INVESTIGATION OF PROGRAM AND PERTURBED MOTIONS OF PARTICLES IN LINEAR ACCELERATOR

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Abstract

Beam control model for program and perturbed motions with interaction account is realized. Quality functional gradient is obtained.

BEAM DYNAMICS EQUATIONS

The problem under study is longitudinal beam dynamics control in linear waveguide electron accelerator. Let coordinate axis Oz be aligned the waveguide symmetry axis. Accelerating wave fundamental mode in device axis neighborhood is described by the expression [1, 2]:

$$E_z(z,t) = E_0(z)\sin\left(\omega\int_0^z \frac{d\eta}{v_{ph}(\eta)} - \omega t + \varphi_0\right).$$

Here z is longitudinal coordinate, t is the time, $\omega = \frac{2\pi c}{\lambda}$, c is velocity of light; λ, $E_0(z)$, $v_{ph}(z)$ and φ_0 are correspondingly wavelength, intensity amplitude, phase velocity value and initial phase of accelerating wave fundamental mode.

Longitudinal beam dynamics equations are as follows [2]:

$$\frac{d\xi}{d\tau} = \frac{p}{\sqrt{1+p^2}},$$
$$\frac{dp}{d\tau} = -\alpha(\xi)\sin\varphi - \frac{e\lambda}{m_0 c^2}E_z^{(\rho)}(\xi\lambda),. \qquad (1)$$
$$\varphi(\tau) = 2\pi\int_0^{\xi(\tau)} \frac{d\eta}{\beta_{ph}(\eta)} - 2\pi\tau + \varphi_0.$$

Independent variable $\tau = \frac{ct}{\lambda}$ is introduced to be analogue of time to provide convenient Coulomb field account; $\xi = \frac{z}{\lambda}$; $p = \frac{mv_z}{m_0 c}$; m_0 and m are correspondingly rest mass and relativistic mass of electron, v_z is particle velocity longitudinal component; $\alpha(\xi) = \frac{e\lambda E_0(\xi\lambda)}{m_0 c^2}$; e is absolute value of electron charge; φ is particle phase; $E_z^{(\rho)}(\xi\lambda)$ is longitudinal component of Coulomb field intensity (integral representation of this field is given hereinafter in Eq. 5); $\beta_{ph}(\xi) = \frac{v_{ph}(\lambda\xi)}{c}$.

The problem of beam control in such a system was treated by Dmitry Ovsyannikov [2] as trajectory ensemble control problem; $\alpha(\xi)$ and $\beta_{ph}(\xi)$ were assumed to be control functions.

PROGRAM AND PERTURBED MOTIONS CONTROL

To continue the research, let us describe beam evolution in terms of program motion and the ensemble of perturbed motions. This approach was suggested by Alexander Ovsyannikov and successfully applied for beam control in RFQ structures [3, 4, 5, 6, 7]. The same approach was used for longitudinal beam dynamics investigation in traveling-wave linear accelerator [8] without Coulomb field account. The idea is to describe the controlled process as a combination of program motion (i.e. the motion of assigned particle) and perturbed motions ensemble (with respect to program motion). Assigned particle is the object with special dynamics equations; its initial position coincides with bunch centre and its velocity coincides with accelerating wave phase velocity $v_{ph}(z)$. Parametrized function $\varphi_a(\xi,\boldsymbol{\theta})$ (where $\boldsymbol{\theta}$ is control parameters vector) characterize the phase of assigned particle; $\varphi_a(\xi,\boldsymbol{\theta})$ is assumed to be control function instead of $\beta_{ph}(\xi)$, which is expressed in terms of assigned particle impulse. Consequently, initial control may be constructed with due account of requirements on program motion and provide rather high quality of controlled process even at optimization start. In particular, $\varphi_a(\xi,\boldsymbol{\theta})$ mathematical form may be chosen to follow synchronous phase variation tendency observed for "good" beam dynamics. Besides, different mathematical descriptions and simultaneous optimization of program motion and the ensemble of perturbed motions provide additional possibilities of beam dynamics optimization by means of control action upon assigned particle and upon beam particles.

In this paper simultaneous optimization approach is applied to time-dependent beam dynamics model with particle interaction account. This model is derived on the base of Eq. 1.

Let coordinates and characteristics of assigned particle have index "a". Program motion is described by the equations:

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$$\frac{d\xi_a}{d\tau} = \frac{p_a}{\sqrt{1+p_a^2}}, \qquad (2)$$
$$\frac{dp_a}{d\tau} = -\alpha(\xi_a,\boldsymbol{\theta})\sin\varphi_a(\xi_a,\boldsymbol{\theta}).$$

Beam dynamics is described by equations for perturbations of particle motions with respect to program motion:

$$\frac{d\tilde{\xi}}{d\tau} = \frac{\tilde{p}+p_a}{\sqrt{1+(\tilde{p}+p_a)^2}} - \frac{p_a}{\sqrt{1+p_a^2}},$$
$$\frac{d\tilde{p}}{d\tau} = -\alpha(\tilde{\xi}+\xi_a,\boldsymbol{\theta})\sin\varphi(\tau) + \alpha(\xi_a,\boldsymbol{\theta})\sin\varphi_a(\xi_a,\boldsymbol{\theta}) -$$
$$-\frac{e\lambda}{m_0c^2}E_z^{(\rho)}\left(\lambda\cdot\left(\tilde{\xi}+\xi_a\right)\right), \qquad (3)$$
$$\varphi(\tau) = 2\pi\int_0^\tau H(p_a(\nu),\tilde{p}(\nu))d\nu + \varphi_0,$$
$$H(x,y) = \frac{x+y}{\sqrt{1+(x+y)^2}}\cdot\frac{\sqrt{1+x^2}}{x} - 1.$$

Here $\tilde{\xi} = \xi - \xi_a$, $\tilde{p} = p - p_a$. Programmed control is introduced to be vector of parameters: $\mathbf{u}(\tau) = \boldsymbol{\theta}$.

Based on beam dynamics simulation experience the following parameterization of functions $\alpha(\xi,\boldsymbol{\theta})$ and $\varphi_a(\xi,\boldsymbol{\theta})$ is suggested:

$$\alpha(\xi,\boldsymbol{\theta}) = \theta_1 arctg(\theta_2\xi+\theta_3)+\theta_4,$$
$$\varphi_a(\xi,\boldsymbol{\theta}) = \frac{-\pi/2\cdot(arctg(\theta_5(\xi-\theta_6))+arctg(\theta_5\theta_6))}{arctg(\theta_5(L/\lambda-\theta_6))+arctg(\theta_5\theta_6)} \qquad (4)$$

Here L is accelerator length. We construct $\varphi_a(\xi,\boldsymbol{\theta})$ to follow synchronous phase variation tendency noted for "good" beam dynamics: $\varphi_a(\xi,\boldsymbol{\theta})$ is assumed to have small negative value in bunching part of the device and monotone decrease until $-\pi/2$ in accelerating part. Due to mathematical form of expressions Eq. 4 there exists some compact $U \subset R^6$ of parameters values providing "rather good" beam evolution. Choosing initial control $\boldsymbol{\theta}_0 \in U$ we obtain "rather good" beam dynamics at optimization start and may improve beam quality using search in the compact U.

Control process quality criterion is introduced as linear combination of two integral functionals. The first functional characterizes program motion and estimates the square of difference between the assigned particle exit energy and required exit energy. The second functional characterizes the behavior of perturbed motions ensemble and presents integral measure of beam trajectories perturbation with respect to program trajectory. Mathematical form of these functionals is given by Eq. 7, 8. The problem is formulated to minimize the criterion.

MATHEMATICAL OPTIMIZATION MODEL

The control problem under study belongs to the class introduces by following mathematical model. Controlled process is described by integro-differential equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(t,\mathbf{x},\mathbf{u}),$$
$$\frac{d\mathbf{y}}{dt} = \mathbf{F}\left(t,\mathbf{x},\mathbf{y},\int_0^t h(\tau,\mathbf{x}(\tau),\mathbf{y}(\tau))d\tau,\mathbf{u}\right) =$$
$$= \mathbf{F}_1\left(t,\mathbf{x},\mathbf{y},\int_0^t h(\tau,\mathbf{x}(\tau),\mathbf{y}(\tau))d\tau,\mathbf{u}\right) + \qquad (5)$$
$$+ \int_{M_{t,\mathbf{u}}} \mathbf{F}_2(t,\mathbf{x},\mathbf{y},\mathbf{z}_t)\rho(t,\mathbf{z}_t)d\mathbf{z}_t,$$
$$\frac{d\rho}{dt} = -\rho(t,\mathbf{y})div_{\mathbf{y}}\mathbf{F}\left(t,\mathbf{x},\mathbf{y},\int_0^t h(\tau,\mathbf{x}(\tau),\mathbf{y}(\tau))d\tau,\mathbf{u}\right).$$

Initial conditions are as follows:

$$\mathbf{x}(0) = \mathbf{x}_0,\ \mathbf{y}(0) = \mathbf{y}_0,\ \rho(0,\mathbf{y}(0)) = \rho_0(\mathbf{y}_0),\ \mathbf{y}_0 \in M_0.$$

Here $t \in T_0 = [0,T]$ is independent variable, T is fixed value; $\mathbf{x} \in R^n$ and $\mathbf{y} \in R^m$ are phase vectors; $\mathbf{u} = \mathbf{u}(t)$ is programmed control; $M_0 \in R^m$ is compact set of initial $\mathbf{y}$ values; $M_{t,\mathbf{u}} = \{\mathbf{y}_t = \mathbf{y}(t,\mathbf{x}(t),\mathbf{y}_0,\mathbf{u}(t)),\ \mathbf{y}_0 \in M_0,\ \mathbf{x}(0) = \mathbf{x}_0\}$ is t-cutset of trajectory ensemble $M_{\mathbf{u}} = \{\mathbf{y}(t,\mathbf{x}(t),\mathbf{y}_0,\mathbf{u}(t)),\ \mathbf{y}_0 \in M_0, \mathbf{x}(0) = \mathbf{x}_0, t \in T_0\}$; $\rho(\tau,\mathbf{y})$ is ensemble phase density defined upon $M_{\mathbf{u}}$; $\rho_0(\mathbf{y}_0)$ is initial phase density.

Main suppositions are: n-vector function $\mathbf{f}(t,\mathbf{x},\mathbf{u})$ and scalar function $h(t,\mathbf{x},\mathbf{y})$ are continuous and have continuous partial derivatives; m-vector functions $\mathbf{F}_1(t,\mathbf{x},\mathbf{y},\nu,\mathbf{u})$ and $\mathbf{F}_2(t,\mathbf{x},\mathbf{y},\mathbf{z}_t)$ have, in addition, continuous second-order derivatives; $\rho_0(\mathbf{y}_0)$ is nonnegative continuous function. Control function $\mathbf{u}(t)$ belongs to class D of piecewise-continuous on T_0 r-dimensional vector-functions assuming values in compact $U \subset R^r$.

Control process quality is estimated by values of functional [7]:

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$$I(\mathbf{u}) = I_1(\mathbf{u}) + I_2(\mathbf{u}), \qquad (6)$$

$$I_1(\mathbf{u}) = \int_0^T \Phi_1(t, \mathbf{x}(t), \mathbf{u}(t))dt + g_1(\mathbf{x}(T)), \qquad (7)$$

$$I_2(\mathbf{u}) = \int_0^T \int_{M_{t,\mathbf{u}}} \Phi_2(t, \mathbf{x}(t), \mathbf{y}_t, v(t), \mathbf{u}(t))\rho(t, \mathbf{y}_t)d\mathbf{y}_t dt + \\ + \int_{M_{T,\mathbf{u}}} g_2(\mathbf{y}_T)\rho(T, \mathbf{y}_T)d\mathbf{y}_T. \qquad (8)$$

We suppose Φ_1, Φ_2, g_1, g_2 to be nonnegative continuously differentiable functions.

In special case under study $\mathbf{x}(t, \mathbf{x}_0, \mathbf{u})$ describes program motion; $\{\mathbf{y}(t, \mathbf{x}(t), \mathbf{y}_0, \mathbf{u}(t)),\ \mathbf{y}_0 \in M_0\}$ present perturbed motions ensemble; $\mathbf{u}(t) = \mathbf{\theta}$, $n = m = 2$; $r = 6$; $v(t) = \int_0^t h(\tau, \mathbf{x}(\tau), \mathbf{y}(\tau))d\tau$ presents the phase of particle (see Eq. 3); the integral $\int_{M_{t,\mathbf{u}}} \mathbf{F}_2(t, \mathbf{x}, \mathbf{y}, \mathbf{z}_t)\rho(t, \mathbf{z}_t)d\mathbf{z}_t$ describes particle interaction [2]. Function $\mathbf{F}_2(t, \mathbf{x}, \mathbf{y}, \mathbf{z})$ is determined by interaction account model. In particular, disk beam model is used and $\mathbf{F}_2(t, \mathbf{x}, \mathbf{y}, \mathbf{z})$ is given by special series [9, 2]. Functional $I_1(\mathbf{u})$ estimates program motion quality and functional $I_2(\mathbf{u})$ estimates the quality of perturbed motions ensemble.

QUALITY FUNCTIONAL GRADIENT

Quality functional derivatives with respect to control parameters are obtained as follows:

$$\frac{\partial I}{\partial \theta_k} = \int_0^T \left(\frac{\partial \Phi_1}{\partial \mathbf{u}} - \psi^* \frac{\partial \mathbf{f}}{\partial \mathbf{u}} \right) \frac{\partial \mathbf{u}}{\partial \theta_k} dt + \\ + \int_0^T \int_{M_{t,\mathbf{u}}} \left(\frac{\partial \Phi_2}{\partial \mathbf{u}} - \boldsymbol{\mu}^* \frac{\partial \mathbf{F}_1}{\partial \mathbf{u}} \right) \frac{\partial \mathbf{u}}{\partial \theta_k} \rho(t, \mathbf{y}_t) d\mathbf{y}_t dt, \quad k = \overline{1, r}. \qquad (9)$$

Auxiliary vector functions $\boldsymbol{\psi}(t, \mathbf{x})$ and $\boldsymbol{\mu}(t, \mathbf{y})$ are defined on trajectories $\mathbf{x}(t, \mathbf{x}_0, \mathbf{u})$ and $\{\mathbf{y}(t, \mathbf{x}(t), \mathbf{y}_0, \mathbf{u}(t)),\ \mathbf{y}_0 \in M_0\}$ correspondingly and satisfy special systems of integro-differential equations. The symbol * signifies transposition of vector.

NUMERICAL RESULTS

Numerical beam dynamics investigation was realized for the device with following characteristics: initial beam energy is $W_{inject} = 40\ keV$; accelerating wavelength is $\lambda = 0.1\ m$; accelerator length is $L = 7.8\ m$; beam current is $I = 3A$.

Optimization fulfilled has provided beam exit characteristics improvement: phase spread has descended by a factor of 1.7, average reduced energy has increased from 8.9 until 10.1. In addition, capture coefficient has grown from 92% until 98%.

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