MULTY FREQUNCY STORED ENERGY RF LINAC

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Abstract

Due to beam loading, accelerating gradient in rf linac is reduced in time if the energy acquired by charged bunches is not compensated by external generator that feeds the linac. Since the bunch energy gain in this mode of operation correlates strongly with bunch number, the energy spectrum of total bunch train might be corrected in order to suppress this additional spectrum widening. This spectrum control might be achieved with the rf cavity that operates at frequency shifted relative the main one in such a way, that any new bunch sees the cavity field in the appropriate phase correlated with bunch number. The first bunch traverses correcting cavity in field node while the last one in the phase, where the energy acquired by this bunch is equal to resulting bunch train energy spread arising from beam loading effect. Measures for suppression of non coherent bunches spectrum widening are suggested leading to insertion additional cavity excited at frequency shifted relative the main and adjacent frequencies.

INTRODUCTION

In RF linac, that might be represented as RF cavity and a train of charged bunches traversing the cavity, the bunches increase their energy due to the work of electric force of electromagnetic field inside the cavity. If the energy being carried out of cavity is compensated by external RF generator one has steady state acceleration. In this case, any new bunch from the bunch train acquires the same energy while traversing the cavity. In stored energy mode, external rf generator is used to excite cavity – to produce definite level of rf field before acceleration takes place while charged bunches get energy from the power stored in the cavity. Any new bunch carries away some portion of stored electromagnetic energy thus resulting in reduction of this energy and hence in corresponding reduction of the field level. The total bunch train is found to have energy spread after acceleration, its width being the function of the train charge accelerated. This seems to be the main shortcoming of stored energy mode of linac operation that limits this method for many applications.

In this paper, we discuss the possibility to avoid or compensate partially this spectrum widening for coherent motion.

QUALITATIVE ESTIMATIONS AND THE MAIN IDEA

The energy W stored inside rf cavity

$$W = QP/\omega \qquad (1)$$

where Q is the cavity quality factor and ω stands for cavity eigen frequency, while P is the power dissipated in cavity walls. Any bunch from bunch train carries out of cavity the portion of energy

$$\Delta w = qU = ITU = IT\sqrt{PR}\,. \qquad (2)$$

Here q, U, I, T denote bunch charge, accelerating voltage, average beam current and RF field period respectively, R is real part of cavity shunt impedance. After acceleration bunch train consisting of N bunches, reduction of accelerating gradient can be estimated as

$$\Delta U = \frac{\Delta E}{e} = \pi N I \frac{R}{Q} \qquad (3)$$

Depending on relation between cavity time constant $\tau = 2Q/\omega$ and beam pulse duration t bunch energy drops down as linear function of time for the case $t << \tau$. This linear energy fall down might be eliminated by the accelerating gap with linear dependence of gap voltage on the number of the bunch entering the gap, the difference between the energy gain of the last bunch and the first one being equal to the energy spread defined by the formula above. A cavity on the beam path that is excited at frequency that differs from the main one by the value defined below provides necessary spectrum correction, and fig,1 serves to illustrate such a correction.

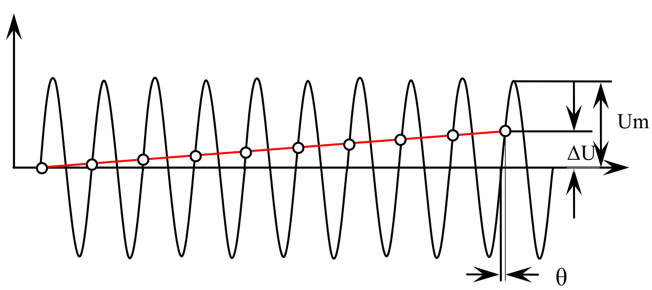

Fig.1 Spectrum correction

It can be calculated from fig.1 that $\theta = \Delta E / eU_m$. From the other hand

$$\theta = \omega(N-1)(T_1 - T_2) = 2\pi(N-1)\frac{\Delta\omega}{\omega}. \qquad (4)$$

It follows from these two expressions that

$$\frac{\Delta\omega}{\omega} = \frac{\Delta E}{2\pi e U_m (N-1)}. \qquad (5)$$

Taking into account formula (3)for cavity voltage reduction after all bunches acceleration one arrives at the next formula for the frequency of correcting cavity

$$\frac{\Delta\omega}{\omega} = \frac{I}{2U_m}\frac{R}{Q} \qquad (6)$$

While deriving this expression, we supposed that $\Delta U << U_m$.

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THE VOLTAGE INDUCED IN A CAVITY BY CHARGED BUNCHES TRAIN

To calculate the field radiated in a cavity by charged bunches traversing this cavity, we will follow standard method described in details in ref. [1]. In this method, vector potential $\vec{A}(\vec{r},t)$ where $\vec{r}$ is radius-vector of a bunch and t denotes time is represented in the form of eigen functions $\vec{A}_\lambda(\vec{r})$ series expansion

$$\vec{A}(\vec{r},t)=\sum_\lambda g_\lambda(t)\vec{A}_\lambda(\vec{r}). \quad (7)$$

$$\Delta\vec{A}_\lambda(\vec{r})+\kappa_\lambda^2\vec{A}_\lambda=0 \quad (8)$$

$$\frac{d^2g_\lambda(t)}{dt^2}+\frac{\omega_\lambda}{Q_\lambda}\frac{dg_\lambda(t)}{dt}+\omega_\lambda^2 g_\lambda(t)=\int_V\vec{j}(\vec{r},t)\vec{A}_\lambda(\vec{r})dV \cdot \quad (9)$$

Here eigen frequencies of cavity modes $\omega_\lambda=c\kappa_\lambda$, c – light velocity, $\vec{j}$ - excitation current density, V- cavity volume, Q_λ - cavity mode quality factor. It is supposed that eigen functions are normalized by the condition

$$\int_V A_\lambda^2=\mu_0c^2=1/\varepsilon_0. \quad (10)$$

Electric and magnetic fields are expressed in terms of following expressions of eigen functions and expansion coefficients

$$\vec{E}_\lambda=-\frac{dg_\lambda}{dt}\vec{A}_\lambda,\ \vec{H}_\lambda=\frac{1}{\mu_0}g_\lambda rot\vec{A}_\lambda \quad (11)$$

For the main mode of pill box form (cylindrical resonator in Russian notation) [2]

$$A^z=\frac{c}{\omega\, a\sqrt{\pi\varepsilon_0L}}\frac{\nu_l}{a}\frac{J_0(\nu_1\frac{r}{a})}{J_1(\nu_1)} \quad (12)$$

In our analysis, we ignore the losses in cavity walls considering time interval of interest mach less cavity time constant $\tau=Q/\omega$. We also consider charge bunches to have point size. Under these assumption we have

$$\frac{d^2g(t)}{dt^2}+\omega^2g(t)=\int_V q\vec{v}(\vec{r},t)\delta(x,y,vt)\eta(vt)\eta(L-vt)\vec{A}(\vec{r})dV, \quad (13)$$

where all values are taken for the main mode, q,v,L denote bunch charge, its velocity and cavity length respectively, while δ and η stand for delta function and step function respectively. Making all substitutions we have

$$\frac{d^2g(t)}{dt^2}+\omega^2g(t)=F(t), \quad (14)$$

$$\text{where } F(t)=\frac{2qvc\nu_1}{\omega a^2J_1(\nu_1)\sqrt{\pi\varepsilon_0L}} \quad (15)$$

for $0<t<L/v$ and $F(t)=0$ for $t>L/v$. (16)

We assume that at the moment t=0 bunch enters the cavity. The solution of this equation that satisfies initial conditions $g(0)=\dot{g}(0)=0$ (corresponding equal to zero magnetic and electrical components of induced field) can be represented in the form [3]

$$g(t)=\frac{2qvc\nu_1}{\omega^3a^2J_1(\nu_1)\sqrt{\pi\varepsilon_0L}}\sin\frac{\omega L}{2v}\sin(\omega t-\frac{\omega L}{2v}) \quad (17)$$

for $t>L/v$.

According to the formula (12) cavity electric field

$$E=-\dot{g}A=-\frac{2qvc^2\nu_1{}^2J_0(\nu_1\frac{r}{a})}{\omega^3a^4\pi\varepsilon_0LJ_1{}^2(\nu_1)}\sin\frac{\omega L}{2v}\cos(\omega t-\frac{\omega L}{2v}) \quad (18)$$

If the bunches from a bunches train traverses a cavity tuned to ω_2 with a frequency ω_1, the electric field that excites k-th bunch is

$$E_k=-\frac{2qvc^2\nu_1{}^2J_0(\nu_1\frac{r}{a})}{\omega_2{}^3a^4\pi\varepsilon_0LJ_1{}^2(\nu_1)}\sin\frac{\omega_2L}{2v}\times\cos\left[\omega_2t-\frac{\omega_2L}{2v}-2\pi(k-1)(\frac{\omega_2}{\omega_1}-1)\right] \quad (19)$$

and the n-th bunch radiates its own field and moves in the field induced by all previous bunches:

$$E^\Sigma=-\sum_{k=1}^{n-1}\frac{2qvc^2\nu_1{}^2J_0(\nu_1\frac{r}{a})}{\omega_2{}^3a^4\pi\varepsilon_0LJ_1{}^2(\nu_1)}\sin\frac{\omega_2L}{2v}\times\cos\left[\omega_2t-\frac{\omega_2L}{2v}-2\pi(k-1)(\frac{\omega_2}{\omega_1}-1)\right] \quad (20)$$

Summation on all bunches gives [4]

$$E^\Sigma=-\frac{2qvc^2\nu_1{}^2J_0(\nu_1\frac{r}{a})}{\omega_2{}^3a^4\pi\varepsilon_0LJ_1{}^2(\nu_1)}\sin\frac{\omega_2L}{2v}\times\cos\left[\omega_2t-\frac{\omega_2L}{2v}-\pi(n-2)(\frac{\omega_2}{\omega_1}-1)\right]\times\sin\left[(n-1)\pi\left(\frac{\omega_2}{\omega_1}-1\right)\right]\frac{1}{\sin\pi(\frac{\omega_2}{\omega_1}-1)} \quad (21)$$

To find out the energy that the n-th bunch looses in the field radiated in the cavity by all previous bunches one has to integrate the last expression over cavity length.

$$U_{lost}=-\frac{2qvc^2\nu_1{}^2}{\omega_2{}^3a^4\pi\varepsilon_0LJ_1{}^2(\nu_1)}\sin\frac{\omega_2L}{2v}\times\frac{\sin\left[(n-1)\pi\left(\frac{\omega_2}{\omega_1}-1\right)\right]}{\sin\pi(\frac{\omega_2}{\omega_1}-1)}\frac{\sin\frac{\omega_2L}{2v}}{\frac{\omega_2}{2v}}\cos\left[\pi(n-2)(\frac{\omega_2}{\omega_1}-1)\right] \quad (22)$$

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It is taken into account that the n-th bunch enters the cavity in the phase $2\pi(n-1)(\omega_1/\omega_2-1)-\omega l/2v$ relative the voltage induced by the first one and in the phase $\pi(n-1)(\omega_1/\omega_2-1)-\omega L/2v, n>2$ relative total induced voltage.

BEAM ENERGY SPECTRUM CORRECTION

Let us consider the simple rf linac consisting of two accelerating cavities. The first one operates at frequency ω_1 that coincides with the bunches repetition rate while the second one is tuned to frequency ω_2. The n-th bunch from bunches train traverses the cavity assembly loosing the energy (with the account of $\pi n(\frac{\omega_1}{\omega_2}-1)<<1$)

$$U_{lost}(n)=\frac{2qvc^2\nu_1^2(n-1)}{\omega_1^3 a_1^4\pi\varepsilon_0 L J_1^2(\nu_1)}\frac{\sin^2\frac{\omega_1 L_1}{2v}}{\frac{\omega_1}{2v}}+ \quad (23)$$

$$\frac{2qvc^2\nu_1^2(n-1)}{\omega_2^3 a_2^4\pi\varepsilon_0 L J_1^2(\nu_1)}\sin\frac{\omega_2 L}{2v}\frac{\sin\frac{\omega_2 L_2}{2v}}{\frac{\omega_2}{2v}}$$

Thus, we have linear dependence of the energy lost with bunch number, and this energy lost might be compensated with linear growth of the accelerated voltage of the second cavity if the following condition takes place:

$$U_2(t)=U_{20}\sin(\omega_2 t), U_{20}>>U_{lost}, \quad U_{20}\times 2\pi(\omega_2/\omega_1-1)=U_{lost}(2), \quad (24)$$

and $\omega_2 t=-L_2/2v$ corresponds to the moment when the first bunch enters the second cavity.

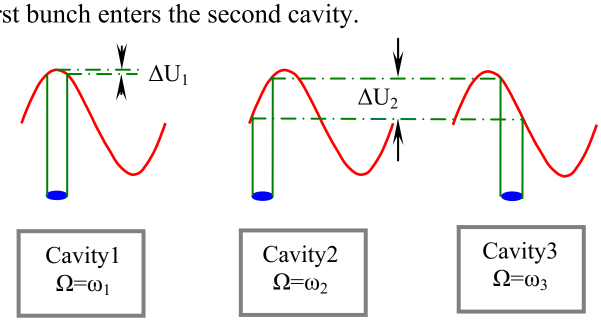

Fig2. Elimination of additional energy spread in sequence of two correcting cavities

Two frequency rf accelerator method just described may result in additional energy spread of individual bunch as in can be seen from fig.3. The bunch positions on sine wave of correction voltage corresponds to large value of $\frac{dU_2}{dt}$ that results in large difference of accelerating gradients of end bunch particles. Three frequency scheme with additional cavity installed on beam path after the first correcting and operating at frequency $\omega_3<\omega_1$ so that

$$U_3(t)=U_{20}\sin(\omega_3 t), U_{20}>>U_{lost}, \quad U_{20}\times 2\pi(\omega_3/\omega_1-1)=-0.5U_{lost}(2), \quad U_{20}\times 2\pi(\omega_2/\omega_1-1)=0.5U_{lost}(2) \quad (25)$$

is free of the shortcoming mentioned. Indeed, the bunch head moves at higher field level than its centre and the tail while traversing the first correcting cavity. In the second correction cavity instead one has quite opposite case that is bunch tail acquires the same energy as its head in the first correction cavity. Thus, in configuration described all bunch particles acquire the same energy after passing consequently two correction cavities.

To provide periodical mode of operation, the relation $T_{rep}=\frac{n\times 2\pi}{\omega_1}=\frac{m\times 2\pi}{\omega_2}$ has to be satisfied. Here m and n are integer numbers, T_{rep} - the period of bunches trains sequence.

DDS (Direct Digital Synthesis) technique might be used to provide necessary frequency relation, their phasing and stability. The details are the subject of separate paper.

CONCLUSION

Due to beam loading, additional energy spread of accelerated bunch train arises in a stored energy linac. It has been shown that this coherent energy spread widening might be eliminated with additional cavities on beam path, tuned to slightly different frequency relative to the main part of rf accelerator. The effect of spectrum correction takes place due to phase slip of successive bunches from bunches train in correcting cavities installed on beam path. The undesired incoherent energy spread widening can be eliminated by adding additional correcting cavity, both correcting cavities being detuned by the values with opposite signs.

The method described might be used in rf superconducting linacs in short current pulse mode.

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