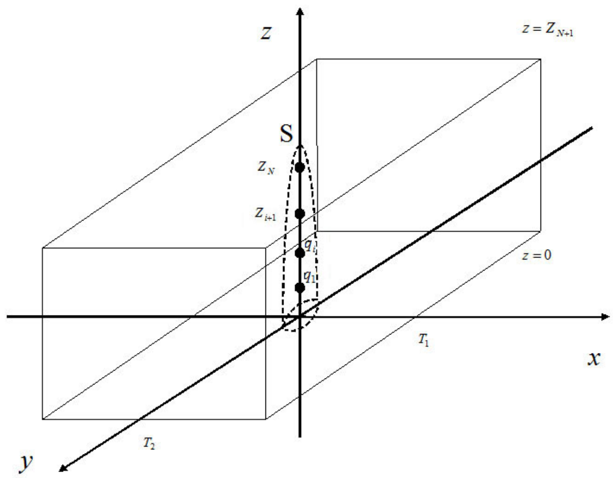

Figure 2: Illustration of a periodic lattice cell.

MATHEMATICAL MODELING AND CALCULATION

So we have to solve the boundary – value problem for the Poisson's equation

$$\begin{cases} \triangle \; V(x,y,z) = -\frac{1}{\varepsilon_0}\rho(x,y,z), \\ V(x,y,0) = 0, \\ V(x,y,Z_{N+1}) = V_0, \\ \left.\frac{\partial V(x,y,z)}{\partial x}\right|_{x=\pm T_1} = 0, \\ \left.\frac{\partial V(x,y,z)}{\partial y}\right|_{y=\pm T_2} = 0. \end{cases} \quad (1)$$

The potential distribution function $V(x,y,z)$ can be represented as series [7]

$$V(x,y,z) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} v_{mn}(z)\cos\frac{\pi n x}{T_1}\cos\frac{\pi m y}{T_2}. \quad (2)$$

Function $V(x,y,z)$ (2) satisfy the boundary – value problem (1) and periodicity conditions.

In compliance with the Poisson's equation, functions $v_{mn}(z)$ are the solutions to the ordinary differential equations [8]

$$v''_{mn}(z) - \lambda^2_{mn}\, v_{mn}(z) = f_{mn}(z), \quad (3)$$

where

$$f_{mn}(z) = \frac{-1}{\varepsilon_0}\int_{-T_1}^{T_1}\int_{-T_2}^{T_2}\rho(x,y,z)\cos\frac{\pi n x}{T_1}\cos\frac{\pi m y}{T_2}dy\,dx,$$

$$\lambda_{mn} = \sqrt{\left(\frac{\pi n}{T_1}\right)^2 + \left(\frac{\pi m}{T_2}\right)^2},\ m \ge 0,\ n \ge 0.$$

The function $\rho(x,y,z)$ in the second member of Poisson's equation will be considered as follows

$$\rho(x,y,z) = \begin{cases}\rho_i, (|x| < \delta_1, |y| < \delta_2, |z - Z_i| < \delta_3), \\ 0,\ (|x| > \delta_1, \text{or}\, |y| > \delta_2, \text{or}\, |z - Z_i| > \delta_3),\end{cases}$$

$$q_i = \lim 8\rho_i\delta_1\delta_2\delta_3\ (\delta_1 \to 0,\ \delta_2 \to 0,\ \delta_3 \to 0),$$
$$\rho_i = \text{const},\ i = \overline{1,N}.$$

Designate

$$\gamma_{mn} = \begin{cases} 1, & m > 0,\ n > 0, \\ 2, & n = 0\ \text{ or } m = 0,\ \ (m^2 + n^2 \ne 0).\end{cases}$$

Using the solutions to the equations (3) and after some transformations the potential distribution $V(x,y,z)$ can be written in an explicit form

$$V(x,y,z) = \frac{V_0}{Z_{N+1}}z + \frac{1}{\varepsilon_0 4T_1T_2}\times$$
$$\times\left(\sum_{i=0}^{k} q_i(Z_{N+1} - z)Z_i + \sum_{i=k+1}^{N} q_i(Z_{N+1} - Z_i)z\right) +$$

$$+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{\cos\lambda_{m0}\,y\,\cos\lambda_{0n}\,x}{\varepsilon_0 T_1 T_2\lambda_{mn}\gamma_{mn}}\times \quad (4)$$

$$\times\left(\sum_{i=0}^{k} q_i\frac{\mathrm{sh}\ \lambda_{mn}(Z_{N+1} - z)\ \mathrm{sh}\ \lambda_{mn}Z_i}{\mathrm{sh}\ \lambda_{mn}Z_{N+1}} + \right.$$
$$\left. + \sum_{i=k+1}^{N} q_i\frac{\mathrm{sh}\ \lambda_{mn}(Z_{N+1} - Z_i)\ \mathrm{sh}\ \lambda_{mn}z}{\mathrm{sh}\ \lambda_{mn}Z_{N+1}}\right),$$
$$Z_k < z < Z_{k+1}.$$

The problem of determining the values of charges q_i is reduced to the solution of the system of linear algebraic equations in order that the tip surface $z_0(x,y)$ is matched with the zero's equipotential (virtual cathode) in N points $(x_l, y_l, z_l = z_0(x_l, y_l))$:

$$\sum_{i=1}^{k} q_i\,A_{il} + \sum_{i=k+1}^{N} q_i\,B_{il} = C_l, \quad (5)$$

where

$$A_{il} = \frac{1}{\varepsilon_0 T_1 T_2}\left(\frac{1}{4}(Z_{N+1} - z_l)Z_i + \right.$$
$$+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\frac{\mathrm{sh}\ \lambda_{mn}(Z_{N+1} - z_l)\ \mathrm{sh}\ \lambda_{mn}Z_i}{\lambda_{mn}\gamma_{mn}\ \mathrm{sh}\ \lambda_{mn}Z_{N+1}}\times \quad (6)$$
$$\left.\times\cos\lambda_{m0}\,y_l\,\cos\lambda_{0n}\,x_l\right),$$

ISBN 978-3-95450-125-0

$$B_{il} = \frac{1}{\varepsilon_0 T_1 T_2}\left(\frac{1}{4}(Z_{N+1} - Z_i) z_l + \right.$$

$$+ \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} \frac{\mathrm{sh}\ \lambda_{mn}(Z_{N+1} - Z_i)\ \mathrm{sh}\ \lambda_{mn} z_l}{\lambda_{mn}\gamma_{mn}\ \mathrm{sh}\ \lambda_{mn} Z_{N+1}} \times \quad (7)$$

$$\left. \times \cos\lambda_{m0}\, y_l\ \cos\lambda_{0n}\, x_l \right),$$

$$C_l = -\frac{V_0}{Z_{N+1}} z_l, \quad (8)$$

$$Z_k < z_l < Z_{k+1},$$

$$k = \overline{0, N}, \quad l = \overline{1, N}.$$

CONCLUSION

In this work the multi-tip field cathode mathematical modeling is considered. The Poissons equation is solved by approximating the point charges. The analytical form of the potential distribution function (4) is obtained over the entire domain of the field emission cathode arrays. The problem of determining the values of charges is reduced to the solving the system of linear algebraic equations (5–8).

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ISBN 978-3-95450-125-0