THE KNIFE-EDGED FIELD EMITTER MATHEMATICAL MODELING

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Abstract

Numerous nano-scale electronic devices are based on the field emitters such as carbon nanotubes. The field emitters are extensively applied in the various domains of an instrument engineering. In the present work the problem of a field emission cathode as the knife-edged field emitter mathematical modeling is solved. The supposed shapes of the emission diode system with the field emitter are the lune's type (as a cathode) and the infinitely thin spherical segment (as an anode). The effect of the space charge is neglected. The boundary – value problem for the Laplace equation in the toroidal coordinate system is presented. To solve the electrostatic problem the variable separation method is used. The potential distribution is represented as the series with respect to Legendre functions. The boundary conditions and the normal derivative continuity conditions lead to the linear algebraic equations system relative to the series coefficients. In this way the distribution of the potentials for the whole region of the considered electro-optical systems was obtained.

INTRODUCTION

Field emitters (FE) have unique parameters for industrial applications in the domain of vacuum micro- and nano-electronics — scanning electron microscopy, x-ray sources, emission displays, parallel e-beam lithography, etc. FE are manufactured of various materials with different morphologies [1–4].

In the present work the rotationally symmetric knife-edged field emitter mathematical modeling is under investigation (see Fig.1).

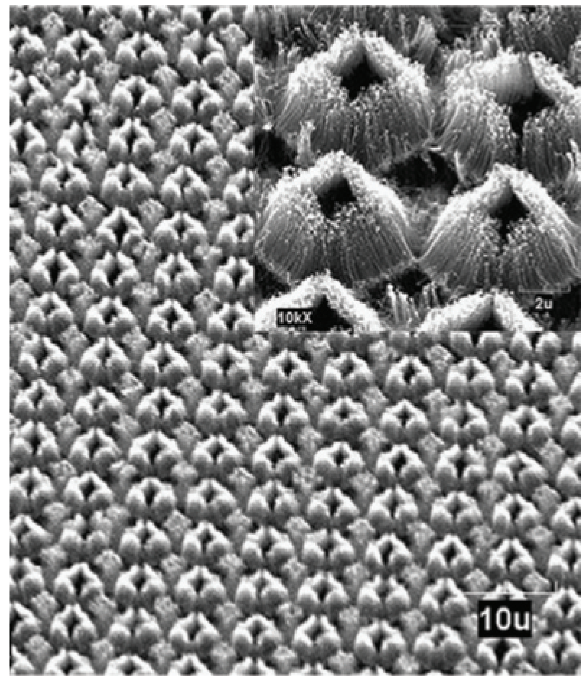

Figure 1: SEM images for aligned and patterned carbon nanotube emitters [1].

PROBLEM BACKGROUND

It is presented the solution to Laplace's equation for the axisymmetric diode systems: cathode is simulated by two spherical segments with the toroidal top (lune's type), an anode modeled by thin spherical segment (see Fig.2).

To solve the rotationally symmetric electrostatic problem the variable separation method is employed. The toroidal coordinate system (α, β, φ) is used.

The parameters of the problem are as follows:

$\beta = \beta_1$ $(0 \leq \alpha \leq \alpha_1)$ — the surface of anode;

$\beta = -\beta_2$, $\beta = 2\pi - \beta_3$ $(0 \leq \alpha \leq \alpha_0)$ and

$\alpha = \alpha_0$, $(\beta_2 \leq \beta \leq 2\pi - \beta_3)$ — the surface of tip;

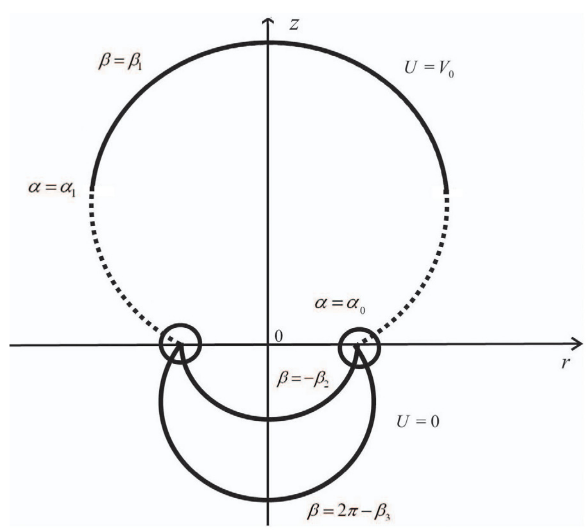

Figure 2: Schematic diagram of the diode systems based on a knife-edged field emitter.

$U(\alpha, \beta_1) = V_0$ $(0 \leq \alpha \leq \alpha_1)$ — the boundary condition on anode.

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The potential of the tip is assumed to be zero without the loss of general character of the problem.

MATHEMATICAL MODELING AND CALCULATION

The potential distribution $U(\alpha,\beta)$ satisfies the Laplace equation. Thus, it is necessary to solve the following boundary – value problem

$$\begin{cases} \triangle U(\alpha,\beta)=0, & \\ U(\alpha,\beta_1)=V_0, & 0\le\alpha<\alpha_1; \\ U(\alpha,-\beta_2)=0, & 0\le\alpha<\alpha_0; \\ U(\alpha,2\pi-\beta_3)=0, & 0\le\alpha<\alpha_0; \\ U(\alpha_0,\beta)=0, & \beta_2\le\beta\le 2\pi-\beta_3. \end{cases} \quad (1)$$

In order to solve the boundary – value problem (1), the entire region of the diode system is split into 3 subregions with the potential distribution $u_i(\alpha,\beta)$ as

$$U(\alpha,\beta)=\begin{cases} u_1(\alpha,\beta), & -\beta_2<\beta<\beta_1, \\ & 0\le\alpha<\alpha_0; \\ u_2(\alpha,\beta), & \beta_1<\beta<2\pi-\beta_3, \\ & 0\le\alpha<\alpha_0; \\ u_3(\alpha,\beta), & -\beta_2<\beta<2\pi-\beta_3, \\ & \alpha_1\le\alpha<\alpha_0. \end{cases}$$

Designate

$$W\left(-\tfrac{1}{2}+\eta_n,\alpha,\alpha_0\right)= = P_{-\frac{1}{2}+\eta_n}(\cosh\alpha)\,Q_{-\frac{1}{2}+\eta_n}(\cosh\alpha_0)- -P_{-\frac{1}{2}+\eta_n}(\cosh\alpha_0)\,Q_{-\frac{1}{2}+\eta_n}(\cosh\alpha),$$

where $P_{-\frac{1}{2}+\eta_n}(\cosh\alpha)$, $Q_{-\frac{1}{2}+\eta_n}(\cosh\alpha)$ — Legendre functions of the first and second kind.

The potential distribution functions $u_i(\alpha,\beta)$ can be represented as series [5,6]

$$u_1(\alpha,\beta)=\sqrt{\cosh\alpha+\cos\beta}\times \times\sum_{l=1}^{\infty}A_l\frac{\sinh(\beta+\beta_2)\tau_l}{\sinh(\beta_1+\beta_2)\tau_l}P_{-\frac{1}{2}+i\tau_l}(\cosh\alpha), \quad (2)$$

$$u_2(\alpha,\beta)=\sqrt{\cosh\alpha+\cos\beta}\times \times\sum_{l=1}^{\infty}A_l\frac{\sinh(2\pi-\beta_3-\beta)\tau_l}{\sinh(2\pi-\beta_3-\beta_1)\tau_l}\times \times P_{-\frac{1}{2}+i\tau_l}(\cosh\alpha). \quad (3)$$

$$u_3(\alpha,\beta)=\sqrt{\cosh\alpha+\cos\beta}\times \times\sum_{n=1}^{\infty}B_n\sin\eta_n(\beta+\beta_2)\times \times\frac{W\left(-\frac{1}{2}+\eta_n,\alpha,\alpha_0\right)}{W\left(-\frac{1}{2}+\eta_n,\alpha_1,\alpha_0\right)}, \quad (4)$$

where

τ_l — the roots of equations $P_{-\frac{1}{2}+i\tau_l}(\cosh\alpha_0)=0$,

$\eta_n=\dfrac{\pi n}{2\pi+\beta_2-\beta_3}$.

Using the boundary conditions (1) and the continuity conditions for the potential distribution functions on the curves separating the subregions, we obtain equations of the form

$$u_1(\alpha,\beta_1)=\begin{cases} V_0, & 0\le\alpha<\alpha_1, \\ u_3(\alpha,\beta_1), & \alpha_1\le\alpha<\alpha_0, \end{cases} \quad (5)$$

$$u_3(\alpha_1,\beta)=\begin{cases} u_1(\alpha_1,\beta), & -\beta_2\le\beta<\beta_1, \\ u_2(\alpha_1,\beta), & \beta_1\le\beta<2\pi-\beta_3. \end{cases} \quad (6)$$

Substituting the potential distribution functions (2) and (4) into equation (5), we can write

$$\sqrt{\cosh\alpha+\cos\beta_1}\sum_{l=1}^{\infty}A_lP_{-\frac{1}{2}+i\tau_l}(\cosh\alpha)= =\begin{cases} V_0, & 0\le\alpha<\alpha_1; \\ \sqrt{\cosh\alpha+\cos\beta_1}\left(\sum_{n=1}^{\infty}C_n\sin\eta_n(\beta_1+\beta_2)\times \times\frac{W\left(-\frac{1}{2}+\eta_n,\alpha,\alpha_0\right)}{W\left(-\frac{1}{2}+\eta_n,\alpha_1,\alpha_0\right)}\right), & \alpha_1\le\alpha<\alpha_0, \end{cases}$$

and after some transformations we arrive the first system of linear algebraic equations relative to coefficients A_l, B_n:

$$A_l=\sum_{n=1}^{\infty}B_nT_{l,n}+R_l, \quad (7)$$

where

$$T_{l,n}=(N_l)^{-1}\int_{\alpha_1}^{\alpha_0}P_{-\frac{1}{2}+i\tau_l}(\cosh\alpha)\times \times\frac{W\left(-\frac{1}{2}+\eta_n,\alpha,\alpha_0\right)}{W\left(-\frac{1}{2}+\eta_n,\alpha_1,\alpha_0\right)}\times \times\sin\eta_n(\beta_1+\beta_2)\sinh\alpha\,d\alpha, \quad (8)$$

$$R_l=V_0(N_l)^{-1}\times \times\int_0^{\alpha_1}\frac{P_{-\frac{1}{2}+i\tau_l}(\cosh\alpha)}{\sqrt{\cosh\alpha+\cos\beta_1}}\sinh\alpha\,d\alpha, \quad (9)$$

$$N_l=\int_0^{\alpha_0}\left(P_{-\frac{1}{2}+i\tau_l}(\cosh\alpha)\right)^2\sinh\alpha\,d\alpha. \quad (10)$$

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Using the expansions of potential distribution functions (2,3,4) as well as equation (6), we can write

$$\sqrt{\cosh\alpha_1+\cos\beta}\sum_{n=1}^{\infty}C_n\sin\eta_n\left(\beta+\beta_2\right)=$$
$$=\begin{cases}\sqrt{\cosh\alpha_1+\cos\beta}\sum\limits_{l=1}^{\infty}A_l\dfrac{\sinh\left(\beta+\beta_2\right)\tau_l}{\sinh\left(\beta_1+\beta_2\right)\tau_l}\times\\ \times P_{-\frac{1}{2}+i\tau_l}\left(\cosh\alpha_1\right), \qquad -\beta_2\le\beta<\beta_1;\\ \sqrt{\cosh\alpha_1+\cos\beta}\sum\limits_{l=1}^{\infty}A_l\dfrac{\sinh\left(2\pi-\beta_3-\beta\right)\tau_l}{\sinh\left(2\pi-\beta_3-\beta_1\right)\tau_l}\times\\ \times P_{-\frac{1}{2}+i\tau_l}\left(\cosh\alpha_1\right), \qquad \beta_1\le\beta<2\pi-\beta_3,\end{cases}$$

and after some transformations we arrive the second system of linear algebraic equations relative to coefficients A_l, B_n:

$$B_n=\sum_{l=1}^{\infty}A_lS_{n,l}, \tag{11}$$

where

$$S_{n,l}=\frac{2}{2\pi-\beta_3+\beta_2}P_{-\frac{1}{2}+i\tau_l}\left(\cosh\alpha_1\right)\times$$
$$\times\left(\frac{-\tau_l}{\tau_l^2+\eta_n^2}\frac{\sin\pi n}{\sinh\tau_l\left(2\pi-\beta_3-\beta_1\right)}+\right.$$
$$+\frac{\tau_l}{\tau_l^2+\eta_n^2}\sin\eta_n\left(\beta_1+\beta_2\right)\times \tag{12}$$
$$\times\left(\coth\tau_l\left(\beta_1+\beta_2\right)+\right.$$
$$\left.\left.+\coth\tau_l\left(2\pi-\beta_3-\beta_1\right)\right)\right).$$

CONCLUSION

In this work the problem of the axisymmetric diode system with a field emission cathode as the knife-edged field emitter mathematical modeling is considered. The cathode is simulated by two spherical segments with the toroidal top, an anode modeled by thin spherical segment. To solve the rotationally symmetric electrostatic problem the variable separation method is employed.

The distribution of electrostatic potential (2–4) is calculated over the entire domain of the diode system. The solution of the boundary value (1) is reduced to solving the system of linear algebraic equations (7–12).

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