MATHEMATICAL MODEL OF BEAM DYNAMIC OPTIMIZATION

V.V. Altsybeyev, SPbSU, Saint-Petersburg, Russia

Abstract

We treat here the process of simulation of charged particle dynamics using so called hybrid system. Hybrid system is a system with continuous and discrete parts, described by differential and difference equations, respectively. In this case new mathematical model of beam dynamics optimization is suggested. The main parameters of optimization are: coefficient of particle capture in the acceleration mode, phase and energy spectra of particles at the exit of the accelerator, the transverse beam characteristics, etc. Optimization was carried out for the drift tubes accelerator.

INTRODUCTION

At present time design of accelerators with accelerating field focusing become important [1–6]. Such a focusing in RFQ accelerators being established, but APF focusing still requires development. For various problems of accelerator physics it is necessary to use a different mathematical models [4, 6–10]. Often it is necessary to build several models, taking into account their hierarchy. For example, it is interesting to consider the model "square wave" approximation of the accelerating field in DTL [5] to optimize its parameters. Such a model allows us to construct a well implemented on a computer optimization algorithm. In this paper, an attempt to optimize APF accelerator of deuterons was considered. Similar problems are discussed, especially in [6]. APF structure considered, as well, in the papers [1, 2], where the selection of the parameters of accelerators was made by using stability diagramm.

MATHEMATICAL MODEL OF OPTIMIZATION

"Square wave" approximation of the accelerating field in DTL allows to accept the following mathematical model of optimization, in which the dynamics of the beam is described by the so-called hybrid system of equations having a continuous (1) and discrete part (2) whith initial conditios (3).

$$\dot{x} = f_1(t, x, u), \quad t \in [\mu_{i-1}, \mu_i), \tag{1}$$

$$x(\mu_{i+1}) = f_2(\mu_i, x(\mu_i), v_i), \quad t \in [\mu_i, \mu_{i+1}), \tag{2}$$

$$x(0) = x_0 \in \overline{M}_0, \tag{3}$$

$$i = 2k + 1, \quad k = 0, \ldots, N - 1.$$

Let us consider beam quality functional. We consider the problem of its minimizing for the admissible controls.

$$I = \sum_{i=0}^{2N} \int_{M_{\mu_i,u}} \varphi(x(\mu_i), \mu_i) dx_{\mu_i} + \\ + \int_{M_{T,u}} g(x_T) dx_T \to min. \tag{4}$$

The variations of functional (4) can be written as

$$\delta I = -\sum_{k=0}^{N-1} \Big(\int_{\mu_{2k}}^{\mu_{2k+1}} \int_{M_{t,u}} \big(\psi(t)^T \Delta_u f_1(t, x, u) + \\ + \lambda(t, x) \Delta_u \mathrm{div}_x f_1(t, x, u)\big) dx_t dt + \\ + \int_{M_{\mu_{2k+2},u}} \big(\psi(\mu_{2k+2})^T \Delta_u f_2(\mu_{2k+1}, x(\mu_{2k+1}), v_{2k+1}) + \\ + \lambda(\mu_{2k+2}, x(\mu_{2k+2})) \Delta_u J(\mu_{2k+1})\big) dx_t \Big). \tag{5}$$

Where ψ and λ are satisfying the following equations

$$\psi(\mu_{i+1} + 0) = \psi(\mu_{i+1} - 0) + \frac{\partial \varphi(x(\mu_{i+1}), \mu_{i+1})}{\partial x}, \tag{6}$$

$$\psi(\mu_i) = J(\mu_i) \Big(\frac{\partial}{\partial x} f_2(\mu_i, x(\mu_i), v_i) \Big)^T \psi(\mu_{i+1}) + \\ + \lambda(\mu_{i+1}) \left(\frac{\partial J(\mu_i)}{\partial x} \right)^T + \\ + \frac{\partial \varphi(x(\mu_i), \mu_i)}{\partial x}, \quad t \in [\mu_{i+1}, \mu_i), \tag{7}$$

$$\frac{d\psi}{dt} = -\Big(\frac{\partial}{\partial x} f_1(t, x, u) + E \mathrm{div}_x f_1(t, x, u) \Big)^T \psi - \\ -\lambda \left(\frac{\partial \mathrm{div}_x f_1(t, x, u}{\partial x} \right), \quad t \in [\mu_i, \mu_{i+1}). \tag{8}$$

$$\lambda(\mu_{i+1} + 0) = \lambda(\mu_{i+1} - 0) + \varphi(x(\mu_{i+1})), \tag{9}$$

$$\lambda(\mu_i) = J(\mu_i) \lambda(\mu_{i+1}) + \\ + \varphi(x(\mu_i)), \quad t \in [\mu_{i+1}, \mu_i), \tag{10}$$

$$\frac{d\lambda}{dt} = -\lambda \mathrm{div}_x f_1(t, x, u), \quad t \in [\mu_i, \mu_{i+1}). \tag{11}$$

$$\lambda(T, x(T)) = -g(x_T), \tag{12}$$

$$\psi(T, x(T)) = -\left(\frac{\partial g(x_T)}{\partial x} \right)^T. \tag{13}$$

So, analitical representation of variation of functional (4) was obtained.

ISBN 978-3-95450-125-0

NUMERICAL OPTIMIZATION

Consider APF deuteron accelerator, consisting of three focusing periods of simple structure with law of changing of synchronous phase, shown in Fig .1. In this case equation (1)–(2) can be written as

$$\frac{d\varphi}{d\xi} = \frac{2\pi\gamma}{\sqrt{\gamma^2-1}},$$
$$\frac{d\gamma}{d\xi} = \alpha(\xi)cos(\varphi), \quad \xi \in [\mu_{i-1}, \mu_i);$$

$$\varphi(\mu_{i+1}) = \varphi(\mu_i) + (\mu_{i+1}-\mu_i)\frac{2\pi\gamma}{\sqrt{\gamma^2-1}},$$
$$\gamma(\mu_{i+1}) = \gamma(\mu_i), \quad \xi \in [\mu_i, \mu_{i+1}).$$

Here $\varphi = \omega t$ is a particle phase, γ is a Lorentz factor, $\alpha(\xi)$ is a function of accelerating field distribution. Based on variation (5) and equations (6)–(13) the optimization algorithm was built. For accelerator optimization the following functionals were used

$$I_1 = \int_{M_{T,u}} \left(a\left(\bar{\varphi}-\varphi_T\right)^2 + b\left(\frac{\gamma_T}{\bar{\gamma}}-1\right)^2 \right) d\varphi_T d\gamma_T,$$
$$I_2 = cF\left(\sum_{i=0}^{2N}\int_{M_{\mu_i,u}} kcos(\varphi(\mu_i))d\varphi_{\mu_i}\gamma_{\mu_i}\right),$$
$$F = \begin{cases} (I-I_{cond})^2, & I < I_{cond}, \\ 0, & I > I_{cond}, \end{cases}$$
$$I = I_1 + I_2.$$

Here a, b, c, I_{cond} are positive constants; k is parameter, that describes a difference of an accelerating field at the gap bound. In an optimization problem of the longitudinal motion it is appropriate to consider parameters, which can provide transverse motion. The functional I_2, that limiting focusing factor, is corresponds to that aim. Functional I_1 evaluates the quality of the beam at the output of the accelerator. Initial conditions of beam are shown in Fig. 2, Fig. 3. Results of the numerical simulation and optimization are shown in Fig. 5–Fig. 6.

CONCLUSION

The paper proposes the mathematical model of beam dynamics optimization in DTL. Optimization was carried out for APF structure. Numerical calculations showed that the optimization of the longitudinal motion including a focusing factor provides APF structure, but simultaneously transverse and longitudinal motion must be optimized. The resulting model can be viewed as the first step, which allows to get an initial approximation of linac paremeters. Later, in particular, the model of approximation field by first of the Fourier harmonics was considered. Also, space charge of the beam was taken into account. Also an effective approach for the optimization was the consideration of the longitudinal dynamics in the equivalent traveling wave.

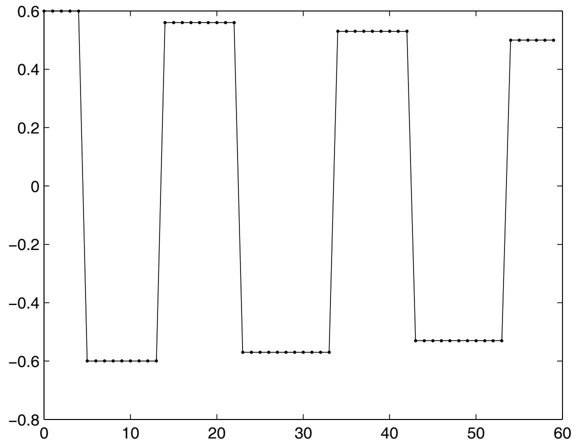

Figure 1: Synchronous phase

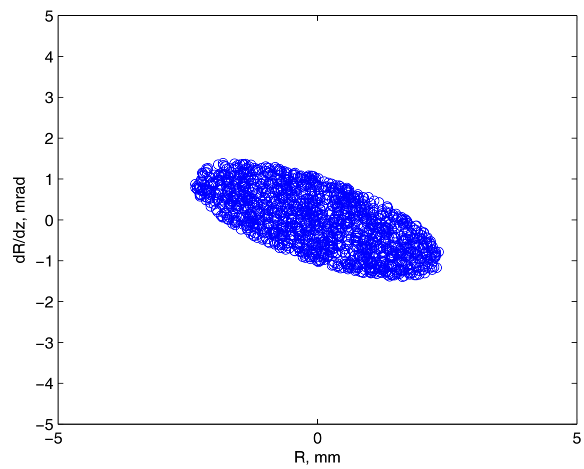

Figure 2: Input transverse emittance

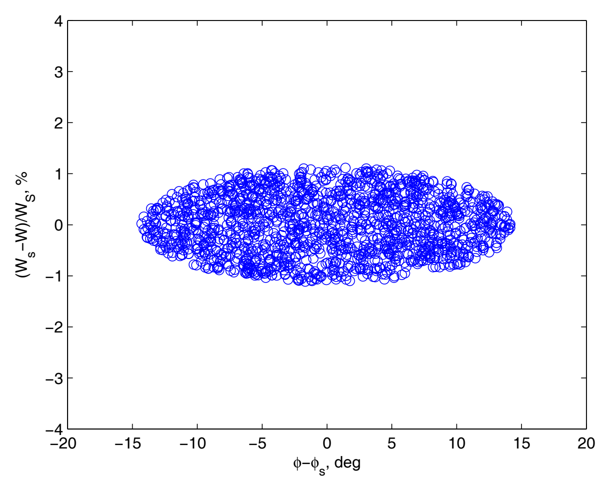

Figure 3: Input longitudinal emittance

ISBN 978-3-95450-125-0

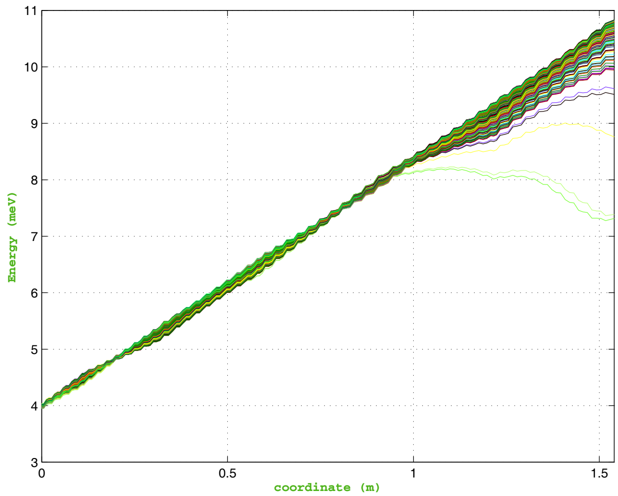

Figure 4: Energy trajectory before optimization

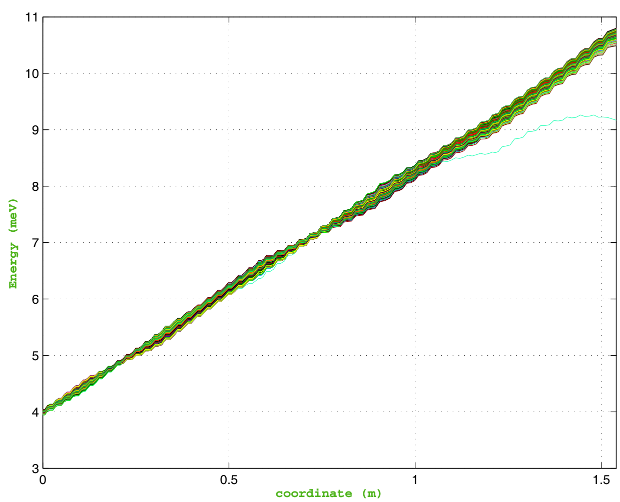

Figure 5: Energy trajectory after optimization

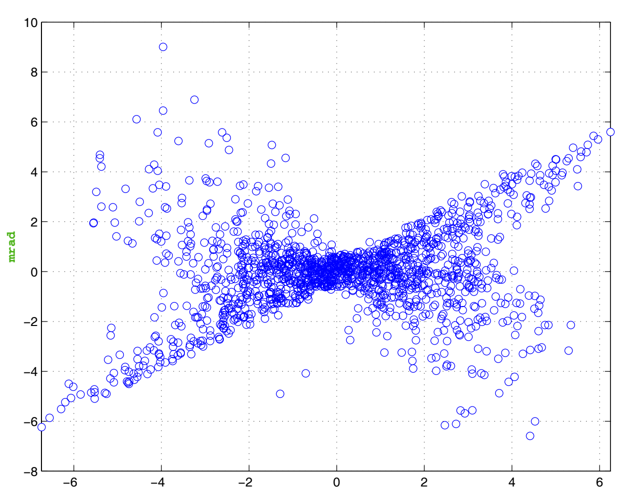

Figure 6: Output transverse emittance

REFERENCES

[1] V.V.Kapin, S.Yamada, Y.Iwata, "Design of APhF-IH Linac for a Compact Medical Accelerator," National Institute of Radiological Sciences, 2003, JAPAN.

[2] Kapin V.V., Nesterovich A.V. "Feasibility of alternative phase focusing for a chain of short independently-phased resonators," Proceedings of RuPAC-2010, Protvino, Russia.

[3] Kushin V.V. "On improving the phase-alternating focusing in linear accelerators," Nuclear Energy, 1970, .29, N2.

[4] D.A. Ovsyannikov, A.D. Ovsyannikov, M.F. Vorogushin, Yu.A. Svistunov, A.P. Durkin, "Beam dynamics optimization: Models, methods and applications," 2006 Nuclear Instruments and Methods in Physics Research, Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 558 (1), pp. 11-19.

[5] I.M.Kapchinsky, *Theory of linear resonance accelerator*, (Moscow, Energoizdat, 1982), pp. 130-165.

[6] D.A.Ovsyannikov, *Modeling and Optimization of Charged Particles Beam Dynamics*, (Lenigrad, 1990, p. 312).

[7] Y.A. Svistunov, A.D.Ovsyannikov "Designing of compact accelerating structures for applied complexes with accelerators," 2010 Problems of Atomic Science and Technology (2), pp. 48-51.

[8] A.D. Ovsyannikov, D.A. Ovsyannikov, S.-L. Chung, "Optimization of a radial matching section," 2009 International Journal of Modern Physics A 24 (5), pp. 952-95.

[9] A.D. Ovsyannikov, D.A. Ovsyannikov, M.Yu. Balabanov, S.-L. Chung, "On the beam dynamics optimization problem," 2009 International Journal of Modern Physics A 24 (5), pp. 941-951.

[10] P. Snopok, C. Johnstone, M. Berz, D.A. Ovsyannikov, A.D. Ovsyannikov, "Muon Collider interaction region simulation and optimization," 2005 International Conference on Physics and Control, PhysCon 2005, Proceedings 2005, art. no. 1513993, pp. 278-281.

ISBN 978-3-95450-125-0