THE FIELD CATHODES WITH THE EFFECT OF SPACE CHARGE MODELING

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Abstract

This work is devoted to the question of the effect of space charge on the field electron emission. The electrostatic potential distributions for the diode emission systems are calculated. The diode systems, which can be readily constructed, are generally used for the characterization of field emission properties of novel materials. They have some effective applications in vacuum nano- and microelectronics. In this work the plane diode emission system and cylindrical diode emission system are investigated. The solutions of Poisson's equation for the electrostatic potential distribution are received for the boundary-value problems. The right side of Poisson's equation is assumed to be the piece-wise constant function. The charge conservation law and the energy conservation law are used. One and two dimensional cases are investigated.

INTRODUCTION

Currently there are great interest in research and practical applications of the field electron emitters, where emission occurs from a nanoscale inclusions of conducting material, carbon nanotubes and fibers, protrusions of nanometer size. Microfabricated field emission arrays (FEAs) have been studied extensively both theoretically and experimentally. FEAs are considered as excellent candidates for use as electron sources operating with high efficiency, high currents for vacuum electronic applications.Much effort has been directed towards the commercial applications of FEAs, including their use as electron sources in various types of visualization equipment, including lithography, x-ray sources, microscopes, high-power microwave amplifiers, transistors and especially for generation high-brightness flat panel displays [1, 2, 3].Field emission diodes and triodes are the most commonly used device architectures for FEAs. The diode structure, which can be readily constructed, is generally used in laboratories for the characterization of field emission properties of novel materials [1].

Electron field emission from a single emitter is a barrier tunneling, quantum mechanical process that can be described by the Fowler-Nordheim equation. At high emission current densities, however, the space charge caused by the cathode may affect the current densityvoltage characteristics predicted by the Fowler-Nordheim theory [1, 4, 5]. This work is devoted to the investigation of the effect of space charge on the field electron emission. Plane diode emission system and cylindrical diode emission system are considered.

MATHEMATICAL MODELING AND CALCULATION

Plane Diode Emission System

The problem is to calculate electrostatic potential distribution in the region between the electrodes of the plane diode emission system. This is a simple case of the electrode configuration of diode system [6]. At first a one-dimensional case was studied. Potential distribution is described by the Poisson's equation

$$\triangle \bar{U}_p(z) = -\bar{\rho}_p(z) \tag{1}$$

with following boundary conditions:

$$\bar{U}_p(z_1) = 0, \quad \bar{U}_p(z_2) = V. \tag{2}$$

Function $\bar{U}_p(z)$ is the potential distribution; $\bar{\rho}_p(z) = \frac{\bar{\rho}_d^*(z)}{\varepsilon_0}$, where $\bar{\rho}_d^*(z)$, is the space charge density, ε_0 is the vacuum dielectric constant. Function $\bar{\rho}_p(z)$ is unknown. In our work we assumed that these function is a piece-wise constant function

$$\bar{\rho}_p(z) = \begin{cases} \widetilde{\rho}_1^1, & z \in [R_1 = z_1, R_2), \\ \widetilde{\rho}_2^1, & z \in [R_2, R_3), \\ \dots \\ \widetilde{\rho}_N^1, & z \in [R_N, R_{N+1} = z_2]. \end{cases}$$

where N is the number of parts for which the region between the electrodes is divided. Solution of (1) with boundary conditions (2) is

$$\bar{U}_p(x,z) = \sum_{s=1}^{k-1} \widetilde{\rho}_s^1 \bar{P}_{s_1}(x,z) + \sum_{s=k+1}^{N} \widetilde{\rho}_s^1 \bar{P}_{s_2}(x,z) + \\ + \widetilde{\rho}_k^1 \bar{P}_k(x,z) + \bar{L}_p(z),$$

where $\bar{P}_{s_1}, \bar{P}_{s_2}, \bar{P}_k, \bar{L}_p$ is a several known functions, $\widetilde{\rho}_i^1$ is unknown values. To find these values we considered the equations

$$\mathrm{div} \vec{j}_p = 0, \tag{3}$$

$$\frac{1}{2} m \vec{V}_p^{\,2} = -e \bar{U}_p(z), \tag{4}$$

where $\vec{j}_p$ is a current density, $\vec{V}_p$ is a speed of the electrons, e is the electron charge. Eq. 3 is the current continuity equation, Eq. 4 is the energy conservation law. Combining these equation gave

$$\bar{\rho}_p(z) \sqrt{\bar{U}_p(z)} = A = \mathrm{const},$$

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where constant A can be determined from the boundary condition at the anode, $A = \widetilde{\rho}_N^1 \sqrt{V}$. From the each interval (R_s, R_{s+1}) was chosen for one z_s^* and was written the system of equations

$$\vec{F}_p^1 = \vec{0}, \quad (5)$$

where $\vec{F}_p^1$ is a vector-function,

$$\vec{F}_p^1 = \begin{pmatrix} \widetilde{\rho}_1^1 \sqrt{\bar{U}(z_1^*)} - \rho_N \sqrt{V} \\ \widetilde{\rho}_2^1 \sqrt{\bar{U}(z_2^*)} - \rho_N \sqrt{V} \\ \vdots \\ \sqrt{\bar{U}(z_N^*)} - \sqrt{V} \end{pmatrix}.$$

System Eq.5 is a system of algebraic nonlinear equations; it was solved by numerically using Newton's method.

Next the two-dimensional case was studied. Field that arises in the region between the electrodes, is described by the Poisson's equation

$$\triangle U_p(x,z) = -\rho_p(z). \quad (6)$$

Simplification was done: the length of the diode system for the x-axis is much greater than the distance between the electrodes. Therefore $\rho_p(z)$ depends only of x. Function $\rho_p(z)$ is unknown. We have assumed that these function is a piece-wise constant function with "pieces" $\widetilde{\rho}_i^2, i = \overline{1,N}$. The boundary conditions

$$U_p(0,z) = V\frac{z-z_1}{z_2-z_1}, \qquad U_p(x_1,z) = V\frac{z-z_1}{z_2-z_1},$$
$$U_p(x,z_1) = 0, \qquad U_p(x,z_2) = V$$

for Eq. 6 was imposed. Solution of the boundary problem for the Poisson's equation was obtained as

$$U_p(x,z) = \sum_{s=1}^{k-1} \widetilde{\rho}_s^2 P_{s_1}(x,z) + \sum_{s=k+1}^{N} \widetilde{\rho}_s^2 P_{s_2}(x,z) + \\ + \widetilde{\rho}_k^2 P_k(x,z) + L_p(z).$$

where $P_{s_1}, P_{s_2}, P_k, L_p$ is a several known functions, $\widetilde{\rho}_i$ is unknown values.

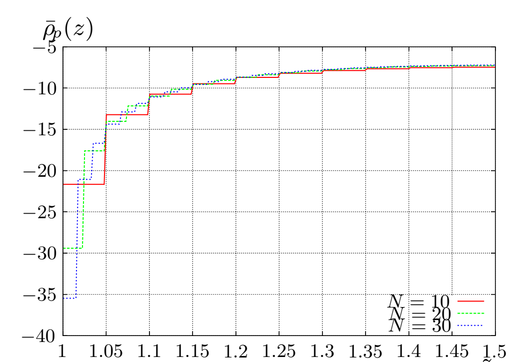

Figure 1: Space charge density, plane diode, one-dimensional case

To find them current continuity equation and energy conservation law were used. In this case according with configuration of the system current density and speed of electrons can be considered depending only of z. Potential distribution U_p can be regarded with a fixed value of x. So

$$\rho_p \sqrt{U_p(x^*, z)} = \widetilde{\rho}_N^2 \sqrt{V}. \quad (7)$$

Using Eq.7 was formed a system of equations for finding the unknowns $\widetilde{\rho}_i^2$ as in a previous case. It was solved by numerically using Newton's method.

Cylindrical diode emission system

Electrodes of these diode system are two axisymmetric cylinders. In one-dimension case equation

$$\triangle \bar{U}_c(r) = -\frac{\bar{\rho}_c(r)}{\varepsilon_0}$$

with boundary conditions

$$\bar{U}_c(r_1) = 0, \quad \bar{U}_c(r_2) = V.$$

was solved. Function $\bar{\rho}_c(r)$ was regarded as piece-wise constant function. Solving of these problem can be derived as

$$\bar{U}_c(r) = \sum_{s=1}^{k-1} \hat{\rho}_s^1 \bar{C}_{s_1}(r) + \sum_{s=k+1}^{N} \hat{\rho}_s^1 \bar{C}_{s_2}(r) + \\ + \hat{\rho}_k^1 \bar{C}_k(r) + \bar{L}_c(z).$$

where $\bar{C}_{s_1}, \bar{C}_{s_2}, \bar{C}_k, \bar{L}_c$ is a known functions. To find the unknown $\hat{\rho}_i^1$ current continuity equation and energy conservation law were used. Through the equation

$$r\bar{\rho}_c(r)\sqrt{\bar{U}_c(r)} = r_2 \hat{\rho}_N^1 \sqrt{V}.$$

the system of nonlinear algebraic equations for the unknowns was composed. To solve these system Newton's method was applied. During the consideration of the two-dimensional case it was supposed that the length of this system is much greater than the distance between the cylinders. Then the function $\rho_c(r) = \frac{\rho_c^*(r)}{\varepsilon_0}$ can be regarded as depending only of r.

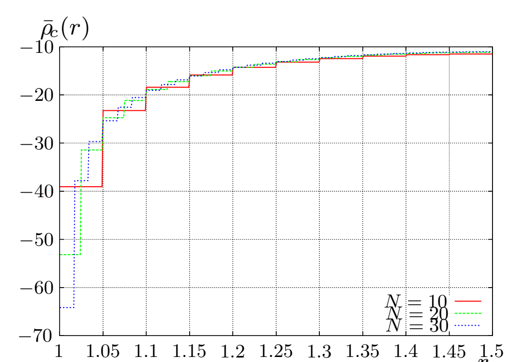

Figure 2: Space charge density, cylindrical diode, one-dimensional case

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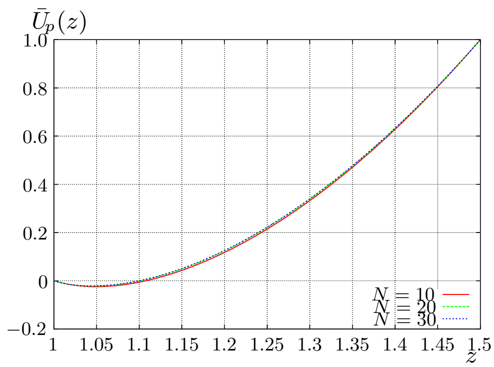

Figure 3: Potential distribution, plane diode, one-dimensional case

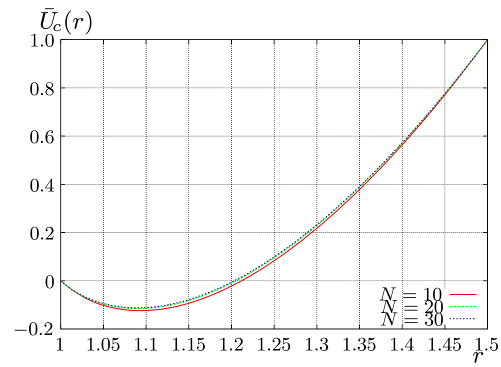

Figure 4: Potential distribution, cylindrical diode, one-dimensional case

At the boundaries of the system was set the following conditions

$$U_c(r,0) = V\frac{\ln\left(\frac{r_1}{r}\right)}{\ln\left(\frac{r_1}{r_2}\right)}, \qquad U_c(r,z_1) = V\frac{\ln\left(\frac{r_1}{r}\right)}{\ln\left(\frac{r_1}{r_2}\right)},$$
$$U_c(r_1,z) = 0, \qquad U_c(r_2,z) = V.$$

Function $\rho_c(r)$ is unknown piece-wise constant function.

The potential distribution was obtained as

$$U_c(r,z) = \sum_{s=1}^{k-1} \hat{\rho}_s^2 C_{s_1}(r,z) + \sum_{s=k+1}^{N} \hat{\rho}_s^2 C_{s_2}(r,z) + \\ + \hat{\rho}_k^2 C_k(r,z) + L_c(r),$$

where C_{s_1}, C_{s_2} and C_k can be defined in explicit form by Bessel function. System of equations for calculation $\hat{\rho}_i^2$ was obtained as in the previous tasks.

RESULTS

Calculations were done using the program that was written on C++. The following parameters were used:
$z_1 = 1, \quad z_2 = 1{,}5, \quad x^* = 0{,}5\,l, \quad l = 100, \quad V = 1$ — for the plane diode emission system;
$r_1 = 1, \quad r_2 = 1{,}5, \quad z^* = 0{,}5\,l, \quad l = 100, \quad V = 1$ — for the cylindrical diode emission system. Parameter l is the length of the diode system.

The Fig. 1 and Fig. 2 shows the space charge density at different values N for the plane and cylindrical diodes correspondingly. Can be seen that in both cases the approximation converge to a function that describes the space charge density in a region between the electrodes. Herewith the space charge density tends to infinity near the cathode. Calculations show that the density of the space charge for one- and two-dimensional cases differ by no more than in fourth decimal place.

The space charge becomes the reason potential drop especially in the region near the cathode. As shown in Fig.3 and Fig. 4 the potential near the cathode is negative. When the anode voltage reduced potential also had a negative values. Comparison distribution potential values for one- and two-dimensional cases shows that the difference in the fifth decimal place (for any fixed values of the second coordinate in the two-dimensional cases).

CONCLUSION

In this paper was considered the problem of modeling the diode emission system with regard to space charge. Poisson's equation with the boundary conditions was solved in one- and two-dimensional cases for the plane and cylindrical diode emission systems. The solution in all cases was obtained analytically. Calculations showed that the piecewise constant approximation converges to a function that describes the distribution of the space charge density in the space between the electrodes. The space charge density tends to infinity near the cathode. Calculations of the electrostatic potential showed that the space charge causes the fall of potential.

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