THE TRIODE-TYPE SYSTEM ON THE BASIS OF THE FIELD EMITTER MODELING

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Abstract

The mathematical model of a cylindrical triode-type system on the basis of the field emitter is under consideration. The internal area of the system is filled of two different dielectrics. Effect of space charge is not considered. The field emitter is modeled by a charged filament of finite length, which located on the system's axis. The modulator has a form of a circular diaphragm. The Poisson equation with the given values of potentials at the electrodes is solved. The variable separation method is used to determine distribution of electrostatic potential. An unknown function of the charge density is approximated by a piecewise constant linear function. The problem of finding unknown coefficients in the potential eigenfunction expansion is reduced to the linear algebraic equations system. Numerical calculations emitter's forms are represented.

INTRODUCTION

Vacuum electronic devices based on the field emission are used in many areas of scientific research. Particularly in the development of new high-precision devices as an electron microscopes, flat panel displays, systems of surface diagnostics, devices of micro- and nano-electronics [1]. The main characteristics of these devices are small dimensions and low consumption of power for efficient operation. The field cathode makes it possible to generate emission of electrons at low values of the potentials in the system. The high current density provided by the small radius of curvature of the tip and does not require in consumption energy for heat the emission region. To improve the emission characteristics into the system usually include an additional electrode called as a modulator. This modulator allows to change the field close by the emitter within a wide range, with a low value of the potential [2].

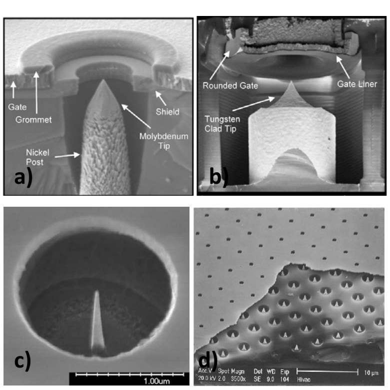

Figure 1: Pictures of cells triode-type systems based on field-emitter.

FORMULATION OF THE PHYSICAL PROBLEM

Consider a cylindrical triode-type system which consists of a substrate on which the field cathode is situated, the modulator in the form of a flat diaphragm and an anode (Fig. 2). The internal part of the system is filled by two different dielectrics with dielectric constants ε_1 and ε_2. One of them serves as a casing of dielectric shell. There is a tip on the axis of the system. This tip is modeled by charged filament which length is z_0. It is assumed that the cross geometrical dimensions of the field cathode are much smaller than the cross dimensions of the system. Cathode has a zero potential, the modulator's potential is V_1 ,the anode's potential is V_2. The main task consists in finding of potential distribution in the triode-type system with a field tip. (System's section with the axial symmetry)

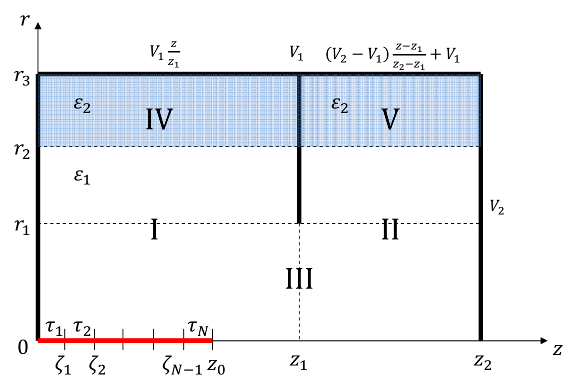

Figure 2: System's section with the axial symmetry.

FORMULATION OF THE MATHEMATICAL PROBLEM

The function of potential distribution must satisfy the Poisson's equation with considering an axial symmetry of the system

$$\triangle U(r,z) = -\frac{\rho(r,z)}{\varepsilon_0}, \tag{1}$$

with boundary conditions

$$\begin{aligned}
&U(r,0) = 0, \quad r \in [0, r_3],\\
&U(r,z_1) = V_1, \quad r \in [r_1, r_3],\\
&U(r,z_2) = V_2, \quad r \in [0, r_3],\\
&U(r_3,z) = V_1\frac{z}{z_1}, \quad z \in [0, z_1],\\
&U(r_3,z) = \frac{V_2 - V_1}{z_2 - z_1}(z - z_1) + V_1, \quad z \in [z_1, z_2],
\end{aligned} \tag{2}$$

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where $\rho(r,z)$ is volume density of the charge created by a charged filament.

The solution of the problem can be represented as a sum of two solutions — i.e. solutions of the Laplace's equation with inhomogeneous boundary conditions and the solution of the Poisson's equation with homogeneous boundary conditions. The solution of the Laplace's equation was found [3] and now it requires solving the Poisson's equation with an unknown right side.

Represent the unknown function $U(r,z)$ in (1) in the form [4]

$$U(r,z)=\sum_{n=1}^{\infty} V_n(z) J_0\left(\frac{\omega_n}{a}r\right), \tag{3}$$

where $V_n(z)$ is an unknown function, a is border of region on r. Define the potential distribution in the first and third areas, because only in these areas located charged filament.

Substitute (3) into (1) and use the orthogonality property of Bessel's functions J_0 will come to an inhomogeneous second order of differential equation

$$V_n{''}(z)-\left(\frac{\omega_n}{a}\right)^2 V_n(z)=$$
$$=-\frac{2}{a^2 {J_1}^2(\omega_n)}\int_0^{r_0} r\frac{\rho(r,z)}{\varepsilon_0}J_0\left(\frac{\omega_n}{a}r\right)dr,$$

where r_0 is the estimated radius of the tip.

Represent the volume density as a linear function

$$\rho_s(r,z)=\frac{\tau_s}{\pi r_s^2},\quad \tau_s=Ash,\quad h=\zeta_s-\zeta_{s-1},$$
$$z\in(\zeta_s,\zeta_{s-1}],\quad s=\overline{1,N}, \tag{4}$$

where A is an unknown coefficient. At sufficiently lesser values of r_0 the integral $\int_0^{r_0} rJ_0(\frac{\omega_n}{a}r)dr \approx \frac{r_0^2}{2}$ [5]. Introduce the notation

$$\varphi(z)=\begin{cases}-\dfrac{1}{a^2J_1^2(\omega_n)}\dfrac{Ash}{\pi\varepsilon_0}, & z\leqslant z_0,\\ 0, \quad z>z_0,\end{cases}$$

$$V_n{''}(z)-\left(\frac{\omega_n}{a}\right)^2 V_n(z)=\varphi(z). \tag{5}$$

Will try solution of equation (5) in the form

$$V_n(z)=a_n(z)\mathrm{ch}\left(\frac{\omega_n}{a}z\right)+b_n(z)\mathrm{sh}\left(\frac{\omega_n}{a}z\right).$$

The coefficients $a_n(z)$ and $b_n(z)$ should be determined from the equations, using the method of variation of constants

$$a_n{'}\mathrm{ch}\left(\frac{\omega_n}{a}z\right)+b_n{'}\mathrm{sh}\left(\frac{\omega_n}{a}z\right)=0,$$
$$\frac{\omega_n}{a}\left(a_n{'}\mathrm{ch}\left(\frac{\omega_n}{a}z\right)+b_n{'}\mathrm{sh}\left(\frac{\omega_n}{a}z\right)\right)=\varphi(z).$$

Determine the form of the function $V_n(z)$ by means of using homogeneous boundary conditions and considering the form (3). Obtain the potential distribution in the regions I and III in the general form

$$U(r,z)=\sum_{n=1}^{\infty}\frac{Ah}{\pi\varepsilon_0\omega_n^2J_1^2(\omega_n)}\frac{1}{\mathrm{sh}\left(\frac{\omega_n}{a}z^*\right)}\times$$
$$\times\left[\mathrm{sh}\left(\frac{\omega_n}{a}(z^*-z)\right)\sum_{s=1}^{m-1}s\left(\mathrm{ch}\left(\frac{\omega_s}{a}\zeta_s\right)-\mathrm{ch}\left(\frac{\omega_s}{a}\zeta_{s-1}\right)\right)-\right.$$
$$-m\mathrm{sh}\left(\frac{\omega_n}{a}(z^*-z)\right)\mathrm{ch}\left(\frac{\omega_s}{a}\zeta_{s-1}\right)+m\mathrm{sh}\left(\frac{\omega_n}{a}z^*\right)-$$
$$-\mathrm{sh}\left(\frac{\omega_n}{a}z\right)\times \tag{6}$$
$$\left.\times\sum_{s=m+1}^{N}s\left(\mathrm{ch}\left(\frac{\omega_n}{a}(z^*-\zeta_s)\right)-\mathrm{ch}\left(\frac{\omega_n}{a}(z^*-\zeta_{s-1})\right)\right)\right]\times$$
$$\times J_0\left(\frac{\omega_n}{a}r\right),\quad z\leqslant z_0,$$

$$U(r,z)=\sum_{l=1}^{\infty}\frac{Ah}{\pi\varepsilon_0\omega_n^2J_1^2(\omega_n)}\frac{\mathrm{sh}\left(\frac{\omega_n}{a}(z^*-z)\right)}{\mathrm{sh}\left(\frac{\omega_n}{a}z^*\right)}\times$$
$$\times\sum_{s=1}^{N}s\left(\mathrm{ch}\left(\frac{\omega_s}{a}\zeta_s\right)-\mathrm{ch}\left(\frac{\omega_s}{a}\zeta_{s-1}\right)\right)\times \tag{6$'$}$$
$$\times J_0\left(\frac{\omega_n}{a}r\right),\quad z>z_0.$$

where z_0 is border of region on z.

As a result, the distribution function of the system area considering linear approximation of density values of the charged wire is obtained. Also introduce two additional equations for the calculation of the coefficient A Using the functions (6) and (6′) set the point in which the zero equipotential will determine the length of the cathode [6].

Solving the system, which binds unknown coefficients, define their values, and obtain a function of the potential distribution in the entire system.

THE NUMERICAL RESULTS

The program is written on the basis of theoretical calculations. This program serves to solve the system which has to do with unknown coefficients. In the calculation of following parameters were used: $r_1=0.5$, $r_2=0,75$, $r_3=1$, $z_1=0.5$, $z_2=1$, $\varepsilon_1=1$, $\varepsilon_2=4.7$, $V_1=1$, $V_2=1$. The length of the charged wire is $z_0=0.4$. The infinite sums were limited by the number 35, which is sufficient for the required accuracy of calculations The values of the geometric parameters and the electrostatic potential are given in relation to the respective maximum values.

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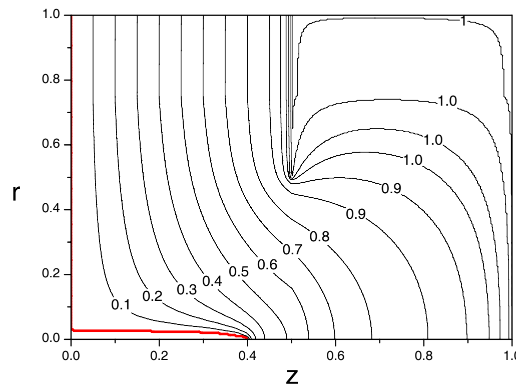

Figure 3: Graph of the potential distribution in the system area.

It is obvious that the boundary conditions is accomplished and form of the field cathode is marked in red.

On the Fig. 4 it makes clear the equipotential lines of the potential distribution close by the tip. Form of the tip is marked in red.

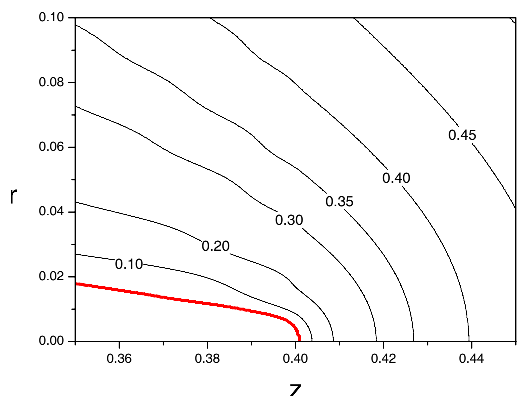

Figure 4: Graph of the potential distribution close by the tip.

On the Fig. 5 graph of the charge density is represented. This graph is calculated by the program.

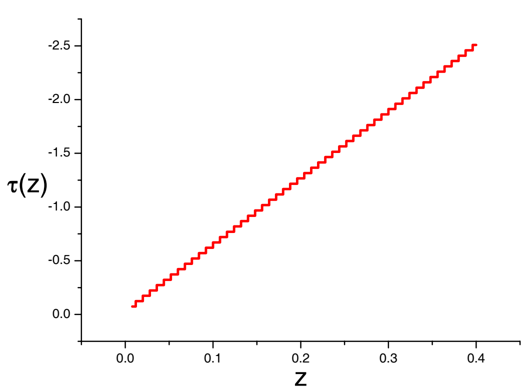

Figure 5: Graph of the charge density.

On the Fig. 6 are represented: the form of field cathode (in red denotes) and the field intensity at the axis (in blue denotes).

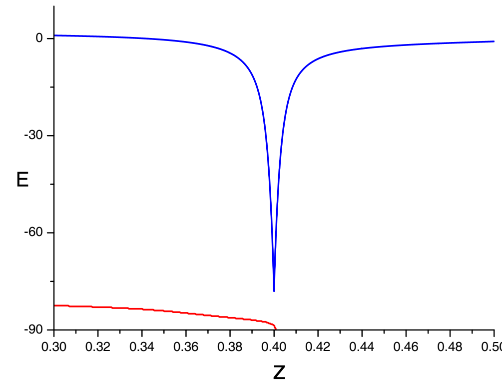

Figure 6: Graph of the field intensity on the systems axis.

It is seen that close by the tip the field intensity rapidly increases in the absolute value.

CONCLUSION

In this work consider the problem of modeling triode-type electron-optical system with a field tip. The system area is filled by two different dielectrics with different dielectric constants. The Poisson's equation is solved by approximating the charge density filaments, which is located on the axis of the system. The analytical form of the potential distribution function is obtained in the system area. According to the results of the numerical experiment the graphs of equipotential lines is represented.

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