MATHEMATICAL MODEL OF BEAM DYNAMICS OPTIMIZATION IN TRAVELING WAVE

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Abstract

In works by B.I. Bondarev, A.P. Durkin, A.D. Ovsyannikov mathematical model of optimization of charged particles dynamics in RFQ accelerators was proposed. In this paper a new mathematical model of optimization of particle dynamics in traveling wave is considered. Joint optimization model of program and disturbed motions is investigated.

INTRODUCTION

In works [1] mathematical model for optimization of RFQ structure was suggested. Transverse and longitudinal motions were investigated separately. But characteristics of transverse motion were considered and analyzed at the stage of longitudinal motion optimization. In particular restrictions were imposed on defocusing factor.

In this paper other model of longitudinal motion based on the selection of program motion (synchronous particle motion) and beam of charged particles (movements in deviations from program motion). This model has been tested for RFQ structure.

Phase of synchronous particle and intensity of accelerating field are considered as control parameters (functions). It should be noted that in paper [1] those parameters also were considered as controls, but mathematical model of optimization was different from the model proposed in this paper.

PROBLEM STATEMENT

Let us investigate the problem of control of longitudinal dynamics of beam in waveguide accelerator as a problem of joint optimization of synchronous particle motion and ensemble of trajectories [3]. As control functions let us choose the laws of changing of dimensionless parameter of the amplitude of the accelerating wave $\alpha(\xi)$ [5] and synchronous phase $\varphi_s(\xi)$ along the structure.

Adopt the following notation $u_1 = \alpha(\xi), u_2 = \varphi_s(\xi)$, where functions $u_1(\xi),\ u_2(\xi)$ are *controls*.

Let the phase of the particle is given by [1]

$$\varphi = \omega \int_0^z \frac{dz}{v(z)} - \omega t + \varphi_s. \tag{1}$$

Under *program motion* (synchronous particle motion) we mean a solution of the system

$$\frac{d\gamma_s}{d\xi} = -\alpha(\xi) \sin\varphi_s,$$
$$\varphi_s = u_2(\xi)$$

with initial condition

$$\gamma_s(0) = \gamma_{s0}.$$

Here γ_s — reduced energy of synchronous particle.

Phase of the beam particles, according to (1) will be considered in the deviation of the phase of the synchronous particle:

$$\widehat{\varphi} = \varphi - \varphi_s.$$

Considering that the longitudinal velocity of the synchronous particle coincides with the phase velocity of the wave, i. e.

$$\beta = \beta_s = \frac{\sqrt{{\gamma_s}^2 - 1}}{\gamma_s}, \tag{2}$$

obtain controlled dynamical system described by the system of ordinary differential equations

$$\frac{d\gamma_s}{d\xi} = -u_1(\xi)\sin(u_2(\xi)), \tag{3}$$
$$\frac{d\gamma}{d\xi} = -u_1(\xi)\sin(\widehat{\varphi} + u_2(\xi)), \tag{4}$$
$$\frac{d\widehat{\varphi}}{d\xi} = 2\pi\left(\frac{\gamma_s}{\sqrt{\gamma_s^2 - 1}} - \frac{\gamma}{\sqrt{\gamma^2 - 1}}\right) \tag{5}$$

with initial conditions

$$\gamma_s(0) = \gamma_{s0}, \tag{6}$$
$$\gamma(0) = \gamma_0, \quad \widehat{\varphi}(0) = \widehat{\varphi}_0. \tag{7}$$

Here $\xi \in T_0 = [0, L]$ — independent variable; $(\gamma_0, \widehat{\varphi}_0)^T \in M_0$, $\gamma_s \in \Omega_x \subseteq R^1$; $(\gamma, \widehat{\varphi})^T \in \Omega_y \subseteq R^2$ — vector of system variables; $(u_1, u_2)^T \in U \subseteq R^2$ — 2-dimensional vector-function of control; L — constant value.

It is assumed that the sets of Ω_x and Ω_y — are open, set U and set of positive measure $M_0 \subset \Omega_y$ — are compact.

We also assume that the admissible controls $u = u(\xi)$, $\xi \in T_0$, constitute a class of piecewise smooth on the interval $[0, L]$ functions with values in a compact set U. By piecewise smooth functions we mean functions, which derivatives have only a finite number of discontinuities of the first kind.

Equations (3)–(5), where $\widehat{\varphi}$ — phase in deviations from the synchronous phase, and γ — complete reduced energy, convenient when considering optimization problems. But in the future equations in deviation of energy of the synchronous particle will be considered.

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THE EQUATION OF THE SEPARATRIX

Choose as dynamic variables, the phase difference of asynchronous and synchronous particle and the difference of the reduced energy

$$\psi = \varphi - \varphi_s, \quad p_\psi = \gamma - \gamma_s.$$

Subtracting equation (3) from (4),

$$\frac{dp_\psi}{d\xi} = -u_1(\xi)\big(\sin(\psi + u_2(\xi)) - \sin(u_2(\xi))\big).$$

Take the derivative with respect to the coordinate of value and make the substitution $\xi = z/\lambda$. Given that $\omega = 2\pi c/\lambda$, $\upsilon = \upsilon_s$, $\frac{\upsilon_z - \upsilon_s}{\upsilon_s} \approx \frac{1}{\gamma_s{}^2}\frac{p_\psi}{p_s \upsilon_s}$ [2], we obtain a system of first order equations describing the dynamics of the longitudinal motion:

$$\frac{dp_\psi}{d\xi} = -u_1(\xi)\big(\sin(\psi + u_2(\xi)) - \sin(u_2(\xi))\big), \quad (8)$$

$$\frac{d\psi}{d\xi} = \frac{2\pi}{\sqrt{(\gamma_s^2 - 1)^3}} p_\psi. \quad (9)$$

From equations (8), (9) we obtain an equation describing the separatrix of the beam in the phase plane ψ, p_ψ [2]:

$$p_\psi = \pm\sqrt{\frac{\sqrt{(\gamma_s{}^2 - 1)^3}}{\pi}}\sqrt{V(-\pi - 2u_2) - V(\psi)},$$

where function

$$V(\psi) = -u_1(\xi)\big(\cos(\psi + u_2(\xi)) + \psi \sin(u_2(\xi))\big)$$

is analogous to the potential energy.

FUNCTIONALS

In accordance with the objectives of optimizing we shall consider the following functionals:

$$I_1(u) = \int_{M_{L,u}} \left(a\left(\frac{\gamma_L}{\gamma_{sL}} - 1\right)^2 + \right.$$

$$\left. + b(\widehat{\varphi}_L - \bar{\varphi}_L)^2 \right) d\gamma_L d\widehat{\varphi}_L, \quad (10)$$

$$I_2(u) = \int_0^L \int_{M_{\xi,u}} h(q,\bar{q}) d\widehat{\varphi} d\gamma d\xi, \quad (11)$$

where

$$h(q,\bar{q}) = \begin{cases} (q - \bar{q})^2, & \text{if} \quad q > \bar{q}; \\ 0, & \text{if} \quad q \leqslant \bar{q}, \end{cases}$$

Here $\widehat{\varphi}_L, \gamma_L$ — phase and reduced energy of disturbed motion at the output of accelerator correspondingly, $\gamma_{sL}, \bar{\varphi}_L$ — reduced energy of synchronous particle and average phase of disturbed motion at the output of accelerator, $q = H(\psi, p_\psi)$ — value of the Hamiltonian of the particles beam system (4), (5), which is given by

$$H(\psi, p_\psi) = V(\psi) + \frac{\pi}{\sqrt{(\gamma_s^2 - 1)^3}} p_\psi{}^2,$$

and $\bar{q}$ — value of Hamiltonian corresponding to the separatrix.

We will consider the minimization of functional (10), (11) by controls $(u_1, u_2)^T$.

Optimization will be performed by the gradient methodon the basis of variation of the [3–6], which is a linear combination of the above functional, equipped with weighting coefficients.

NUMERICAL RESULTS

Software tool for considered problem was implemented in Matlab as a unit of the BDO-RFQ 1.6 system, developed by the department of theory of control systems of electrophysical equipment in SPbSU.

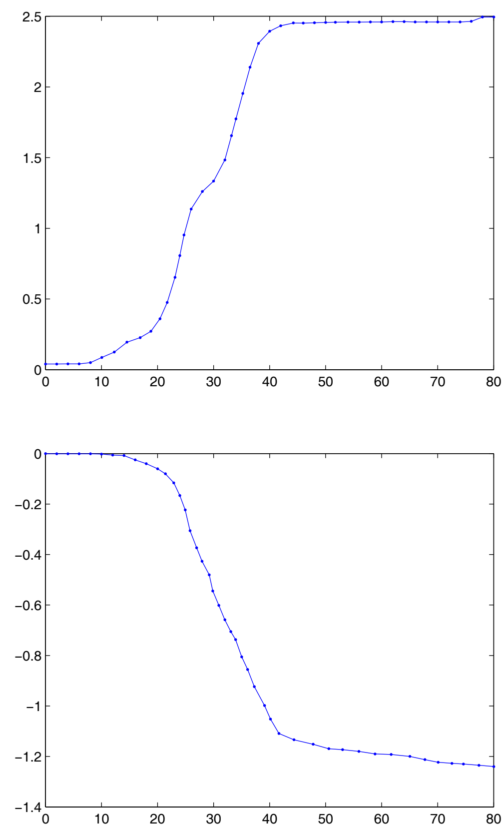

Figure 1: The change of reduced amplitude of intensity (upper plot) and phasesynchronous particle (lower plot) after optimization

Initial controls $u_1 = \alpha(\xi)$ and $u_2 = \varphi_s(\xi)$ were defined by 40 points and interpolating between them with splines.

The calculations were carried out for the structure with the following parameters: injection energy — 80 keV, ac-

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celerating wave length — 1 cm, length of structure — 80 cm.

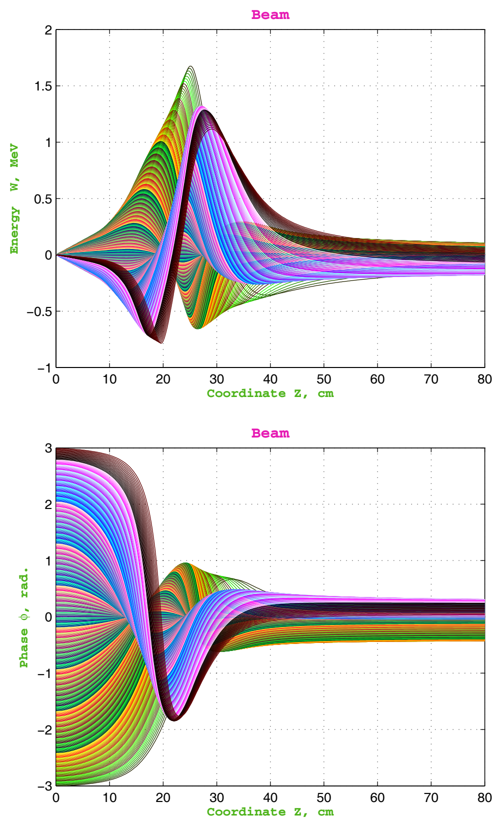

Figure 2: The relative deviation of the energy of the beam particles (upper plot) and the phases of the beam particles (lower plot)

The solution was carried out in two stages. At the first stage capture coefficient equal to 0.95 was achieved. At the second stage the width of the energy and phase spectra was minimized at the output of accelerator.

Numerical optimization was performed on 500 particles uniformly located in between the phases of -3 to 3 rad., which corresponds to obtained coefficients of capture. This resulted in the controls $\alpha^{(1)}(\xi)$ and $\varphi_s^{(1)}(\xi)$ (Fig. 1). These controls allow the output structure with the energy spread 28%and the width of the phase spectrum 0,73 rad. (Fig. 2). The average energy at output of structure is 5,6 MeV.

It should be noted that all of the particles were in the acceleration mode. It is seen in Fig. 3, since the separatrix limits the capture of particles into the acceleration mode.

CONCLUSION

In this paper a mathematical model to optimize the beam dynamics in the accelerator with traveling wave was developed. The separatrix equation was used for the construction of mathematical optimization model of beam dynamics.

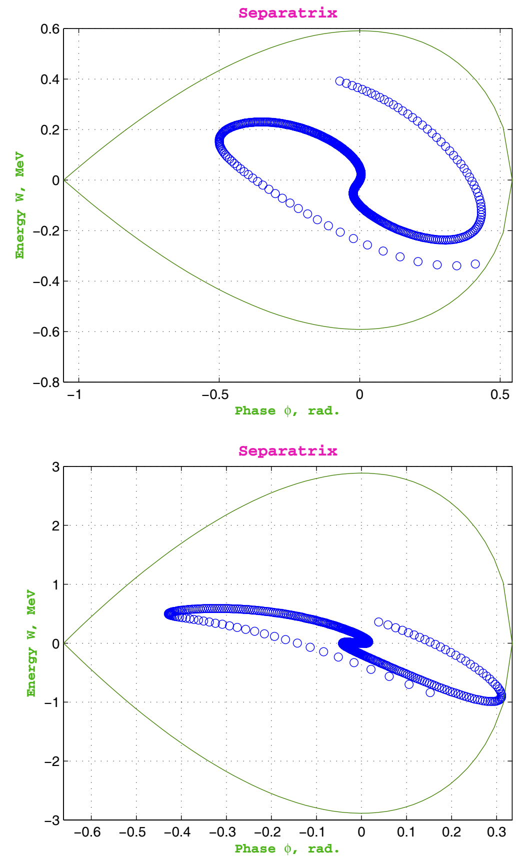

Figure 3: Separatrix and energy-phase distribution of the particle beam at the output of buncher (upper plot) and output of structure (lower plot)

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